System Specification, Verification and Synthesis (SSVS) – CS 4830/7485, Fall 2019

4: Formal System Modeling: Transition Systems

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Outline

- Transition systems
- Fundamental concepts: reachable states, deadlocks, pre/post, invariants, inductive invariants
- Labeled transition systems and Kripke structures
- Transition systems as the underlying semantics of many formalisms
- Modeling transition systems in Spin/Promela

TRANSITION SYSTEMS

Transition Systems

- The fundamental mathematical model in modern system theory
- A very basic model, capturing the essence of systems
- Reminder: **System** = state + dynamics
- **Transition system** = states + transitions

Transition Systems

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- A very basic model, capturing the essence of systems
- Reminder: **System** = state + dynamics
- **Transition system** = states + transitions (+ labels)

Transition Systems: version 1 (without labels)

A tuple:

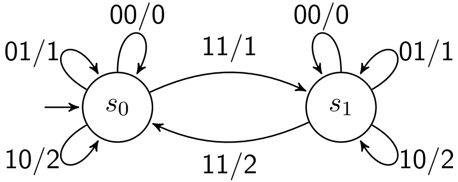
$$(S, S_0, R)$$

- S: set of states (perhaps infinite)
- $S_0 \subseteq S$: set of initial states
- R: transition relation

$$R \subseteq S \times S$$

Example: from an FSM to a transition system

Let's build the transition system for this Mealy machine:



Main idea:

- In an FSM, the next state depends on the input.
- For different inputs, different transitions $\Rightarrow$ non-determinism!

Transition Systems: crucial features

- Sets of states/transitions **possibly infinite**:
  - This allows to model everything! Finite-state systems, infinite-state systems, discrete systems, continuous systems, hybrid systems, ...
- **Non-determinism**:
  - This allows to avoid having to model everything!
  - E.g., omit modeling the inputs (abstraction).

Example: transition system for a digital circuit

Figure from [Baier and Katoen, 2008]:

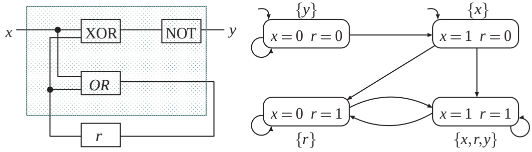


Figure 2.2: Transition system representation of a simple hardware circuit.

How would the transition system look if we didn't include the input x in the state?

Example: transition system for a program

```
var int x;
x := 0
while (x<100) {
  x := x+1;
}
```

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var int x;
x := 0
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```

What if the loop condition was `true` instead?

Example: transition system for a program

Figure from [Clarke et al., 2018]:

```
Var n:Integer initially n = 10
l0 : while (n > 0) {
l1 :     n = n - 1;
l2 : }
l3 :
```

What if the program was this one?

```
var int x;
x := read_input();
while (x>1) {
  if (x % 2 = 0)
    x := x/2;
  else
    x := 3*x+1;
}
```

What if the program was this one?

```
var int x;
x := read_input();
while (x>1) {
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```

Collatz conjecture: the above program terminates for every `x`.
Open problem in mathematics.
Nevertheless, the transition system is well-defined.

What if we had a real program?

~/private/tools/open-kronos/Matrix

```c
#include "matrix.h"
#include "matrix_hash.h"

MATRIX  guarded_matrix,
        temp_matrix,
        reset_matrix,
        ExtrapoMatrix;

MATRIX  EMPTY_MATRIX;

MATRIX  create_matrix(int dimension)
{
  MATRIX marx;
  int i, j;

  if (dimension <= 0)
    return(NULL);
  marx = (BOUND_TYPE **) malloc(sizeof(BOUND_TYPE *) * dimension);
  for (i=0; i<dimension; i++) {
    marx[i] = (BOUND_TYPE *) malloc(sizeof(BOUND_TYPE) * dimension);
    for (j=0; j<dimension; j++)
      marx[i][j] = 0;
  }
  matrix_dimension(marx) = dimension;
  return(marx);
}

void  destroy_matrix(MATRIX marx)
{
  int i;
  if (is_not_trivial_matrix(marx)) {
    for (i = 0; i < matrix_dimension(marx); i++)
      free(marx[i]);
    free(marx);
  }
}

MATRIX  create_init_extrapolation_matrix()
{
  MATRIX marx;
  int i, j;

  marx = create_matrix(N_OF_CLOCKS);
  for (i=1; i<N_OF_CLOCKS; i++) {
    marx[i][i] = i;
    marx[0][i] = ZERO_LE;
"matrix.c" 756L, 20293C                                   1,0-1         Top
```

Transition systems: some fundamental concepts

Transition system: (S, S_0, R), also called a **state space** (sometimes state space refers only to S).

- We often write $s \to s'$ instead of $(s, s') \in R$.
  Note that the notation $s \to s'$ leaves R implicit.
- s' is a **successor** of s, and s a **predecessor** of s'.
- A (finite or infinite) sequence of states/transitions:
  $s_1 \to s_2 \to s_3 \to \cdots$ is called a **trace**.
- A state s is **reachable** if there exists a trace
  $s_0 \to s_1 \to s_2 \to \cdots \to s$, such that $s_0 \in S_0$.
  Otherwise s is called **unreachable**.

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- Note: every initial state is reachable by definition. Why?
- Quiz: are all states in S reachable?

Transition systems: some fundamental concepts

Transition system: (S, S_0, R).

- A state s is a **deadlock** if $\nexists s' \in S : s \to s'$ (i.e., if s has no successor). Deadlocks are sometimes called **trap states** or just **traps**.
- A **cycle** is a trace $s_1 \to s_2 \to \cdots \to s_n$ such that $s_n = s_1$.
- A **lasso** is a trace $s_1 \to s_2 \to \cdots \to s_n \to s_{n+1} \to \cdots \to s_m$ such that $s_n \to s_{n+1} \to \cdots \to s_m$ is a cycle.

Transition systems: some fundamental concepts

Transition system: (S, S_0, R).

- The transition system is called **deterministic** if every state has at most one successor:

$$\forall s, s', s'' \in S : (s \to s' \land s \to s'') \Rightarrow s' = s''$$

- Otherwise it is called **non-deterministic**.

Transition systems: some fundamental concepts

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- Otherwise it is called **non-deterministic**.

- **Quiz**: can deadlocks introduce non-determinism?

Transition systems: some fundamental concepts

Transition system: (S, S_0, R).
Let $X \subseteq S$ be a set of states.

- One-step **predecessors**:

$$\mathsf{pre}(X) = \{s \in S \mid \exists s' \in X : s \to s'\}$$

- One-step **successors**:

$$\mathsf{post}(X) = \{s \in S \mid \exists s' \in X : s' \to s\}$$

Transition systems: some fundamental concepts

Transition system: (S, S_0, R).
Let $X \subseteq S$ be a set of states.

- X is an **invariant** if every reachable state is in X
- i.e., an invariant is a superset of the set of reachable states
- X is **inductive** if post$(X) \subseteq X$
- i.e., X is inductive if when starting at any state in X, we cannot leave X
- X is an **inductive invariant** if X is both an invariant and inductive

Transition systems with labels

- Kripke Structures: labels on states
- Labeled Transition Systems: labels on transitions

Kripke Structures

A Kripke structure is a tuple:

$$(\mathsf{AP}, S, S_0, L, R)$$

- AP: set of atomic propositions (modeling state properties)
- S: set of states (perhaps infinite)
- $S_0 \subseteq S$: set of initial states
- L: labeling function on states

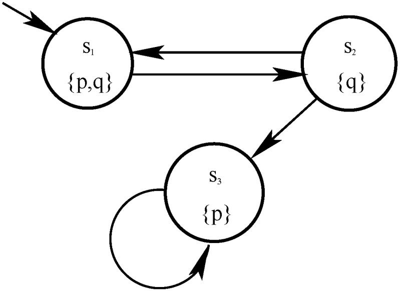
  $$L : S \to 2^{\mathsf{AP}}$$

  2^{AP}: the **powerset** (set of all subsets) of AP.
  For $p \in \mathsf{AP}$ and $s \in S$: "s has property p" iff $p \in L(s)$.
- R: transition relation

  $$R \subseteq S \times S$$

Example: Kripke Structure

In a Kripke structure the labels are on the states. Each state is labeled with a **set of atomic propositions** (those facts that are true when system is in that state).



In the example above, AP $= \{p, q\}$. Atomic proposition p holds at states s_1 and s_3, while atomic proposition q holds at states s_1 and s_2.

Labeled Transition Systems

An LTS is a tuple:

$$(\Sigma, S, S_0, R)$$

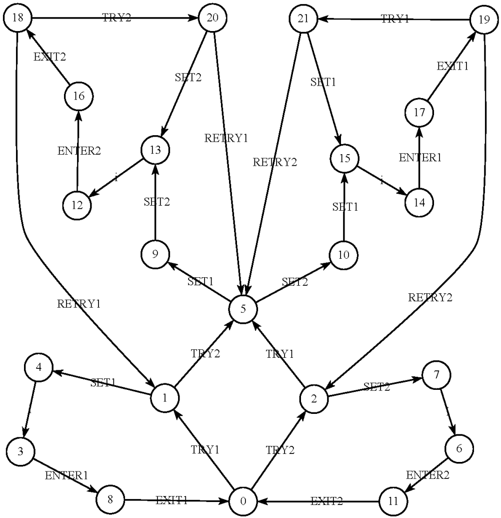
- Σ: set of labels (modeling events, actions, ...)
- S: set of states (perhaps infinite)
- $S_0 \subseteq S$: set of initial states
- R: transition relation

$$R \subseteq S \times (\Sigma \cup \{\epsilon\}) \times S$$

ϵ (sometimes τ): internal, unobservable action (used in composition, simulation/bisimulation equivalences, ...).

Example: LTS

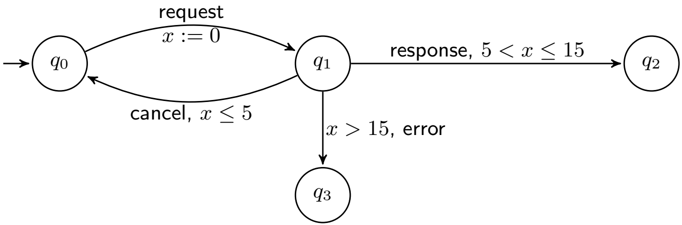
In a LTS the labels are on the transitions:



TRANSITION SYSTEMS: SEMANTIC FOUNDATIONS OF OTHER FORMALISMS

Transition systems of timed automata [Alur and Dill, 1994]

Timed automaton = finite automaton + clocks
Clocks = continuous (real) variables measuring time



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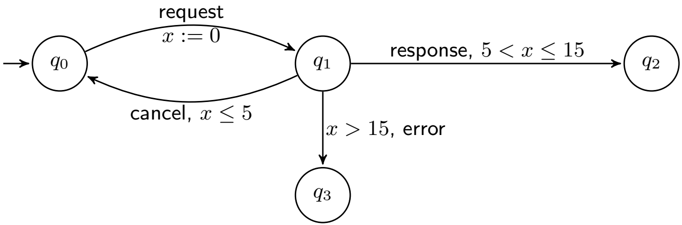
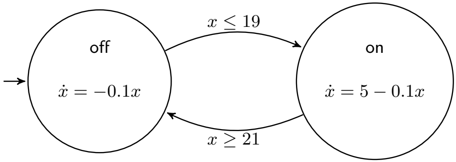


Transition system of timed automaton:
States: $\{(q_1, x = 0), (q_1, x = 1), (q_1, x = 0.5), (q_1, x = 0.1), ..., (q_2, x = 0), ...\}$
Transitions: discrete (change q_i) or continuous (change x, time elapses)
State space, transition space: infinite and uncountable!

Transition systems of hybrid automata [Alur et al., 1995]

Hybrid automaton = finite automaton + continuous (real) variables
Continuous dynamics: differential equations
Example: thermostat:



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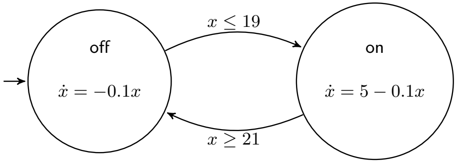
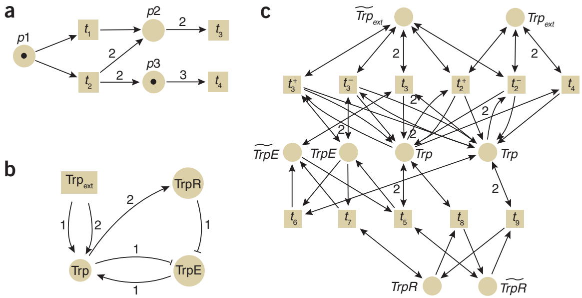


Transition system of hybrid automaton:
State space: $\{\text{on, off}\} \times \mathbb{R}$
Transitions: discrete or continuous
Both state and transition spaces: infinite and uncountable!

Transition system of a biological system

Modeling biological networks with **Petri nets**:



(a) A simple, standard Petri net. The circles denote places, whereas the boxes denote transitions. The distribution of tokens (black dots) in the places at a given time defines a marking. Transitions change the marking by removing a token from each incoming arrow and adding a token to each outgoing arrow. (b) Simplified logical regulatory graph for the biosynthesis of tryptophan in E. coli. Each node of the regulatory graph represents an active component: tryptophan (Trp), the active enzyme (TrpE) and the active repressor (TrpR). The node marked by a rectangle accounts for the import of Trp from external medium. All nodes are binary (that is, can take the value 0 or 1), except Trp, which is represented by a ternary variable (taking the values 0, 1, 2). Arrows represent activation and bars denote inhibition. (c) Petri net of the Trp regulatory network. Each of the four components of b is represented by two complementary places and all the different situations that lead to a change of the state of the system are modeled by one of the nine transitions.
Figure and text taken from [Fisher and Henzinger, 2007]. For more info on Petri nets, see [Petri and Reisig, 2008].

Challenge – Extra credit!

Can you think of a system that **cannot** be modeled as a transition system?

Spin

The model-checker Spin

- Widespread explicit-state (enumerative) model checker
- Created by Gerard J. Holzmann at Bell Labs in the 1980s
- Open source, numerous extensions, continuously evolving
- Two books [Holzmann, 1991, Holzmann, 2003], online course
- ACM System Software Award 2001
- Asynchronous systems (mostly)

Modeling discrete transition systems in Spin

```
// a small example spin model
// Peterson's solution to the mutual exclusion problem (1981)

bool turn, flag[2];          // the shared variables, booleans
byte ncrit;                  // nr of procs in critical section

active [2] proctype user()  // two processes
{
    assert(_pid == 0 || _pid == 1);
again:
    flag[_pid] = 1;
    turn = _pid;
    (flag[1 - _pid] == 0 || turn == 1 - _pid);

    ncrit++;
    assert(ncrit == 1);     // critical section
    ncrit--;

    flag[_pid] = 0;
    goto again
}
// analysis:
// $ spin -run peterson.pml
```

Summary

- Transition systems = our first formal model for systems!
  - FSMs can be seen as special cases
  - We will see later that transition systems cannot capture everything we want (e.g., composition, fairness), but OK for now
- Many kinds of TSs: finite/infinite, discrete/continuous, labeled in various ways, ...
- Serve as the semantic foundation of many higher-level formalisms

Notes

- State machines, transition systems, nuXmv models, Spin models, ...: these are not the same like your typical Java or Python programs!
- Why?

Notes

- State machines, transition systems, nuXmv models, Spin models, ...: these are not the same like your typical Java or Python programs!
- Why?
- The former are declarative and formal models of systems: they have formal semantics, and they represent sets or trees of behaviors.
- So what can I do with a model like the one above?
  - Right now: **simulation**! E.g.:
    - Load a model (in nuXmv, Spin, ...)
    - Set the initial state
    - Print the current state
    - Print the set of successor states
    - Choose a successor and move on step forward
    - Repeat
  - You are "unfolding" paths in the transition system of the model: like **debugging** your program with a debugger!
  - Do this also by hand (paper and pencil)! You might get this in homeworks, exams, etc.
  - Later, we will also do: model-checking (verification), synthesis, ...

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