

Semantic Solutions to Program Analysis Problems

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A talk in three parts.

1. A provocative claim. (The thought)
2. A idea about modular program analysis. (The idea)
3. And a demo! (The fun)

The claim

Program analysis should focus on semantics

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Interesting
Semantics



Computable
Analysis

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Three reasons

Why focus on semantics?

1. Semantics is easier to get right
2. Off-the-shelf approximation techniques exist
3. Semantics by itself is interesting

Getting it wrong

$$\begin{aligned} ((\lambda x^\beta . e)^{\ell_\lambda} v^{\ell_v})^{\ell_a} &\longrightarrow e[v^{\ell_v}/x^\beta] \\ (n^{\ell_n} v^{\ell_v})^{\ell_a} &\longrightarrow (\text{blame } \lambda \mathcal{R})^{\ell_a} \\ (\text{if0 } 0^{\ell_0} e_1 e_2)^{\ell} &\longrightarrow e_1 \\ (\text{if0 } v^{\ell_v} e_1 e_2)^{\ell} &\longrightarrow e_2 \\ (\text{int}_f^{\ell \ell'} \Leftarrow n^{\ell_n})^{\ell_c} &\longrightarrow n^{\ell} \\ (\text{int}_f^{\ell \ell'} \Leftarrow \bar{v}^{\ell_v})^{\ell_c} &\longrightarrow (\text{blame } f \mathcal{R})^{\ell'} \\ ((c_1 \rightarrow c_2)_f^{\ell \ell'} \Leftarrow \bar{v}^{\ell_v})^{\ell_c} &\longrightarrow ((c_1 \hat{\rightarrow} c_2)_f^{\ell \ell'} \Leftarrow \bar{v}^{\ell_v})^{\ell_c} \\ ((c_1 \rightarrow c_2)_f^{\ell \ell'} \Leftarrow n^{\ell_n})^{\ell_c} &\longrightarrow (\text{blame } f \mathcal{R})^{\ell'} \\ (((c_1 \hat{\rightarrow} c_2)_f^{\ell \ell'} \Leftarrow \bar{v}^{\ell_v})^{\ell_c} w^{\ell_w})^{\ell_a} &\longrightarrow (c_2 \Leftarrow (\bar{v}^{\ell_v} (c_1 \Leftarrow w^{\ell_w})) \mathcal{L}^+(c_1) \mathcal{L}^-(c_2)) \mathcal{L}^+(c_2) \end{aligned}$$

Getting it wrong

<i>Source</i> \ <i>Sink</i>	$\text{int}_h^{\ell_5^+ \ell_5^-}$
n^{ℓ_n}	
$\text{int}_f^{\ell_1^+ \ell_1^-}$	
$(\lambda x^\beta . e^\ell)^\ell$	$\{\ell_\lambda\} \subseteq \varphi(\ell_5^-) \Rightarrow \{\langle h, \mathcal{R} \rangle\} \subseteq \psi(\ell_5^-)$
$(c_g^{\ell_1^+ \ell_1^-} \rightarrow c_f^{\ell_2^+ \ell_2^-})_f^{\ell_3^+ \ell_3^-}$	$\{\ell_3^+\} \subseteq \varphi(\ell_5^-) \Rightarrow \{\langle h, \mathcal{R} \rangle\} \subseteq \psi(\ell_5^-)$

<i>Source</i> \ <i>Sink</i>	$(e^{\ell_5} e^{\ell_6})^{\ell_a}$	$(c_i^{\ell_7^+ \ell_7^-} \rightarrow c_h^{\ell_8^+ \ell_8^-})_h^{\ell_5^+ \ell_5^-}$
n^{ℓ_n}	$\{\ell_n\} \subseteq \varphi(\ell_5) \Rightarrow \{\langle \lambda, \mathcal{R} \rangle\} \subseteq \psi(\ell_a)$	$\{\ell_n\} \subseteq \varphi(\ell_5^-) \Rightarrow \{\langle h, \mathcal{R} \rangle\} \subseteq \psi(\ell_5^-)$
$\text{int}_f^{\ell_1^+ \ell_1^-}$	$\{\ell_1^+\} \subseteq \varphi(\ell_5) \Rightarrow \{\langle \lambda, \mathcal{R} \rangle\} \subseteq \psi(\ell_a)$	$\{\ell_1^+\} \subseteq \varphi(\ell_5^-) \Rightarrow \{\langle h, \mathcal{R} \rangle\} \subseteq \psi(\ell_5^-)$
$(\lambda x^\beta . e^\ell)^\ell$	$\{\ell_\lambda\} \subseteq \varphi(\ell_5) \Rightarrow \varphi(\ell_6) \subseteq \varphi(\beta)$ $\{\ell_\lambda\} \subseteq \varphi(\ell_5) \Rightarrow \varphi(\ell) \subseteq \varphi(\ell_a)$	$\{\ell_\lambda\} \subseteq \varphi(\ell_5^-) \Rightarrow \varphi(\ell_7^+) \subseteq \varphi(\beta)$ $\{\ell_\lambda\} \subseteq \varphi(\ell_5^-) \Rightarrow \varphi(\ell) \subseteq \varphi(\ell_8^-)$
$(c_g^{\ell_1^+ \ell_1^-} \rightarrow c_f^{\ell_2^+ \ell_2^-})_f^{\ell_3^+ \ell_3^-}$	$\{\ell_3^+\} \subseteq \varphi(\ell_5) \Rightarrow \varphi(\ell_6) \subseteq \varphi(\ell_1^-)$ $\{\ell_3^+\} \subseteq \varphi(\ell_5) \Rightarrow \varphi(\ell_2^+) \subseteq \varphi(\ell_a)$	$\{\ell_3^+\} \subseteq \varphi(\ell_5^-) \Rightarrow \varphi(\ell_7^+) \subseteq \varphi(\ell_1^-)$ $\{\ell_3^+\} \subseteq \varphi(\ell_5^-) \Rightarrow \varphi(\ell_2^+) \subseteq \varphi(\ell_8^-)$

Getting it wrong

Source\Sink	$\text{int}_h^{e_1^1 e_5^1}$	$(\dots e_5 \text{int}_h^{e_1^1 e_5^1} e_6^1 e_6^1)$	$\text{any}_h^{e_1^1 e_5^1}$	$(\dots e_5 \text{any}_h^{e_1^1 e_5^1} e_6^1 e_6^1)$
$n_{e_1^1 \dots}$	$\{e_1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$	$\{e_1 \dots \sqsubseteq e_5\}$	$\{e_1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$	$\{e_1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$
$\text{int}_f^{e_1^1 e_1^1}$				
$(\dots e_1 \text{int}_f^{e_1^1 e_1^1} e_2^1 e_2^1)$	$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$	$\{e_1^1 \dots \sqsubseteq e_5^1\}$	$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$	$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$
$\text{any}_f^{e_1^1 e_1^1}$				
$(\dots e_1 \text{any}_f^{e_1^1 e_1^1} e_2^1 e_2^1)$	$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{R})\} \subseteq \psi(e_5^1)$		$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$	$\{e_1^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$
$(\lambda x^\beta . e^1 e_{e_1^1}^1 \dots)$	$\{e_\lambda\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{R})\} \subseteq \psi(e_5^1)$		$\{e_\lambda\} \subseteq \varphi(e_5^1) \Rightarrow \varphi(e_5^1) \subseteq \varphi(\beta)$ $\{e_\lambda\} \subseteq \varphi(e_5^1) \Rightarrow \varphi(\ell) \subseteq \varphi(e_5^1)$ $\{e_\lambda\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$	
$(e_9^{e_1^1 e_1^1} \rightarrow e_f^{e_2^1 e_2^1})_f^{e_1^1 e_1^1}$	$\{e_9^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{R})\} \subseteq \psi(e_5^1)$	$\{e_9^1 \dots \sqsubseteq e_5^1\}$	$\{e_9^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$	$\{e_9^1\} \subseteq \varphi(e_5^1) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5^1)$
$(\dots e_3 (e_9^{e_1^1 e_1^1} \rightarrow e_f^{e_2^1 e_2^1})_f^{e_1^1 e_1^1} e_4^1 e_4^1)$				

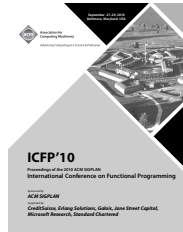
Source\Sink	$(e^1 e_1^1) e_\lambda^1$	$(e_1^1 e_1^1 \rightarrow e_h^{e_1^1 e_1^1} e_5^1 e_5^1)$	$(\dots e_5 (e_1^1 e_1^1 \rightarrow e_h^{e_1^1 e_1^1} e_5^1 e_5^1) e_6^1 e_6^1)$
$n_{e_1^1 \dots}$	$\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \{(\lambda, \mathcal{R})\} \subseteq \psi(e_\lambda)$		$\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{R})\} \subseteq \psi(e_5)$
$\text{int}_f^{e_1^1 e_1^1}$	$\{e_1^1\} \subseteq \varphi(e_5) \Rightarrow \{(\lambda, \mathcal{R})\} \subseteq \psi(e_5)$		$\{e_1^1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{R})\} \subseteq \psi(e_5)$
$(\dots e_1 \text{int}_f^{e_1^1 e_1^1} e_2^1 e_2^1)$			
$\text{any}_f^{e_1^1 e_1^1}$			
$(\dots e_1 \text{any}_f^{e_1^1 e_1^1} e_2^1 e_2^1)$			
$(\lambda x^\beta . e^1 e_{e_1^1}^1 \dots)$	$\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \varphi(e_\lambda) \subseteq \varphi(\beta)$ $\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \varphi(\ell) \subseteq \varphi(e_\lambda)$		$\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \varphi(e_5^1) \subseteq \varphi(\beta)$ $\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \varphi(\ell) \subseteq \varphi(e_5)$ $\{e_\lambda\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$
$(e_9^{e_1^1 e_1^1} \rightarrow e_f^{e_2^1 e_2^1})_f^{e_1^1 e_1^1}$	$\{e_9^1\} \subseteq \varphi(e_5) \Rightarrow \varphi(e_\lambda) \subseteq \varphi(e_1^1)$ $\{e_9^1\} \subseteq \varphi(e_5) \Rightarrow \varphi(e_5^1) \subseteq \varphi(e_\lambda)$		$\{e_9^1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$ $\{e_9^1\} \subseteq \varphi(e_5) \Rightarrow \varphi(e_5^1) \subseteq \varphi(e_5)$ $\{e_9^1\} \subseteq \varphi(e_5) \Rightarrow \{(h, \mathcal{O})\} \subseteq \psi(e_5)$
$(\dots e_3 (e_9^{e_1^1 e_1^1} \rightarrow e_f^{e_2^1 e_2^1})_f^{e_1^1 e_1^1} e_4^1 e_4^1)$			

Table 1. Constraints creation for source-sink pairs.

The shelf

Generic abstraction techniques exist.

- ▶ Nielsen, Nielsen, and Hankin, '99
- ▶ Cousot, '02
- ▶ Van Horn and Might, '10: Abstracting Abstract Machines



Semantics as verification

Once you have a semantics that answers interesting questions, try running it.

A Modular Semantics

Modularity matters.

Modularity of analysis matters.

Modularity matters.

- ▶ Some programs are open (c.f.: the web).

```
// dynamically load any javascript file.
load.getScript = function(filename) {
  var script = document.createElement('script')
  script.setAttribute("type","text/javascript")
  script.setAttribute("src", filename)
  if (typeof script!="undefined")
  document.getElementsByTagName("head")[0]
    .appendChild(script)
}
```

Modularity matters.

- ▶ Good components are written in bad languages.

```
#include "escheme.h"
Scheme_Object *scheme_initialize(Scheme_Env *env) {
    Scheme_Env *mod_env;
    mod_env = scheme_primitive_module(scheme_intern_symbol("hi"),
                                      env);

    scheme_add_global("greeting",
                    scheme_make_utf8_string("hello"),
                    mod_env);
    scheme_finish_primitive_module(mod_env);
    return scheme_void;
}

Scheme_Object *scheme_reload(Scheme_Env *env) {
    return scheme_initialize(env); /* Nothing special for reload */
}

Scheme_Object *scheme_module_name() {
    return scheme_intern_symbol("hi");
}
```

Modularity matters.

- ▶ Libraries matter.

```
;; To use: (require (planet dvanhorn/ralist))
;; Purely Functional Random-Access Lists.
;; Implementation based on Okasaki, FPCA '95.
#lang racket
(provide (all-defined-out))

(struct tree      (val))
(struct (leaf tree) ())
(struct (node tree) (left right))

;; X [RaListof X] -> [RaListof X]
(define (ra:cons x ls)
  (match ls
    [(list-rest (cons s t1) (cons s t2) r)
     (cons (cons (+ 1 s s) (make-node x t1 t2)) r)]
    [else
     (cons (cons 1 (make-leaf x)) ls)]))
...
```

An idea:

reduction semantics + abstract values
= *abstract* reduction semantics

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$$(\lambda x.E) V \triangleright \{V/x\}E$$

An idea:

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= *abstract* reduction semantics

$$\frac{(\lambda x.E) V \triangleright \{V/x\}E}{(\lambda x.E) : A \rightarrow B}$$

An idea:

reduction semantics + abstract values
= *abstract* reduction semantics

$$\frac{\begin{array}{l} (\lambda x.E) V \triangleright \{V/x\}E \\ (\lambda x.E) : A \rightarrow B \end{array}}{(A \rightarrow B) V \blacktriangleright B}$$

```
(module fact (int/c -> int/c)
  (lambda (x)
    (if (= x 0)
        1
        (* x (fact (sub1 x)))))))
```

```
(module input int/c 0)
```

```
(fact input)
```

```
▷* ((lambda (x) ...) 0)
```

```
▷* 1
```



```
(module fact (int/c -> int/c)
  ●)
```

```
(module input int/c 0)
```

```
(fact input)
```

```
▶* ((int/c -> int/c) 0)
```

```
▶* int/c
```

```
(module fact (int/c -> int/c)
  (lambda (x)
    (if (= x 0)
        1
        (* x (fact (sub1 x))))))
```

```
(module input int/c ●)
```

```
(fact input)
```

- ▶* ((lambda (x) ...) int/c) ▶* (if (= int/c 0) 0 ...)
- ▶* (if bool 1 ...)
- ▶* 1, int/c

```
(module * (int/c int/c -> int/c) •)
(module sub1 (int/c -> int/c) •)
(module fact (int/c -> int/c)
  (lambda (x)
    (if (= x 0)
        1
        (* x (fact (sub1 x)))))))
```

```
(module input int/c 0)
```

```
(fact input)
```

```
▶* ((lambda (x) ...) 0)
```

```
▶* 1
```

```
(module * (int/c int/c -> int/c) ●)
(module sub1 (int/c -> int/c) ●)
(module fact (int/c -> int/c)
  (lambda (x)
    (if (= x 0)
        1
        (* x (fact (sub1 x)))))))
```

```
(module input int/c ●)
```

```
(fact input)
```

```
▶* ((lambda (x) ...) int/c)
```

```
▶* int/c
```

```
(module * (any/c any/c -> int/c) •)
(module sub1 (any/c -> int/c) •)
(module fact any/c
  (lambda (x)
    (if (= x 0)
        1
        (* x (fact (sub1 x)))))))
```

```
(module input int/c •)
```

```
(fact input)
```

```
▶* ((lambda (x) ...) int/c)
```

```
▶* int/c
```

Demo

- ▶ Focus on semantics.
- ▶ Abstract reduction provides modularity.
- ▶ A semantics can be a verifier.

<http://bit.ly/abstract-reduction>

<http://redex.racket-lang.org>

