Resolution

- basis for first (less successful) resolution based DP
- can be extended to first order logic
- helps to explain learning

Resolution Rule

$$\frac{C \cup \{v\} \qquad D \cup \{\neg v\}}{C \cup D} \qquad \{v, \neg v\} \cap C = \{v, \neg v\} \cap D = \emptyset$$

Read: resolving the clause $C \cup \{v\}$ with the clause $D \cup \{\neg v\}$, both above the line, on the variable v, results in the clause $D \cup C$ below the line.

Correctness of Resolution Rule

Usage of such rules: if you can derive what is above the line (premise) then you are allowed to deduce what is below the line (conclusion).

Theorem. (premise satisfiable $\Rightarrow$ conclusion satisfiable)

$$\sigma(C \cup \{v\}) = \sigma(D \cup \{\neg v\}) = 1 \quad \Rightarrow \quad \sigma(C \cup D) = 1$$

Proof.

let $c \in C$, $d \in D$ with $(\sigma(c) = 1$ or $\sigma(v) = 1)$ and $(\sigma(d) = 1$ or $\sigma(\neg v) = 1)$

if $\sigma(c) = 1$ or $\sigma(d) = 1$ conclusion follows immediately

otherwise $\sigma(v) = \sigma(\neg v) = 1$ $\Rightarrow$ contradiction

q.e.d.

Completeness of Resolution Rule

Theorem. (conclusion satisfiable $\Rightarrow$ premise satisfiable)

$$\sigma(C \cup D) = 1 \quad \Rightarrow \quad \exists \sigma' \quad \text{with} \quad \sigma'(C \cup \{v\}) = \sigma'(D \cup \{\neg v\}) = 1$$

Proof.

with out loss of generality pick $c \in C$ with $\sigma(c) = 1$

define $\quad \sigma'(x) = \begin{cases} 0 & \text{if } x = v \\ \sigma(x) & \text{else} \end{cases}$

since v and $\neg v$ do not occur in C, we still have $\sigma'(C) = 1$ and thus $\sigma'(C \cup \{v\}) = 1$

by definition $\sigma'(\neg v) = 1$ and thus $\sigma'(D \cup \{\neg v\}) = 1$

q.e.d.

Resolution Based DP

Idea: use resolution to *existentially* quantify out variables

1. if empty clause found then terminate with result **unsatisfiable**

2. find variables which only occur in one phase (only positive or negative)

3. remove all clauses in which these variables occur

4. if no clause left then terminate with result **satisfiable**

5. choose x as one of the remaining variables with occurrences in both phases

6. add results of all possible resolutions on this variable

7. remove all trivial clauses and all clauses in which x occurs

8. continue with **1.**