CS3800 Theory of Computation September 11, 2013

Problem Set 1

Lecturer: Daniel Wichs *Due: Thursday, September 19*

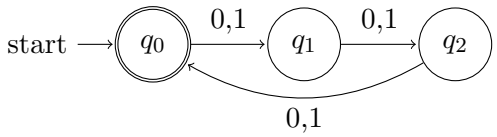
Due at the beginning of class on Thursday, 9/19.
When solving a problem you can use all the results seen in class. You should not use other sources. You are allowed to discuss problems with other students. However, each student must write up his or her own solutions and must not read or copy the solutions of others. You should try working on each problem on your own before discussing with others. If you work with others on a problem, you must note with whom you discussed the problem at the beginning of your solution write-up.

Problem 1 (Practice with Sets, Functions, Notation) 10 points

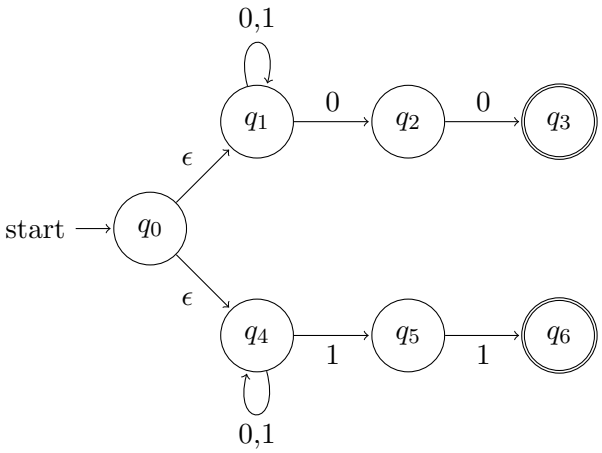
1. (3 pts) Give an example of an element $s \in S$ from each of the following sets S:
   (a) $S = \{0,1\}^3$
   (b) $S = \{a,b,c\} \times \{1,2,3\}$
   (c) $S =$Powerset($\{a,b,c\}$)
2. (4 pts) If A is a set of size $|A| = 5$, and B is a set of size $|B| = 3$, what is the size of each of the following sets (you can write your answer as e.g., 2^{20} or 10^3 instead of an exact number):
   (a) $A \times B$
   (b) Powerset(A)
   (c) Powerset($A \times B$)
   (d) Powerset(A)$\times$ Powerset(B)
3. (3 pts) Is the following statement true or false? Think about it carefully and explain your answer:
   "For all sets S, if Powerset(S) contains exactly 7 elements, then S is the empty set."

Problem 2 (What Language is Recognized?) 10 Points

1. (5 pts) What language does the following DFA recognize? Write out a formal description of this DFA using the notation $M = (Q, \Sigma, \delta, q_0, F)$ that we saw in class.



2. (5 pts) What language does the following NFA recognize? Write out a formal description of this NFA using the notation $M = (Q, \Sigma, \delta, q_0, F)$ that we saw in class. Use the formal definition of what it means for an NFA to "accept" a string to w to show that this NFA accepts $w = 100$.



Problem 3 (Construct DFAs and NFAs) 20 Points

Fix the alphabet to be $\Sigma = \{0, 1\}$:

1. Construct a DFA that recognizes: $L = \{w : w \neq 01\}$.
2. Construct a DFA that has 4 states and recognizes: $L = \{w : w$ ends with a 00 or a 01$\}$.
3. Construct any two different DFAs M, M' that recognize the same language $L(M) = L(M')$.
4. Construct a NFA that has 3 states and recognizes: $L = \{w : w$ ends with a 00 or a 01$\}$.
5. Construct a NFA that recognizes: $L = \{w : w$ starts with a 0 or ends with a 1$\}$.

Problem 4 (Finite Languages Are Regular) 10 Points

Let L be any language containing a finite number of strings (i.e., $|L|$ is finite). Show that L is regular by describing how to construct a DFA that recognizes L.