Lagrange Multipliers

$\vec{p} = (p_1, p_2, \ldots p_n)$

Where is $H(\vec{p})$ maximized? $\quad H(\vec{p}) = -\sum_{i=1}^{n} p_i \lg p_i = \frac{-1}{\ln 2} \sum_{i=1}^{n} p_i \ln p_i$

$\frac{1}{\ln 2}$ doesn't affect where max occurs.

① First try..

$$\frac{\partial H(\vec{p})}{\partial p_j} = \frac{\partial}{\partial p_j}\left(-\left(p_1 \ln p_1 + p_2 \ln p_2 + \ldots p_j \ln p_j + \ldots p_n \ln p_n \right)\right)$$

$$= -\left(p_j \cdot \frac{1}{p_j} + \ln p_j \right)$$

$$= -(1 + \ln p_j)$$

set to 0 .. $\quad -(1+\ln p_j) = 0$

$$\iff \ln p_j = -1$$

$$p_j = e^{-1} \approx .368$$

— not a distribution!

② Lagrange multipliers

$$J(\vec{p}) = \underbrace{-\sum_{i=1}^{n} p_i \ln p_i}_{\text{max this}} + \lambda \underbrace{\sum_{i=1}^{n} p_i}_{\text{constraint}}$$

$$\frac{\partial J(\vec{p})}{\partial p_j} = -p_j \cdot \frac{1}{p_j} - \ln p_j \cdot 1 + \lambda$$

$$= -1 - \ln p_j + \lambda$$

· Set to zero.. $\quad -1 - \ln p_j + \lambda = 0$

(fixes p_i's) $\quad \iff \ln p_j = \lambda - 1$

$$\iff p_j = e^{\lambda - 1}$$

- Satisfy constraint (fixes λ)

$$\sum_{j=1}^{n} p_j = 1$$

$$\Leftrightarrow \quad \sum_{j=1}^{n} e^{\lambda-1} = 1$$

$$n e^{\lambda-1} = 1$$

$$e^{\lambda-1} = \frac{1}{n}$$

$$\lambda - 1 = \ln 1/n$$

$$\lambda = 1 + \ln 1/n$$

- plug back in

$$p_j = e^{\lambda-1} = e^{(1+\ln 1/n)-1} = e^{\ln 1/n} = \underline{\underline{1/n}}$$

$\Rightarrow$ **uniform distribution**

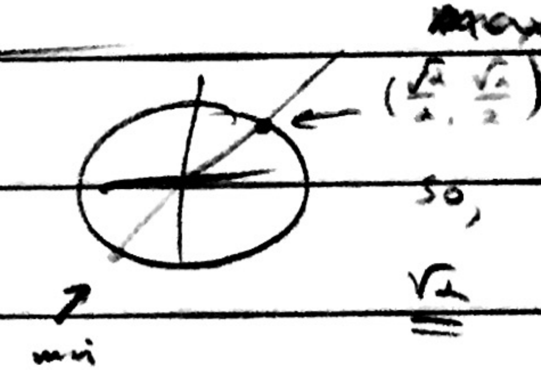
$$H(1/n, 1/n, \ldots 1/n) = -\sum_{i=1}^{n} \frac{1}{n} \lg 1/n$$

$$= \frac{1}{n} \sum_{i=1}^{n} \lg n$$

$$= \lg n$$

[illegible] I

max $\quad x+y \quad$ s.t. $\quad x^2+y^2=1$



so, $\underline{\underline{\sqrt{2}}}$

$$J(x,y) = x+y + \lambda\cdot(x^2+y^2)$$

$$\frac{\partial J}{\partial x} = 1 + 2\lambda x \qquad \frac{\partial J}{\partial y} = 1 + 2\lambda y$$

$1+2\lambda x = 0 \Rightarrow x = -\frac{1}{2\lambda}$

$1+2\lambda y = 0 \Rightarrow y = \frac{-1}{2\lambda}$

1 form of x & y

Constraint: $x^2+y^2=1$

$$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1$$

$$\frac{1}{2\lambda^2} = 1$$

$$\lambda^2 = 1/2$$

$$\lambda = \pm \frac{1}{\sqrt{2}}$$

2 Solve for λ

3 plug in λ: $\quad -\frac{1}{2(-\frac{1}{\sqrt{2}})} = \frac{\sqrt{2}}{2}$ ✓