Indexing

Index Construction

Motivation: Scale

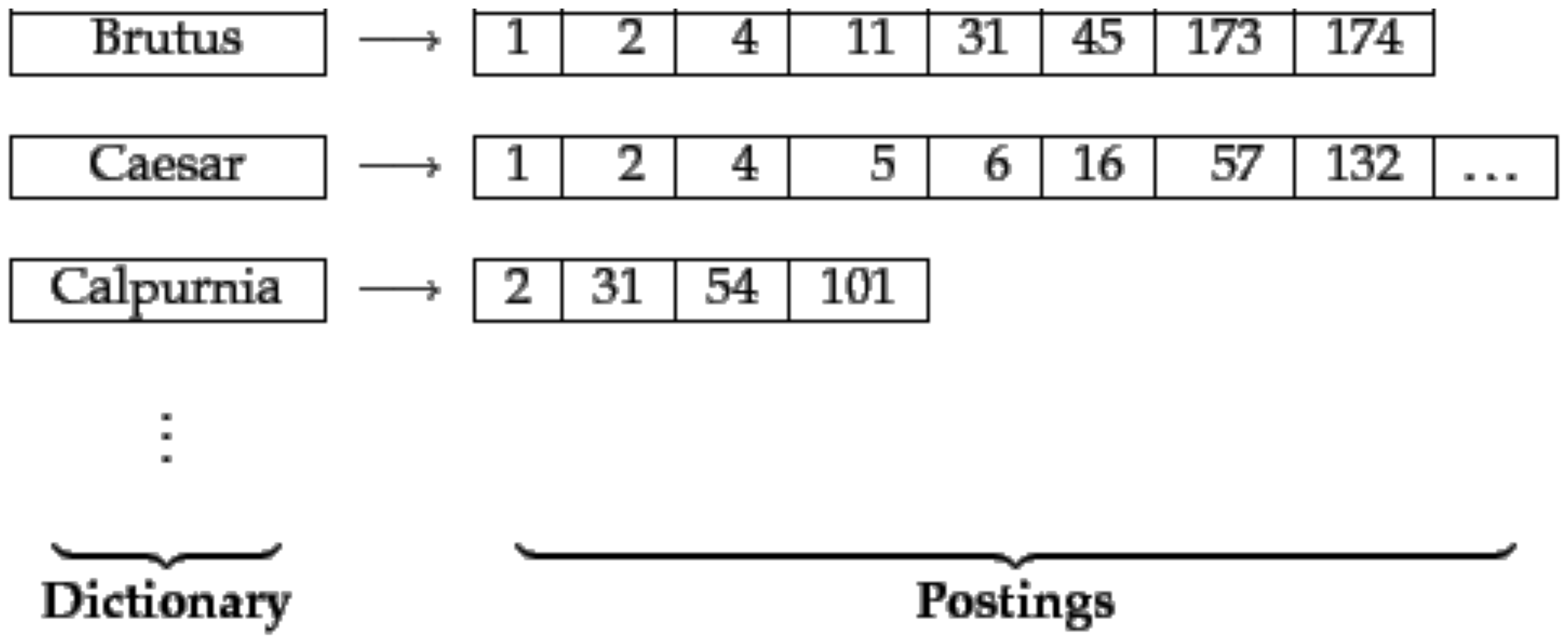
Corpus	Terms	Docs	Entries
Shakespeare's Plays	~31,000	37	~1.1 million
English Wikipedia	~1.7 million	~4.5 million	~7.65 trillion
English Web	>2 million	>1.7 billion	>3.4x10

- A term incidence matrix with V terms and D documents has O(V x D) entries.

- Shakespeare used around 31,000 distinct words across 37 plays, for about 1.1M entries.

- As of 2014, a collection of Wikipedia pages comprises about 4.5M pages and roughly 1.7M distinct words. Assuming just one bit per matrix entry, this would consume about 890GB of memory.

Inverted Indexes - Intro



Brutus → 1 | 2 | 4 | 11 | 31 | 45 | 173 | 174

Caesar → 1 | 2 | 4 | 5 | 6 | 16 | 57 | 132 | …

Calpurnia → 2 | 31 | 54 | 101

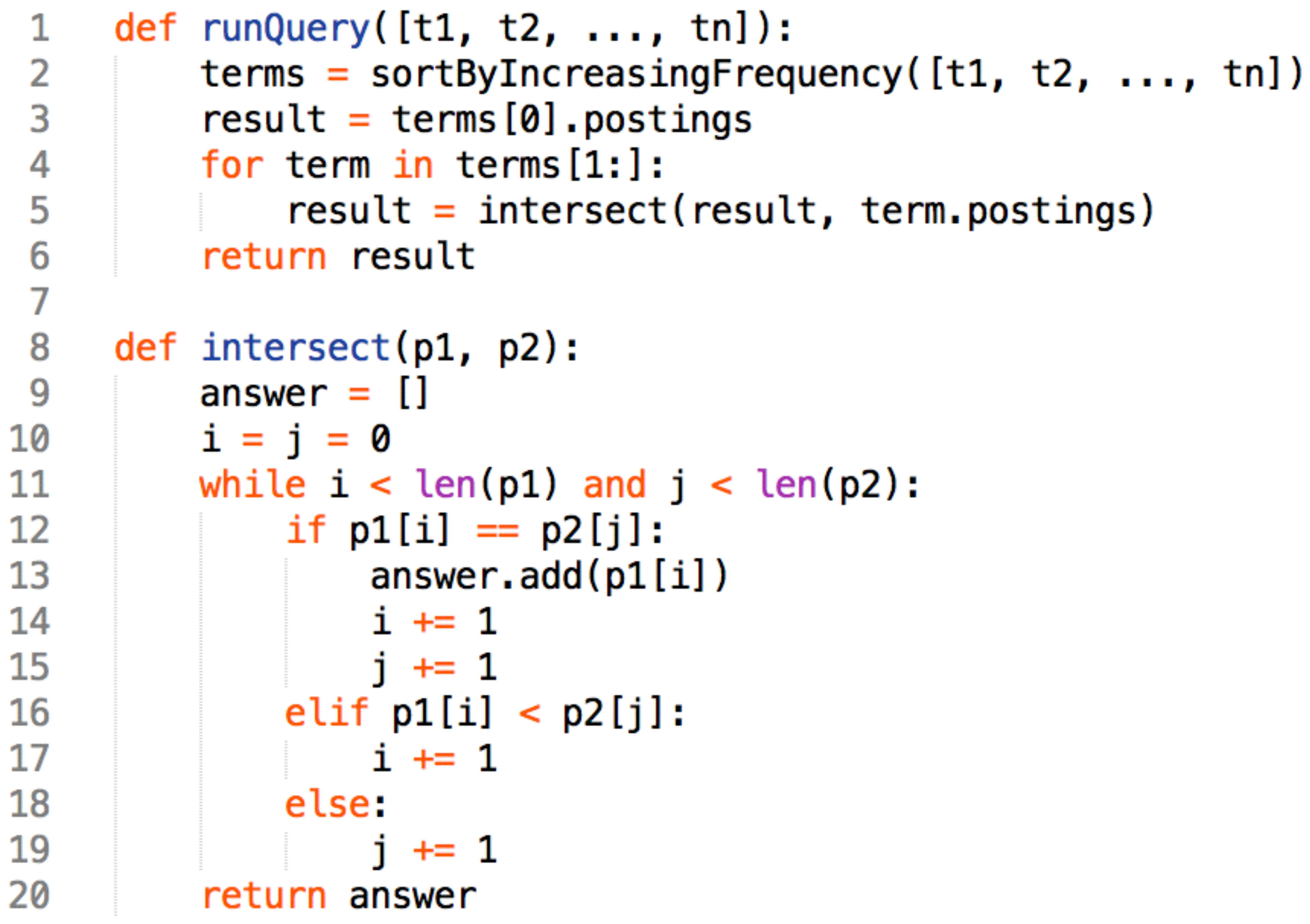
► **Figure 1.2** The two parts of an inverted index. The dictionary is commonly kept in memory, with pointers to each postings list, which is stored on disk.

- Two insights allow us to reduce this to a manageable size:

    1. The matrix is *sparse* – any document uses a tiny fraction of the vocabulary.
    2. A query only uses a handful of words, so we don't need the rest.

- We use an *inverted index* instead of using a term incidence matrix directly.
- An inverted index is a map from a term to a *posting list* of documents which use that term.

Search Algorithm



```
def runQuery([t1, t2, ..., tn]):
    terms = sortByIncreasingFrequency([t1, t2, ..., tn])
    result = terms[0].postings
    for term in terms[1:]:
        result = intersect(result, term.postings)
    return result

def intersect(p1, p2):
    answer = []
    i = j = 0
    while i < len(p1) and j < len(p2):
        if p1[i] == p2[j]:
            answer.add(p1[i])
            i += 1
            j += 1
        elif p1[i] < p2[j]:
            i += 1
        else:
            j += 1
    return answer
```

- Consider queries of the form:

  t_1 AND t_2 AND … AND t_n

- In this simplified case, we need only take the intersections of the term posting lists.

- This algorithm, inspired by merge sort, relies on the posting lists being sorted by length.

- We save time by processing the terms in order from least common to most common. (Why does this help?)

Motivation

- All modern search engines rely on inverted indexes in some form. Many other data structures were considered, but none has matched its efficiency.

- The entries in a production inverted index typically contain many more fields providing extra information about the documents.

- The efficient construction and use of inverted indexes is a topic of its own, and will be covered in a later module.

Motivation

A reasonably-sized index of the web contains many billions of documents and has a massive vocabulary.

Search engines run roughly 10^5 queries per second over that collection.

We need fine-tuned data structures and algorithms to provide search results in much less than a second per query. $O(n)$ and even $O(\log n)$ algorithms are often not nearly fast enough.

The solution to this challenge is to run an inverted index on a massive distributed system.

Inverted Indexes

Inverted Indexes are primarily used to allow fast, concurrent query processing.

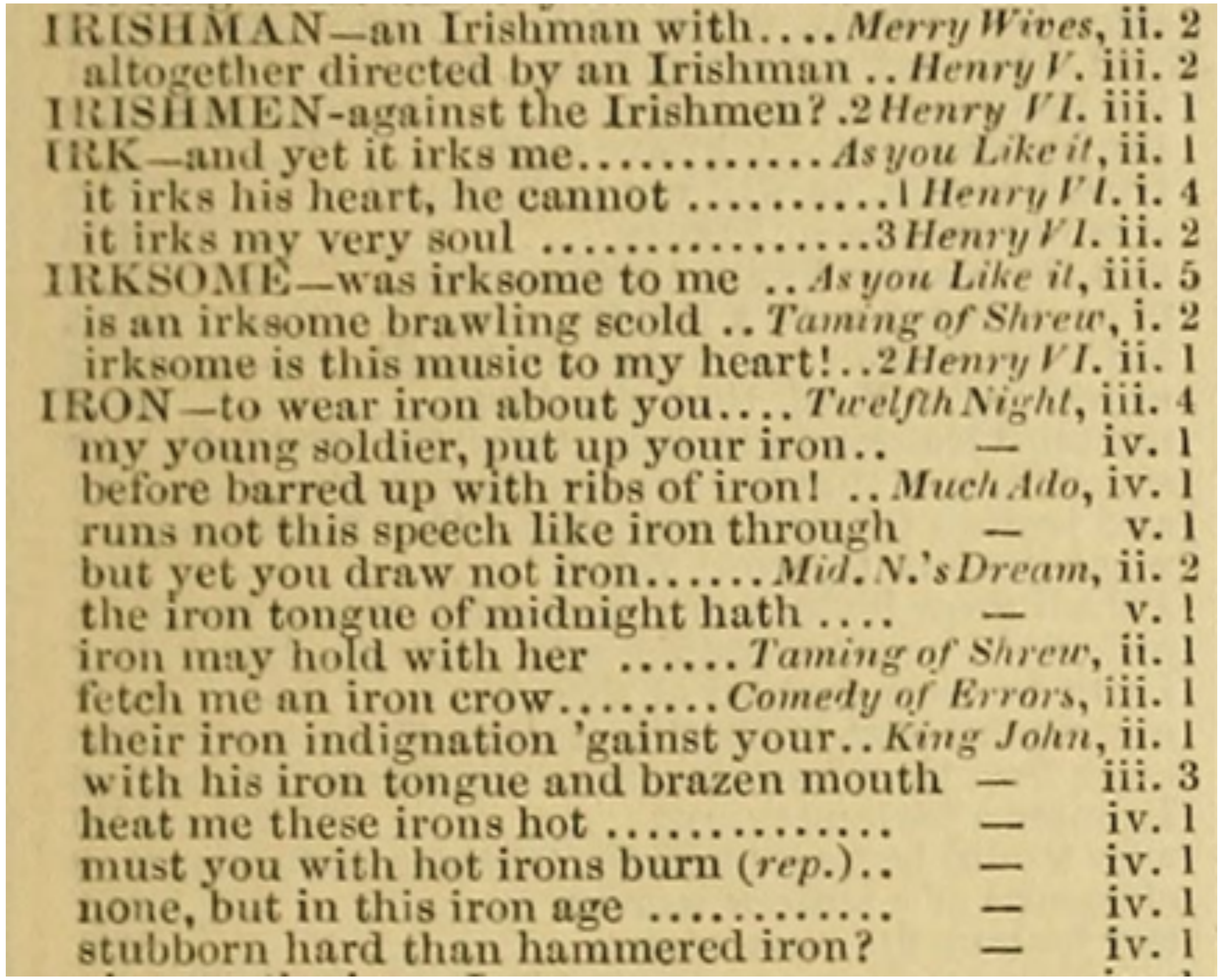
Each term found in any indexed document receives an independent inverted list, which stores the information necessary to process that term when it occurs in a query.

Indexes

The primary purpose of a search engine index is to store whatever information is needed to minimize processing at query time.

Text search has unique needs compared to, e.g., database queries, and needs its own data structures – primarily, the inverted index.

- A **forward index** is a map from documents to terms (and positions). These are used when you search within a document.
- An **inverted index** is a map from terms to documents (and positions). These are used when you want to find a term in any document.



IRISHMAN—an Irishman with.... *Merry Wives*, ii. 2
altogether directed by an Irishman .. *Henry V.* iii. 2
IRISHMEN-against the Irishmen? .2 *Henry VI.* iii. 1
IRK—and yet it irks me............ *As you Like it*, ii. 1
it irks his heart, he cannot ....1 *Henry VI.* i. 4
it irks my very soul ....3 *Henry VI.* ii. 2
IRKSOME—was irksome to me .. *As you Like it*, iii. 5
is an irksome brawling scold .. *Taming of Shrew*, i. 2
irksome is this music to my heart!..2 *Henry VI.* ii. 1
IRON—to wear iron about you.... *Twelfth Night*, iii. 4
my young soldier, put up your iron.. — iv. 1
before barred up with ribs of iron! .. *Much Ado*, iv. 1
runs not this speech like iron through — v. 1
but yet you draw not iron...... *Mid. N.'s Dream*, ii. 2
the iron tongue of midnight hath .... — v. 1
iron may hold with her ...... *Taming of Shrew*, ii. 1
fetch me an iron crow......... *Comedy of Errors*, iii. 1
their iron indignation 'gainst your.. *King John*, ii. 1
with his iron tongue and brazen mouth — iii. 3
heat me these irons hot .... — iv. 1
must you with hot irons burn (*rep.*).. — iv. 1
none, but in this iron age .... — iv. 1
stubborn hard than hammered iron? — iv. 1

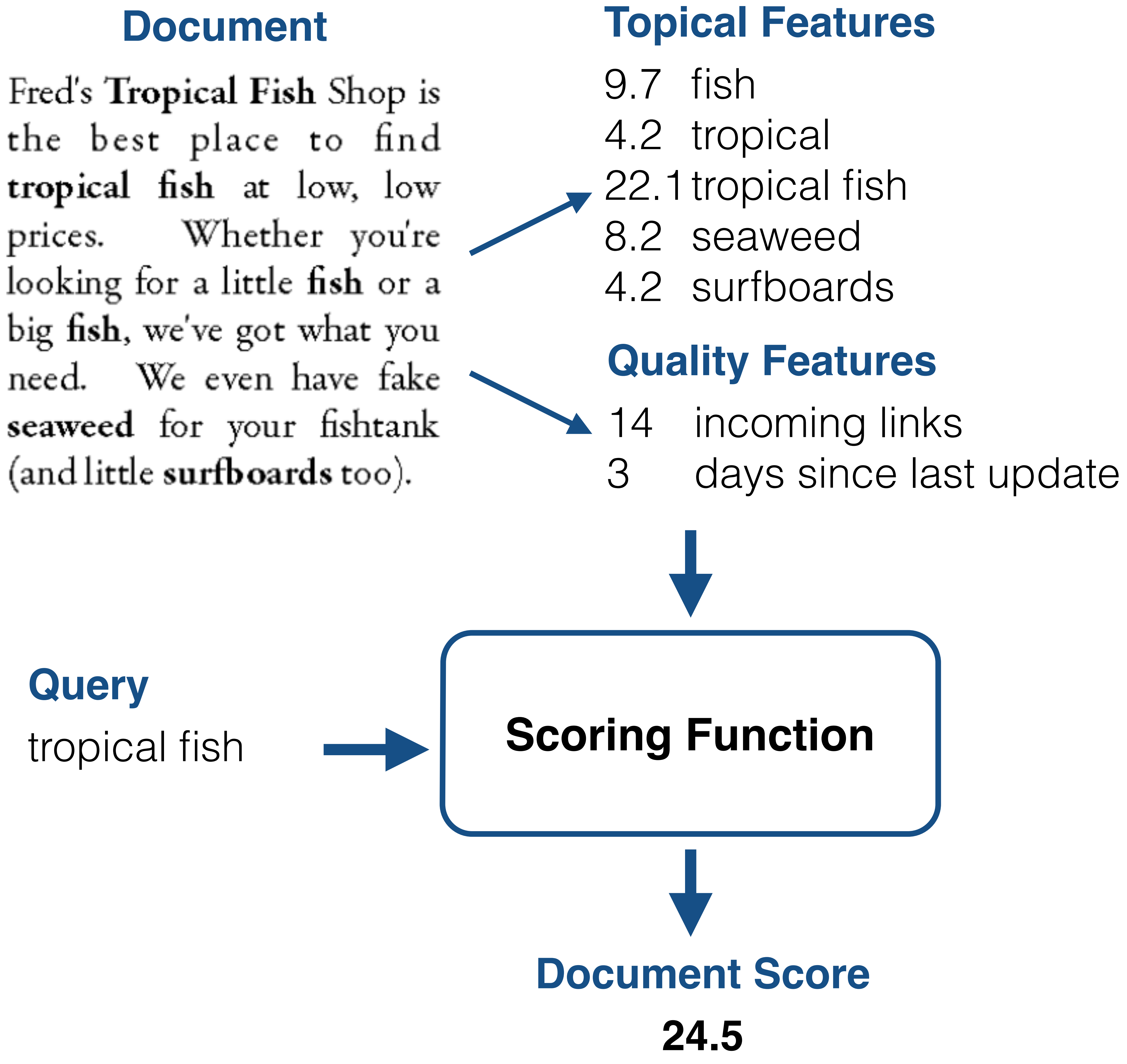
Is this a forward or an inverted index?

Abstract Model of Ranking

Indexes are created to support search, and the primary search task is *document ranking*.

We sort documents according to some scoring function which depends on the terms in the query and the document representation.

In the abstract, we need to store various document features to efficiently score documents in response to a query.



Document

Fred's **Tropical Fish** Shop is the best place to find **tropical fish** at low, low prices. Whether you're looking for a little **fish** or a big **fish**, we've got what you need. We even have fake **seaweed** for your fishtank (and little **surfboards** too).

Topical Features

9.7	fish
4.2	tropical
22.1	tropical fish
8.2	seaweed
4.2	surfboards

Quality Features

14	incoming links
3	days since last update

Query

tropical fish

Scoring Function

Document Score

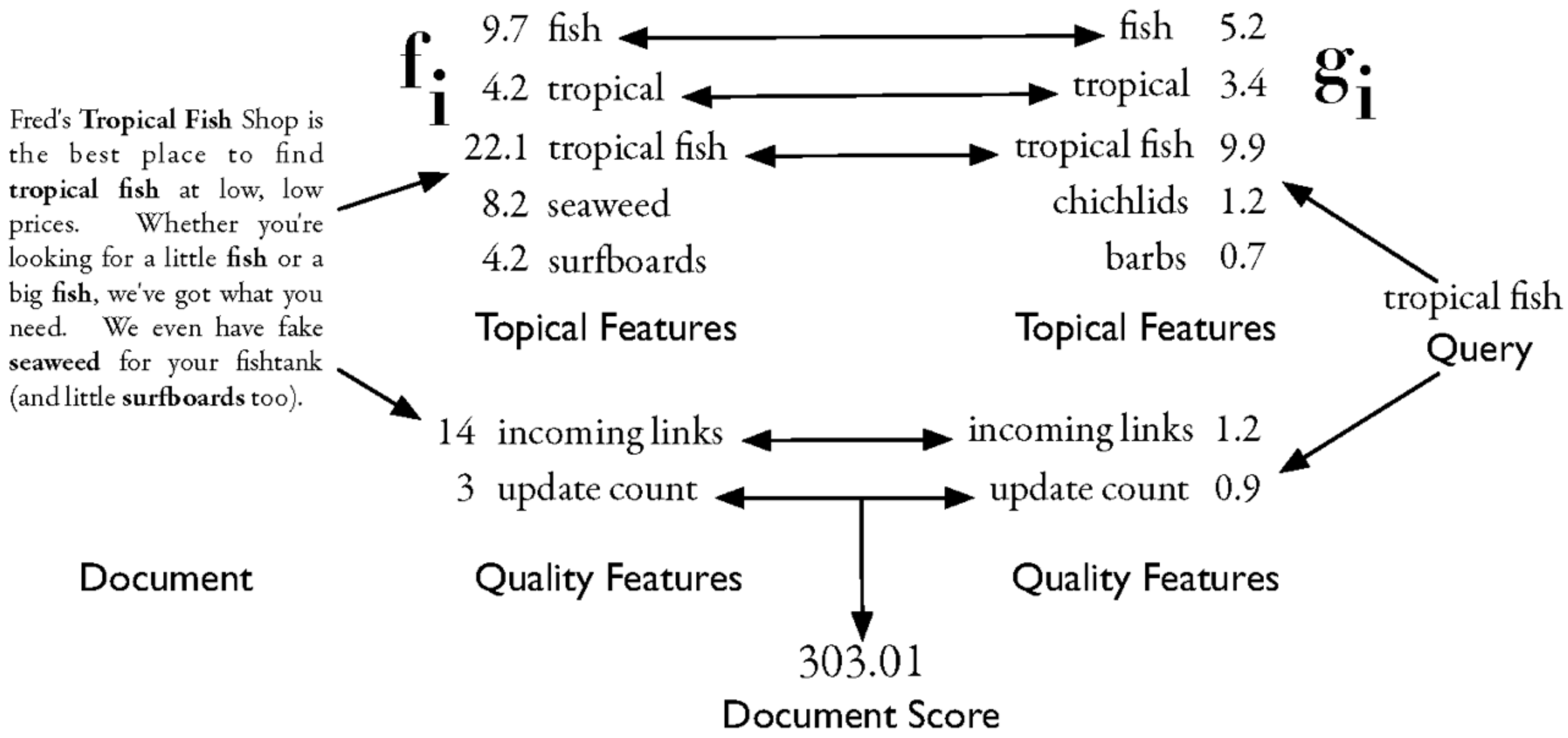
24.5

More Concrete Model

$$R(Q, D) = \sum_{i} g_i(Q) f_i(D)$$

f_i is a document feature function

g_i is a query feature function



Fred's **Tropical Fish** Shop is the best place to find **tropical fish** at low, low prices. Whether you're looking for a little **fish** or a big **fish**, we've got what you need. We even have fake **seaweed** for your fishtank (and little **surfboards** too).

Document

$\mathbf{f_i}$

9.7 fish

4.2 tropical

22.1 tropical fish

8.2 seaweed

4.2 surfboards

Topical Features

14 incoming links

3 update count

Quality Features

$\mathbf{g_i}$

fish 5.2

tropical 3.4

tropical fish 9.9

chichlids 1.2

barbs 0.7

Topical Features

incoming links 1.2

update count 0.9

Quality Features

tropical fish

Query

303.01

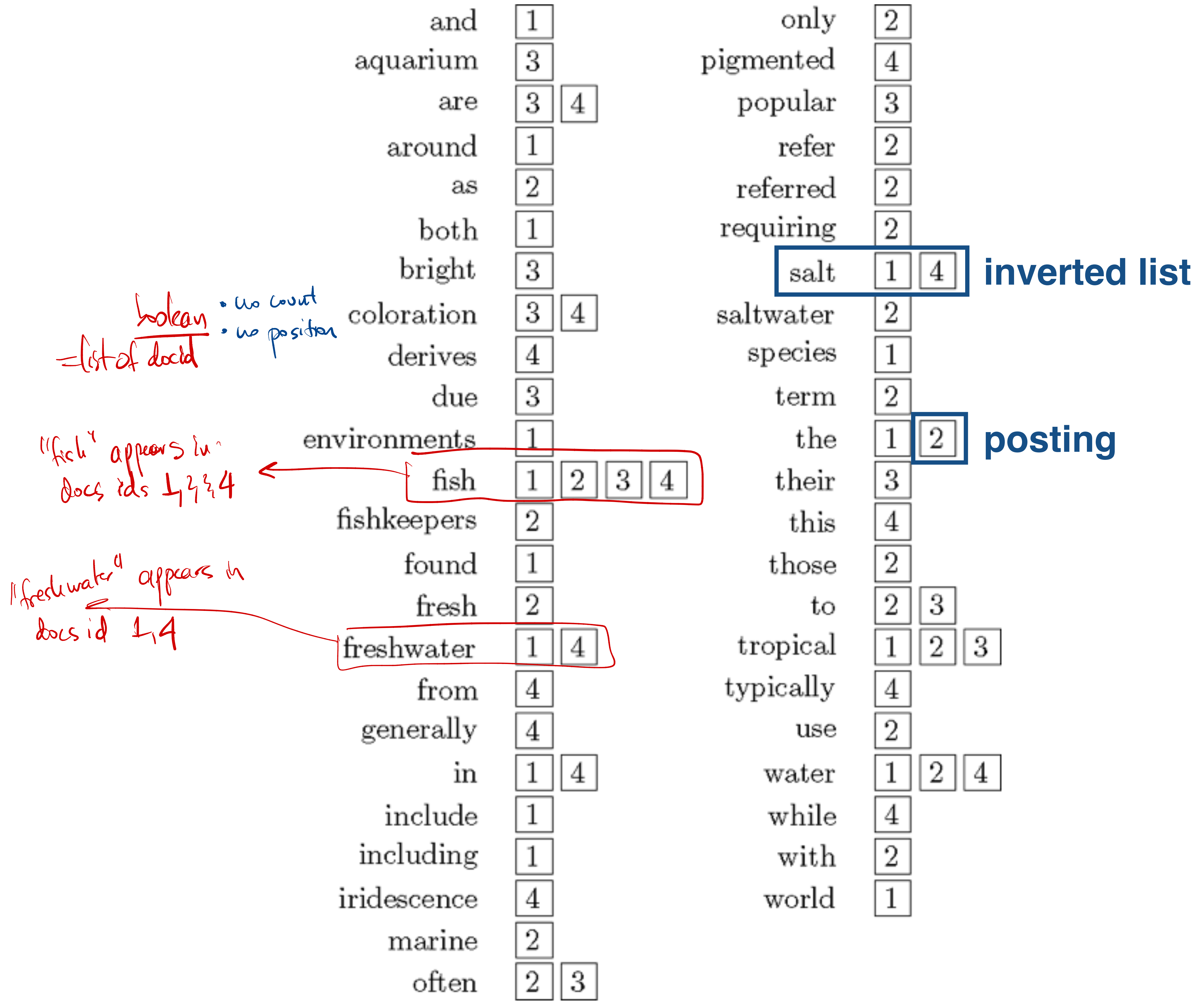
Document Score

Inverted Lists

In an inverted index, each term has an associated **inverted list**.

At minimum, this list contains a list of identifiers for documents which contain that term.

Usually we have more detailed information for each document as it relates to that term. Each entry in an inverted list is called a **posting**.



Term	Postings
and	1
aquarium	3
are	3 4
around	1
as	2
both	1
bright	3
coloration	3 4
derives	4
due	3
environments	1
fish	1 2 3 4
fishkeepers	2
found	1
fresh	2
freshwater	1 4
from	4
generally	4
in	1 4
include	1
including	1
iridescence	4
marine	2
often	2 3
only	2
pigmented	4
popular	3
refer	2
referred	2
requiring	2
salt	1 4
saltwater	2
species	1
term	2
the	1 2
their	3
this	4
those	2
to	2 3
tropical	1 2 3
typically	4
use	2
water	1 2 4
while	4
with	2
world	1

inverted list
posting
boolean
= list of docid
• no count
• no position
"fish" appears in docs ids 1,2,3,4
"freshwater" appears in docs id 1,4

Simple Inverted Index

Inverted Index with Counts

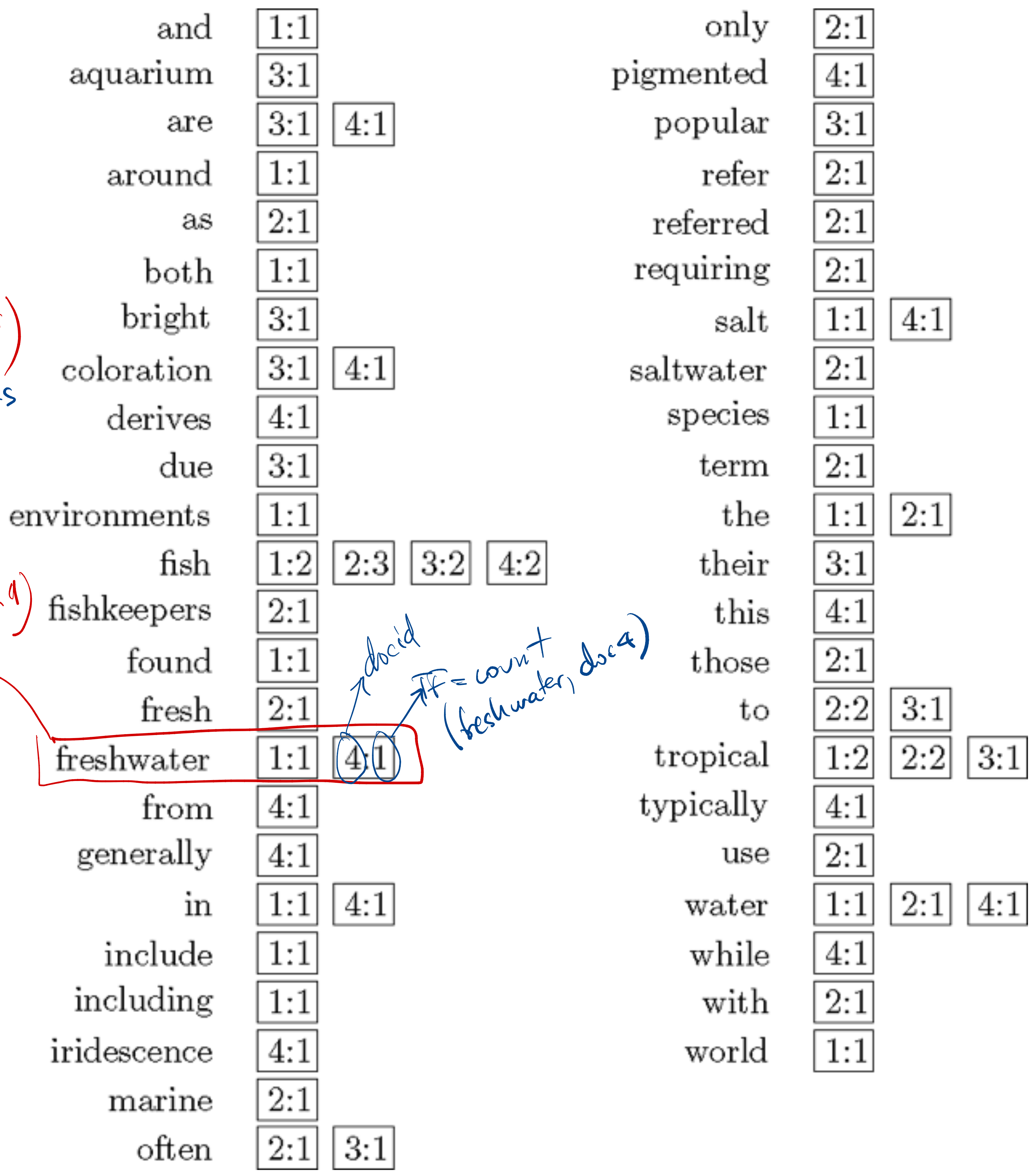
Document postings can store any information needed for efficient ranking.

For instance, they typically store term counts for each document – $tf_{w,d}$.

Depending on the underlying storage system, it can be expensive to increase the size of a posting. It's important to be able to efficiently scan through an inverted list, and it helps if they're small.

(default)
TF inv list (docids counts)
• no positions

inv. list ("freshwater")
docid : count



and	1:1			
aquarium	3:1			
are	3:1	4:1		
around	1:1			
as	2:1			
both	1:1			
bright	3:1			
coloration	3:1	4:1		
derives	4:1			
due	3:1			
environments	1:1			
fish	1:2	2:3	3:2	4:2
fishkeepers	2:1			
found	1:1			
fresh	2:1			
freshwater	1:1	4:1		
from	4:1			
generally	4:1			
in	1:1	4:1		
include	1:1			
including	1:1			
iridescence	4:1			
marine	2:1			
often	2:1	3:1		

only	2:1		
pigmented	4:1		
popular	3:1		
refer	2:1		
referred	2:1		
requiring	2:1		
salt	1:1	4:1	
saltwater	2:1		
species	1:1		
term	2:1		
the	1:1	2:1	
their	3:1		
this	4:1		
those	2:1		
to	2:2	3:1	
tropical	1:2	2:2	3:1
typically	4:1		
use	2:1		
water	1:1	2:1	4:1
while	4:1		
with	2:1		
world	1:1		

docid
TF = count (freshwater, doc4)

Inverted Index with Counts

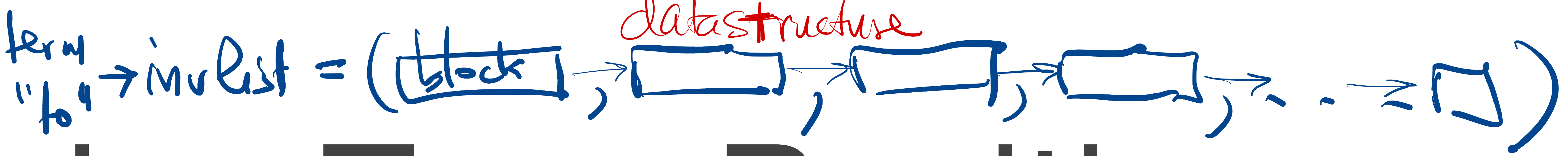
Indexing Additional Data

The information used to support all modern search features can grow quite complex.

Locations, dates, usernames, and other metadata are common search criteria, especially in search functions of web and mobile applications.

When these fields contain text, they are ultimately stored using the same inverted list structure.

Next, we'll see how to compress inverted lists to reduce storage needs and filesystem I/O.



Indexing Term Positions

block content?

Many scoring functions assign higher scores to documents containing the query terms in closer proximity.

Some query languages allow users to specify proximity requirements, like "tropical NEAR fish."

In the inverted lists to the right, the word "to" has a DF of 993,427. It is found in five documents; its TF in doc 1 is 6, and the list of positions is given.

pos-inv-list
docids, count, positions

→ term
→ termid = integer = hashing value
→ docid
→ count TF(term, docid)
→ positions of term in doc

posting
block
block(term, doc)

to, 993427:
⟨1, 6: ⟨7, 18, 33, 72, 86, 231⟩;
2, 5: ⟨1, 17, 74, 222, 255⟩;
4, 5: ⟨8, 16, 190, 429, 433⟩;
5, 2: ⟨363, 367⟩;
7, 3: ⟨13, 23, 191⟩; . . .⟩

be, 178239:
⟨1, 2: ⟨17, 25⟩;
4, 5: ⟨17, 191, 291, 430, 434⟩;
5, 3: ⟨14, 19, 101⟩; . . .⟩

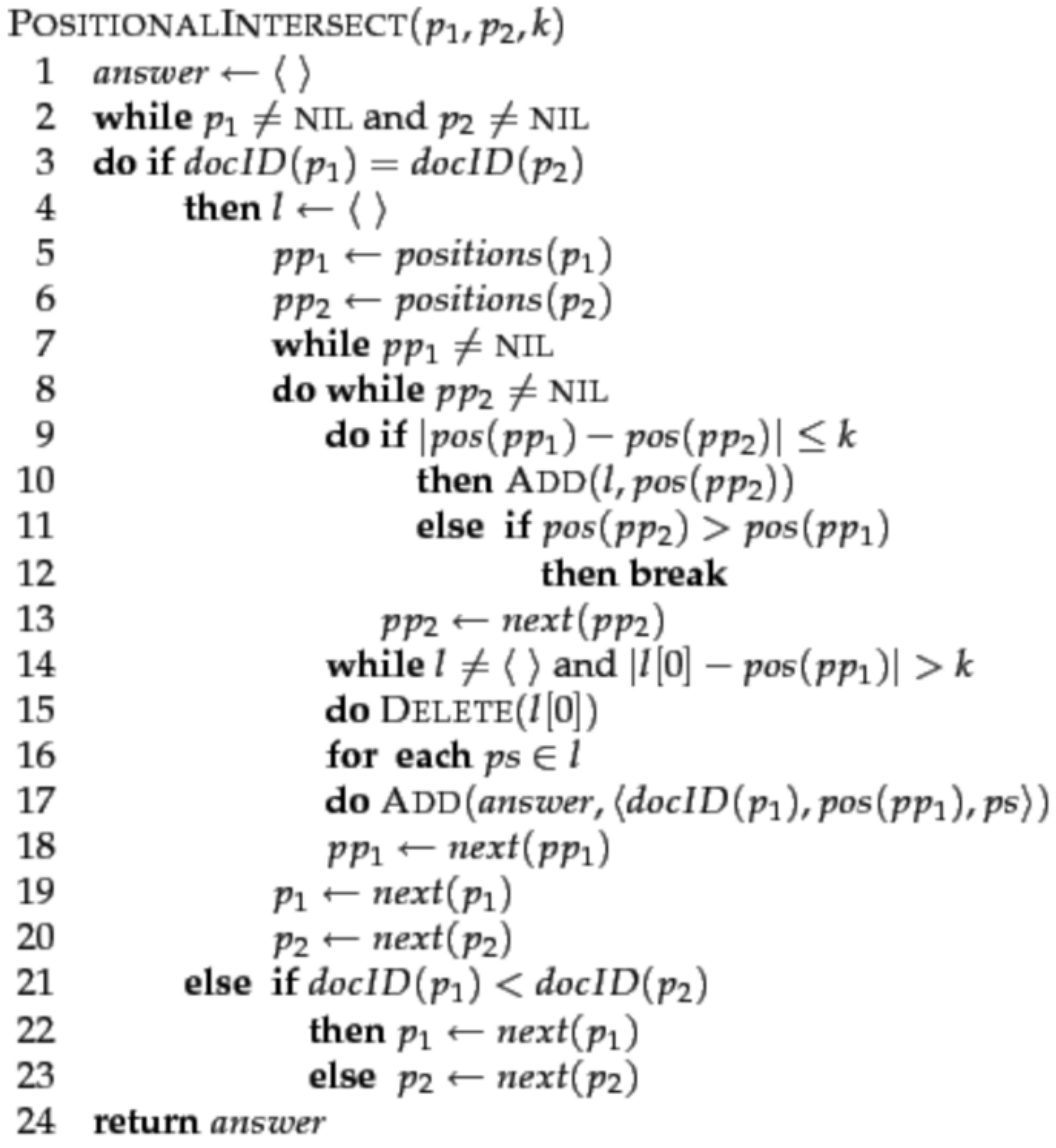
Postings with DF, TF, and Positions

Proximity Searching

In proximity search, you search for documents where terms are sufficiently close to each other.

We process terms from least to most common in order to minimize the number of documents processed.

The algorithm shown here finds documents from two inverted lists where the terms are within k words of each other.



```
POSITIONALINTERSECT($p_1, p_2, k$)
  $answer \leftarrow \langle\, \rangle$
  while $p_1 \neq$ NIL and $p_2 \neq$ NIL
  do if $docID(p_1) = docID(p_2)$
        then $l \leftarrow \langle\, \rangle$
             $pp_1 \leftarrow positions(p_1)$
             $pp_2 \leftarrow positions(p_2)$
             while $pp_1 \neq$ NIL
             do while $pp_2 \neq$ NIL
                  do if $|pos(pp_1) - pos(pp_2)| \leq k$
                       then ADD($l, pos(pp_2)$)
                       else if $pos(pp_2) > pos(pp_1)$
                              then break
                     $pp_2 \leftarrow next(pp_2)$
                while $l \neq \langle\, \rangle$ and $|l[0] - pos(pp_1)| > k$
                do DELETE($l[0]$)
                for each $ps \in l$
                do ADD($answer, \langle docID(p_1), pos(pp_1), ps \rangle$)
                $pp_1 \leftarrow next(pp_1)$
             $p_1 \leftarrow next(p_1)$
             $p_2 \leftarrow next(p_2)$
        else if $docID(p_1) < docID(p_2)$
               then $p_1 \leftarrow next(p_1)$
               else $p_2 \leftarrow next(p_2)$
  return $answer$
```

Algorithm for Proximity Search

Indexing Scores

For some search applications, it’s worth storing the document’s matching score for a term in the posting list.

Postings may be sorted from largest to smallest score, in order to quickly find the most relevant documents. This is especially useful when you want to quickly find the approximate-best documents rather than the exact-best.

Indexing scores makes queries much faster, but gives less flexibility in updating your retrieval function. It is particularly efficient for single term queries.

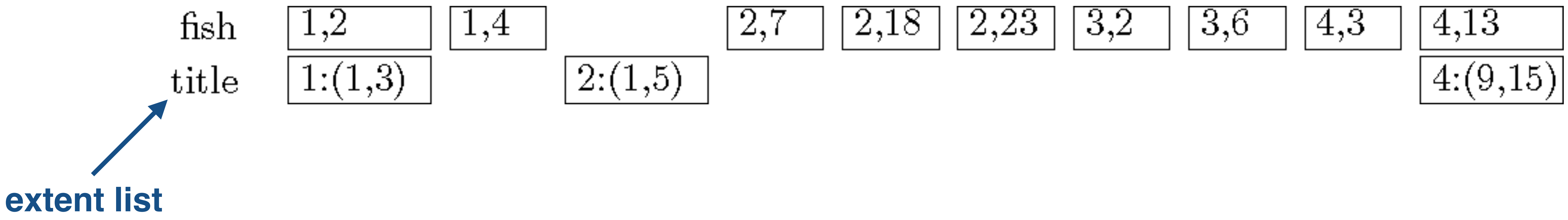
For Machine Learning based retrieval, it’s common to store per-term scores such as BM25 as features.

Fields and Extents

Some indexes have distinct *fields* with their own inverted lists. For instance, an index of e-mails may contain fields for common e-mail headers (from, subject, date, …).

Others store document regions such as the title or headers using *extent lists*.

Extent lists are contiguous regions of a document stored using term positions.



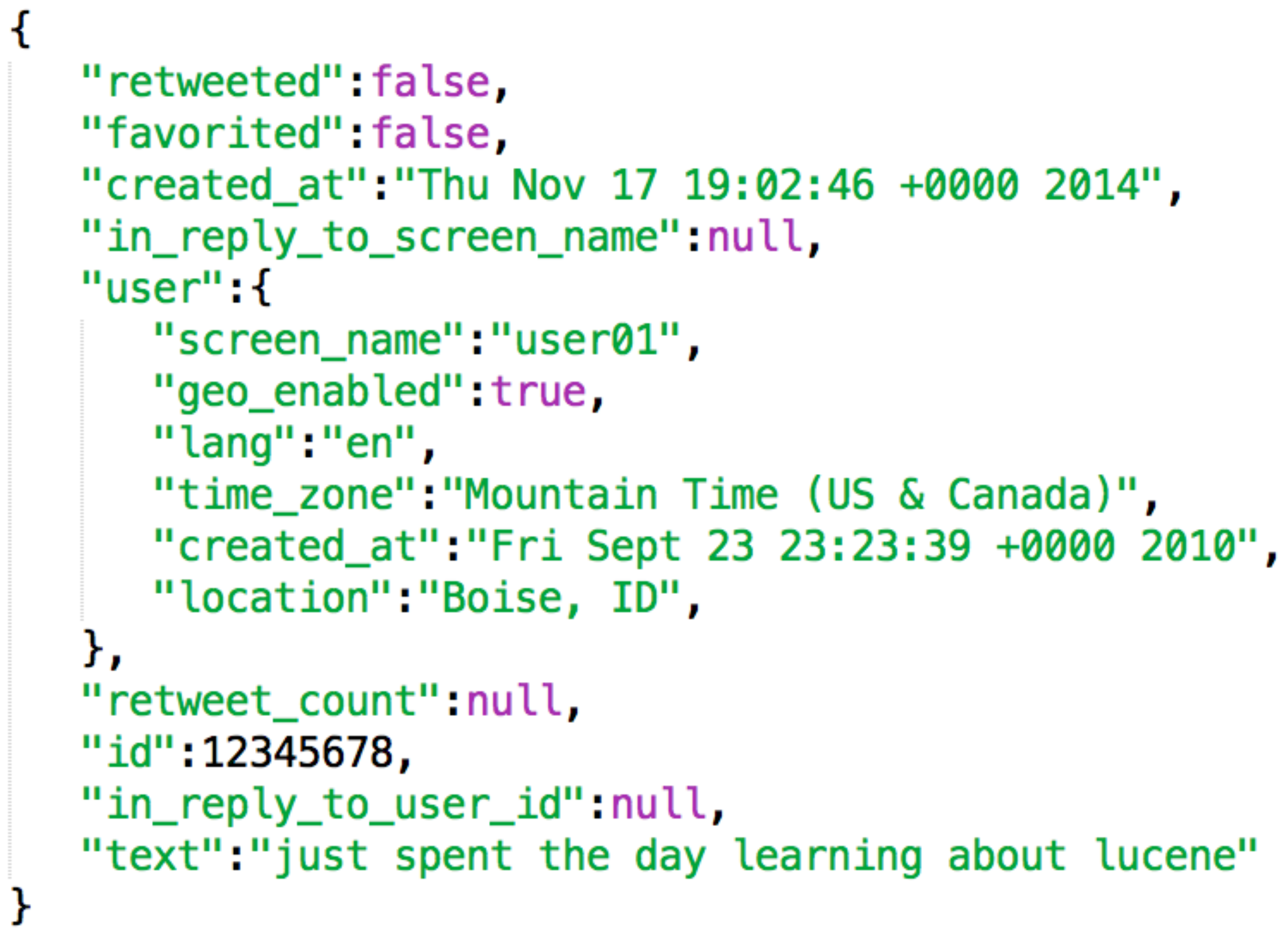
fish	1,2	1,4		2,7	2,18	2,23	3,2	3,6	4,3	4,13
title	1:(1,3)		2:(1,5)							4:(9,15)

Index Schemas

As the information stored in an inverted index grows more complex, it becomes useful to represent it using some form of schema.

However, we normally don't use strict SQL-type schemas, partly due to the cost of rebuilding a massive index. Instead, flexible formats such as `<key, value>` maps with field names arranged by convention are used.

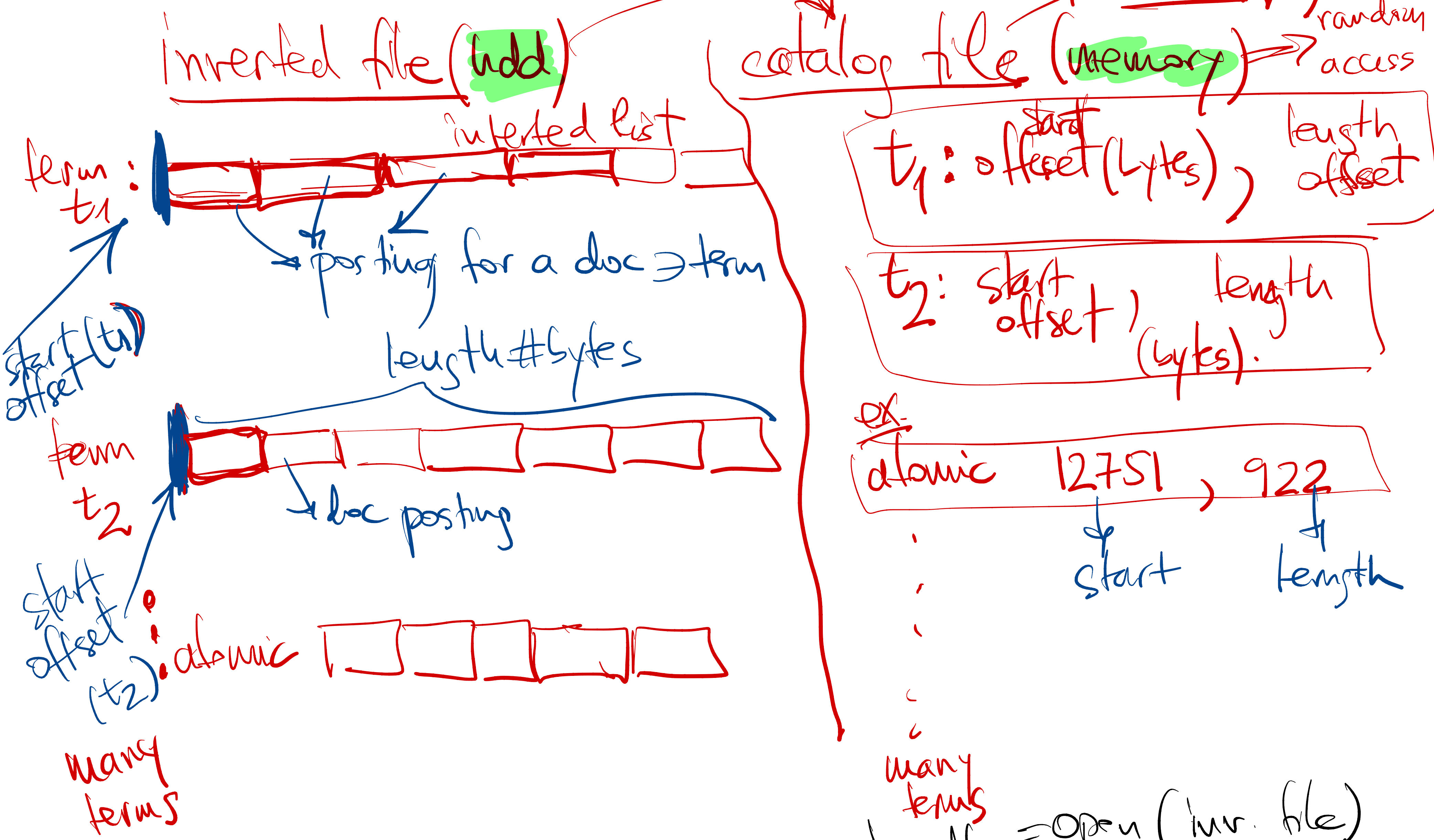
Each text field in the schema typically gets its own inverted lists.



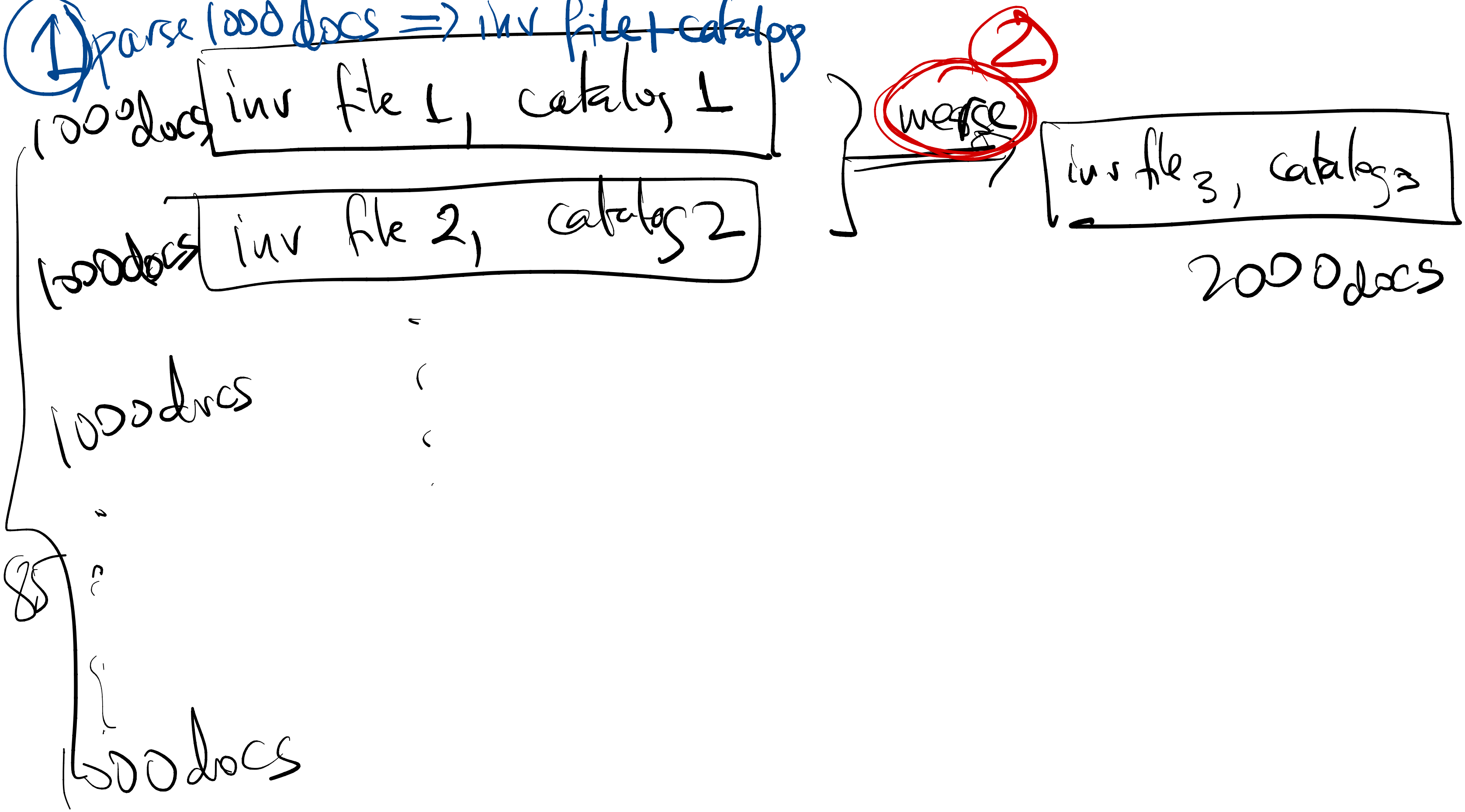
```
{
   "retweeted":false,
   "favorited":false,
   "created_at":"Thu Nov 17 19:02:46 +0000 2014",
   "in_reply_to_screen_name":null,
   "user":{
      "screen_name":"user01",
      "geo_enabled":true,
      "lang":"en",
      "time_zone":"Mountain Time (US & Canada)",
      "created_at":"Fri Sept 23 23:23:39 +0000 2010",
      "location":"Boise, ID",
   },
   "retweet_count":null,
   "id":12345678,
   "in_reply_to_user_id":null,
   "text":"just spent the day learning about lucene"
}
```

Partial JSON Schema for Tweets

(conceptually) inverted index

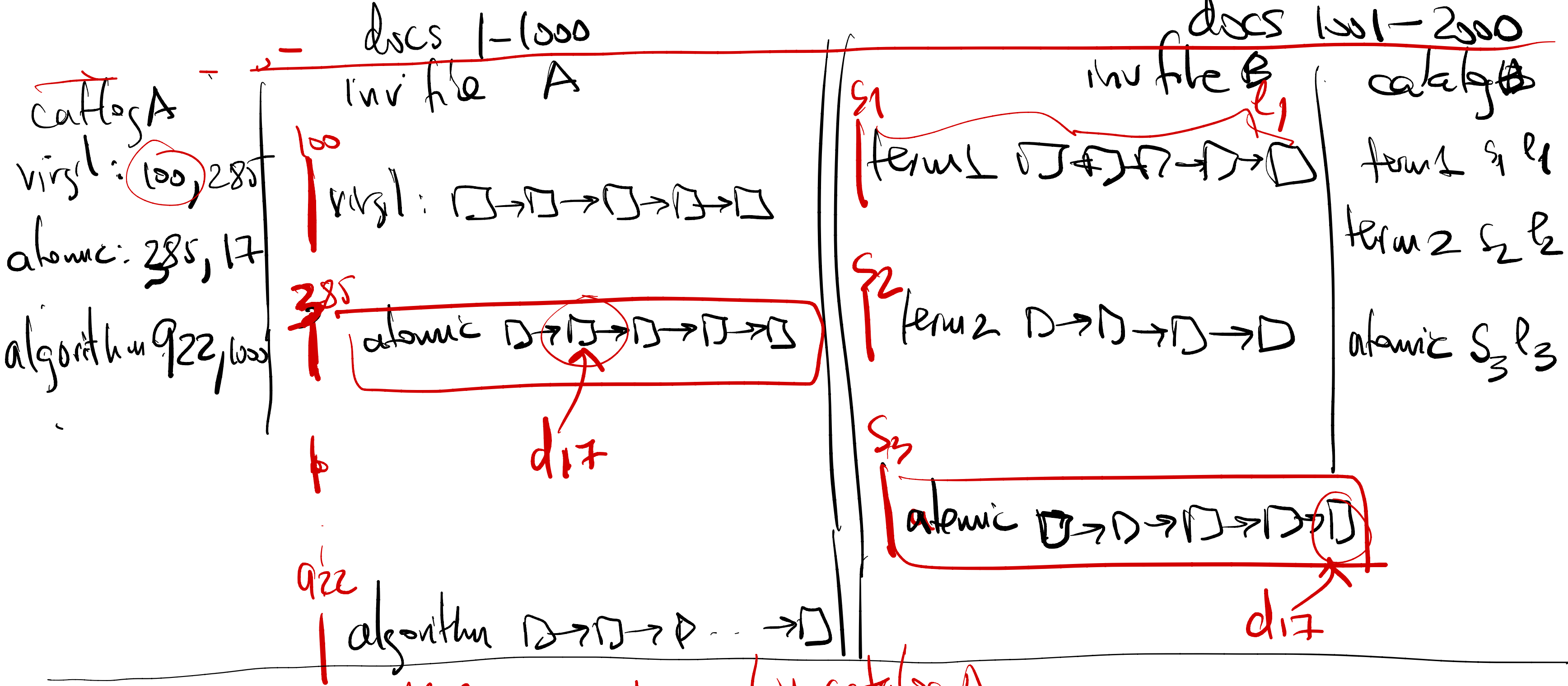


```
handle = open(inv. file)
 - move(handle, start)
 - read( length bytes)
```


1) parse 1000 docs => inv file + catalog
1000 docs
inv file 1, catalog 1
1000 docs
inv file 2, catalog 2
merge
2
inv file 3, catalog 3
2000 docs
1000 docs
85
1000 docs

1 parse docs → Inv file+catalog
1000

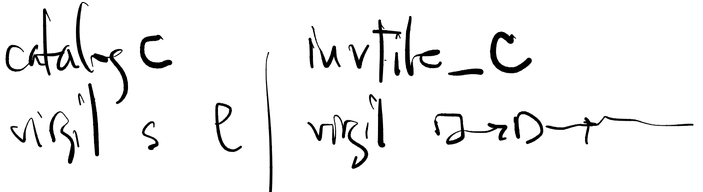
```
for each doc d ∈ set-of 1000
    for each term t ∈ d        // parse the document
        ignore t? yes → next
        block   hash(t)(d)   update → add current position to pos_list
                                    → update TF count
                first key   second key
                                        block operation
done parsing
term: output inv_list(t)
    for each d ∈ keys(hash(t))
        write block hash(t)(d)
    hdd inv file
    t : [d1] → [d2] → [d3] → [ ] → [ ]
    write catalog entry for term
        t, startoffset, length)
    catalog
```



MERGE → recap MERGE procedure from MERGESORT

by catalog A

```
for each term ∈ catalog A
    invlist_A = read_inv_list(t, A)
    if t ∈ catalog B then
        invlist_B = read_inv_list(t, B)
```



atomic	s	e
algorithms	s	e
term1	s	e
term2	s	e

atomic D→D→D

merged

```
inv list_C = MERGE(invlist_A, invlist_B)
write invlist_C to new merge file
  update catalog_C  → on the fly
else write invlist_A into new merge file
  update catalog_C  don't count on cat s,e ∈ A,B
for each term t ∈ catalog B
  if t ∈ catalog A → next;
  inv list_B = read_inv_list(t, B)
  write invlist B, to new merged file
  update catalog C
```

MERGE(invlistA, invlistB)
simply concat works HW2

assuming: no shared docs

not realistic in practice!

- we might not control inv_file A, inv_file B (production)

no assumption on docs indexed:

- concat no good.
- sort docs ids, apply MERGE procedure similar to Mergesort

↳ close (file) ?

Index Construction

We have just scratched the surface of the complexities of constructing and updating large-scale indexes. The most complex indexes are massive engineering projects that are constantly being improved.

An indexing algorithm needs to address hardware limitations (e.g., memory usage), OS limitations (the maximum number of files the filesystem can efficiently handle), and algorithmic concerns.

When considering whether your algorithm is sufficient, consider how it would perform on a document collection a few orders of magnitude larger than it was designed for.

Basic Indexing

Given a collection of documents, how can we efficiently create an inverted index of its contents?

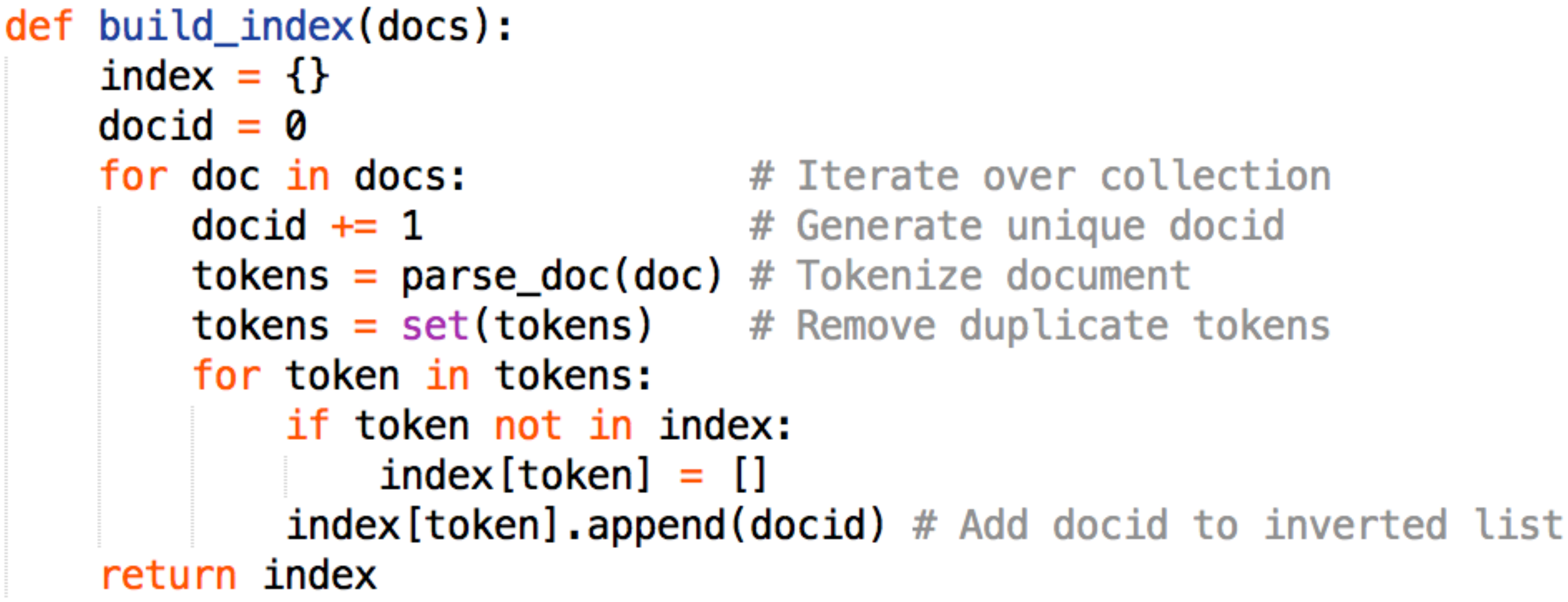
The basic steps are:

1. Tokenize each document, to convert it to a sequence of terms

2. Add doc to inverted list for each token

This is simple at small scale and in memory, but grows much more complex to do efficiently as the document collection and vocabulary grow.

Basic In-Memory Indexer



```
def build_index(docs):
    index = {}
    docid = 0
    for doc in docs:                # Iterate over collection
        docid += 1                  # Generate unique docid
        tokens = parse_doc(doc) # Tokenize document
        tokens = set(tokens)        # Remove duplicate tokens
        for token in tokens:
            if token not in index:
                index[token] = []
            index[token].append(docid) # Add docid to inverted list
    return index
```

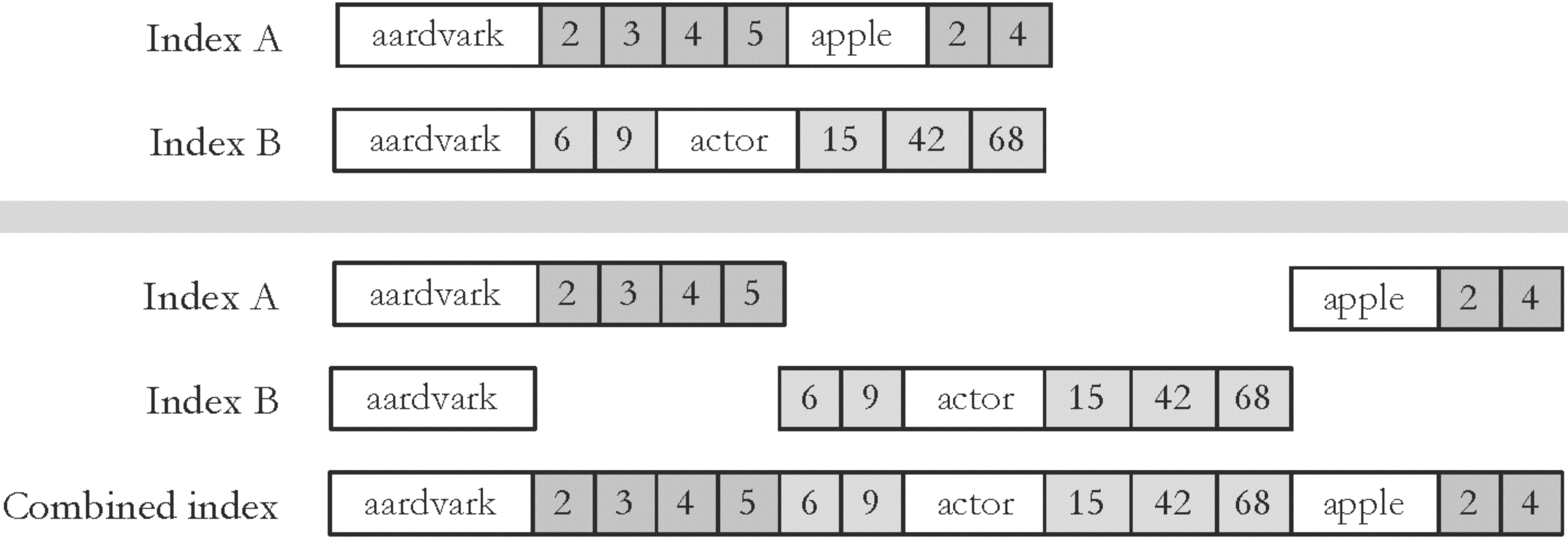
Merging Lists

The basic indexing algorithm will fail as soon as you run out of memory.

To address this, we store a partial inverted list to disk when it grows too large to handle. We reset the in-memory index and start over. When we’re finished, we merge all the partial indexes.

The partial indexes should be written in a manner that facilitates later merging. For instance, store the terms in some reasonable sorted order. This permits merging with a single linear pass through all partial lists.

Merging Example



Index A: aardvark 2 3 4 5 apple 2 4

Index B: aardvark 6 9 actor 15 42 68

Index A: aardvark 2 3 4 5 apple 2 4

Index B: aardvark 6 9 actor 15 42 68

Combined index: aardvark 2 3 4 5 6 9 actor 15 42 68 apple 2 4

Admin 6/3/2020

1 lecture behind => push HW2 deadline next week Wed 6/10?

- Anjur demo ✓
- index construction: encoding / compression / positions bigrams / freq analysis

what to write

≃ zip?

how to write/compress

?

bigrams → index bigrams as tokens

bigrams →
- index words pos
- compute bigrams on the fly

how difficult comput?

HW2
- encoding

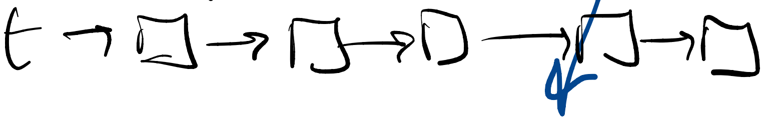
- positions n grams
- freq (natural freq) Heap's Zipf's Laws
- compression

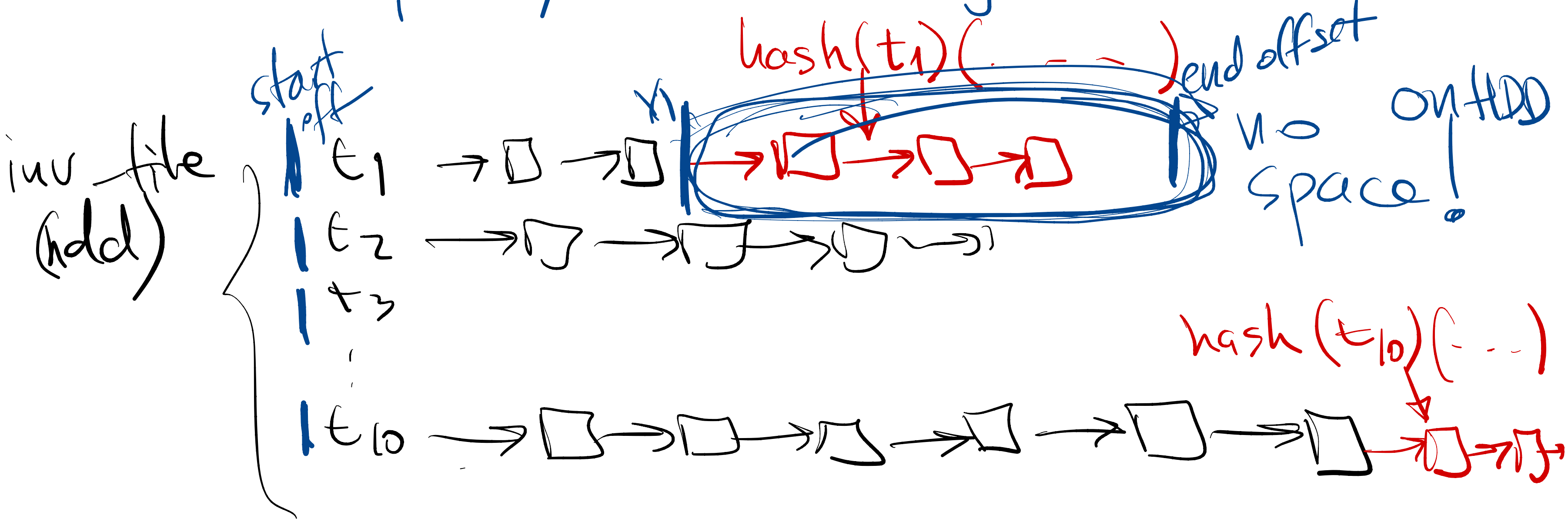
WRONG!

HW2 naive non-merge?

inv_file = empty
catalog = empty

```
For each batch of 1000 docs          85 batches
    hash = empty
    - ( parse, store data in hash(t)(d) = inv_list_block(t,d)
        //same last time
    - add hash(t)(d) to the existing invfile
    - update/rewrite catalog
```





Result Merging

An index can be updated from a new batch of documents by merging the posting lists from the new documents. However, this is inefficient for small updates.

Instead, we can run a search against both old and new indexes and merge the *result lists* at search time. Once enough changes have accumulated, we can merge the old and new indexes in a large batch.

In order to handle deleted documents, we also need to maintain a *delete list* of docids to ignore from the old index. At search time, we simply ignore postings from the old index for any docid in the delete list.

If a document is modified, we place its docid into the delete list and place the new version in the new index.

Updating Indexes

If each term's inverted list is stored in a separate file, updating the index is straightforward: we simply merge the postings from the old and new index.

However, most filesystems can't handle very large numbers of files, so several inverted lists are generally stored together in larger files. This complicates merging, especially if the index is still being used for query processing.

There are ways to update live indexes efficiently, but it's often simpler to simply write a new index, then redirect queries to the new index and delete the old one.

Compressing Indexes

The best any compression scheme can do depends on the entropy of the probability distribution over the data. More random data is less compressible.

Huffman Codes meet the entropy limit and can be built in linear time, so are a common choice. Other schemes can do better, generally by interpreting the input sequence differently (e.g. encoding sequences of characters as if they were a single input symbol – different distribution, different entropy limit).

Index Size

Inverted lists often consume a large amount of space.

- e.g., 25-50% of the size of the raw documents for TREC collections with the Indri search engine

- much more than the raw documents if n-grams are indexed

Compressing indexes is important to conserve disk and/or RAM space. Inverted lists have to be decompressed to read them, but there are fast, lossless compression algorithms with good compression ratios.

frequency → 1) efficiency 2) security

vs good + coding theory: pipeline

term value object → number → write/encode number (bits)

fixed length vs

restricted variable length codes

chars encode

= ability to decode.

- an extension of multicase encodings ("shift key") where different code lengths are used for each case. Only a few code lengths are chosen, to simplify encoding and decoding.

Choice ≤ code (theory) principle

- high freq(x) ⇒ short encode (x)

- low freq(x) ⇒ long encode (x)

- Use first bit to indicate case.

control

e a r

0000, 0001, 0010, 0011,

2^3 =

- 8 most frequent characters fit in 4 bits (0xxx). 0100, 0101, 0110, 0111 3 bits for value

2^7 =

- 128 less frequent characters fit in 8 bits (1xxxxxxx)

1000000, 1000001, 1000010, . . - 7 bits for value

- In English, 7 most frequent characters are 65% of occurrences
- Expected code length is approximately 5.4 bits per character, for a 32.8% compression ratio.

control control

- average code length on WSJ89 is 5.8 bits per character, for a 27.9% compression ratio

decode(110010 , 0010100111001 - - -)

next 7 bits 3 bits

restricted variable length codes: more symbols

- Use more than 2 cases.
- 1xxx for 2^3 = 8 most frequent symbols, and
- 0xxx1xxx for next 2^6 = 64 symbols, and
- 0xxx0xxx1xxx for next 2^9 = 512 symbols, and
- ...
- average code length on WSJ89 is 6.2 bits per symbol, for a 23.0% compression ratio.
- Pro: Variable number of symbols.
- Con: Only 72 symbols in 1 byte.

schema/code

1xxx ← 8 most freq (4 bits)

0xxx1xxx ← 6 bits value => 64 objects

0 xxx 0xxx 1xxx 9 bits val => 512 obj

0xxx0xxx0xxx1xxx (12 bits => 2^{12} obj

0 xxx 0xxx 0xxx 0xxx 1xxx ← least freq

DECODE

0101 1001 1110 0101 0110 1101 - - ——

advance value bits | stop 101001 | stop value 110 | adv adv 1=stop value bits 101110101

UBYTE: store all control bits first 0001 xxx xxx xxx xxx control value

restricted variable length codes : numeric data

- 1xxxxxxx for 2^7 = 128 most frequent symbols
- 0xxxxxxx1xxxxxxx for next 2^{14} = 16,384 symbols
- ...
- average code length on WSJ89 is 8.0 bits per symbol, for a 0.0% compression ratio (!!).
- Pro: Can be used for integer data
    - Examples: word frequencies, inverted lists

restricted variable –length codes : word based encoding

- Restricted Variable-Length Codes can be used on words (as opposed to symbols)
- build a dictionary, sorted by word frequency, most frequent words first
- Represent each word as an offset/index into the dictionary
- Pro: a vocabulary of 20,000-50,000 words with a Zipf distribution requires 12-13 bits per word
    - compared with a 10-11 bits for completely variable length
- Con: The decoding dictionary is large, compared with other methods.

restricted variable-length codes: summary

- Four methods presented. all are
    - simple
    - very effective when their assumptions are correct
- No assumptions about language or language models
- all require an unspecified mapping from symbols to numbers (a dictionary)
- all but the basic method can handle any size dictionary

next time : Zip (Lempel-Ziv) ; Huffman better than RVL

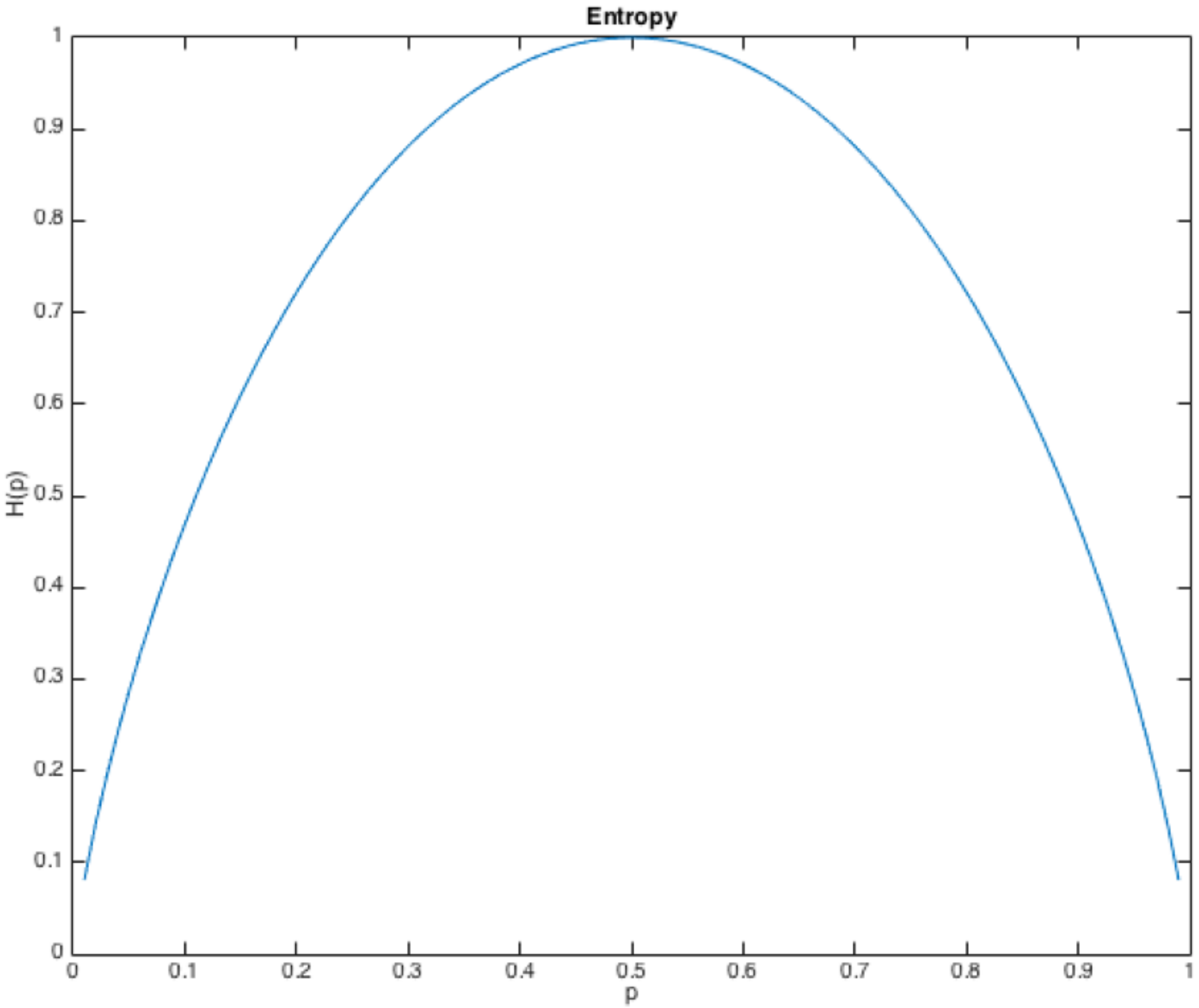
Entropy and Compressibility

The **entropy** of a probability distribution is a measure of its randomness.

$$H(p) = -\sum_i p_i \log p_i$$

The more random a sequence of data is, the less predictable and less compressible it is.

The entropy of the probability distribution of a data sequence provides a bound on the best possible compression ratio.



Entropy of a Binomial Distribution

greedy

Huffman Codes

- no loss encoding
- able to decode

variable length => prefix free code

In an ideal encoding scheme, a symbol with probability p_i of occurring will be assigned a code which takes $\log(p_i)$ bits.

The more probable a symbol is to occur, the smaller its code should be. By this view, UTF-32 assumes a uniform distribution over all unicode symbols; UTF-8 assumes ASCII characters are more common.

Huffman Codes achieve the best possible compression ratio when the distribution is known and when no code can stand for multiple symbols.

Symbol	p	Code	$\mathbb{E}[length]$
a	1/2	0	0.5
b	1/4	10	0.5
c	1/8	110	0.375
d	1/16	1110	0.25
e	1/16	1111	0.25

Plaintext: aedbbaae (64 bits in UTF-8)

Ciphertext: 0111111101010001111

a e d

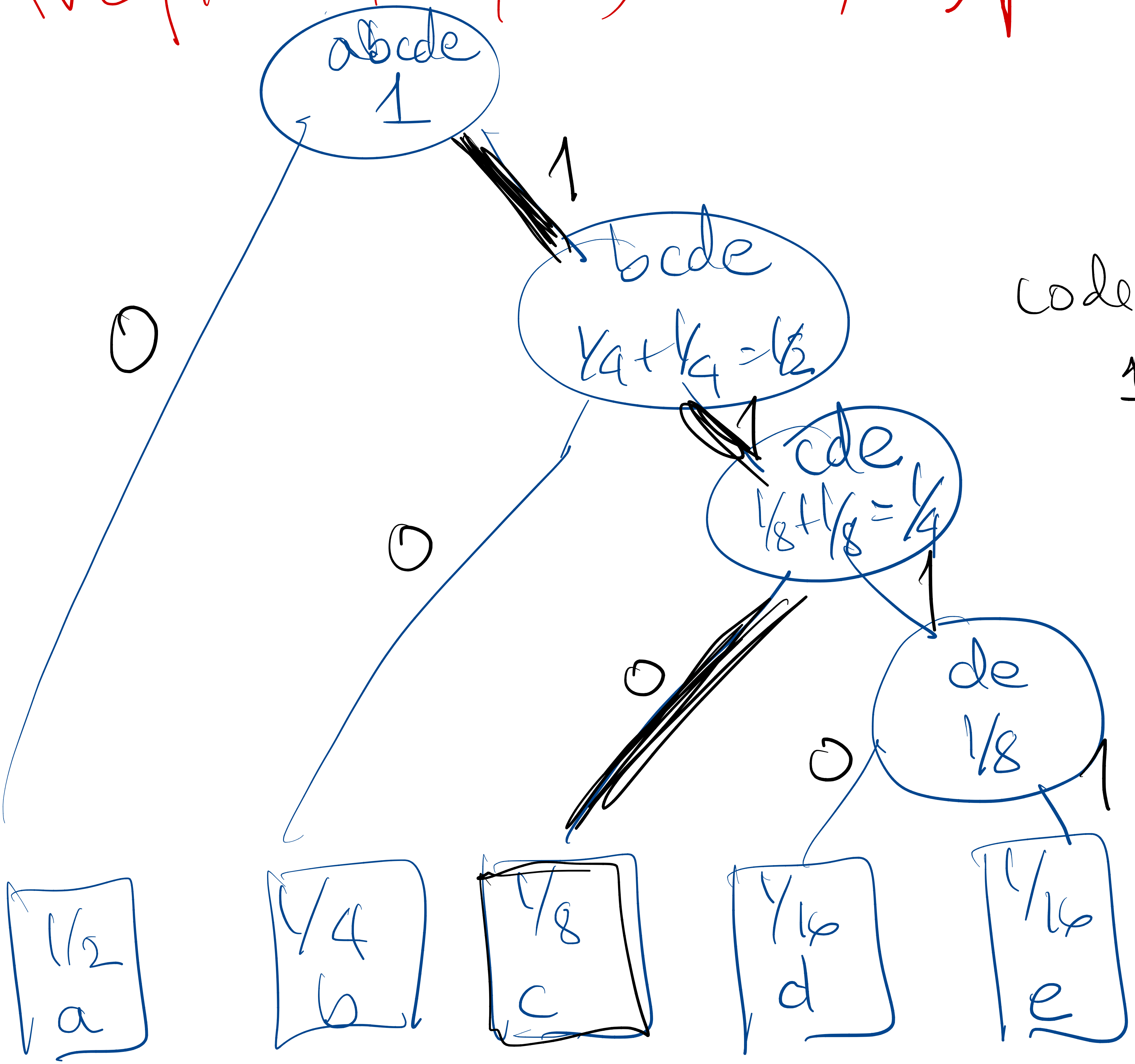
no code is prefix to another code!

not nec. easy
- proof of optimality

AVG(#bits per symbol) $\simeq$ H(symbols) entropy

easy
- proof of correctness

⇒ prefix free



- left branch = 0
- right branch = 1

code(c) = path to c

110

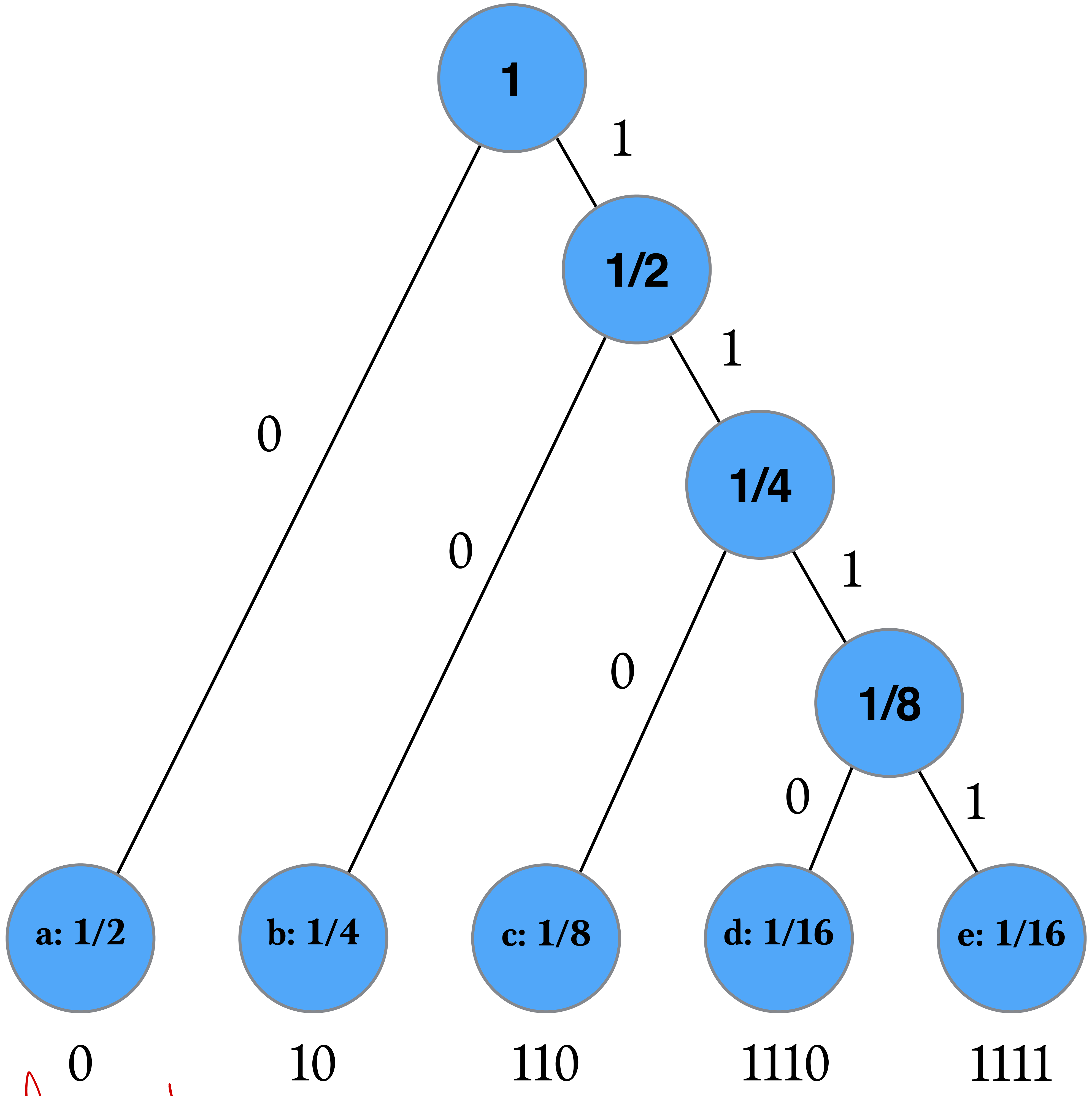
greedy
~~bottom-up~~

- add up the 2 smalles

Building Huffman Codes

Huffman Codes are built using a binary tree which always joins the least probable remaining nodes.

1. Create a leaf node for each symbol, weighted by its probability.
2. Iteratively join the two least probable nodes without a parent by creating a parent whose weight is the sum of the childrens' weights.
3. Assign 0 and 1 to the edges from each parent. The code for a leaf is the sequence of edges on the path from the root.



- we need dictionary map symbl ↔ code both at encode and decodings

Can We Do Better?

Huffman codes achieve the theoretical limit for compressibility, assuming that the size of the code table is negligible and that each input symbol must correspond to exactly one output symbol.

Other codes, such as Lempel-Ziv encoding, allow variable-length sequences of input symbols to correspond to particular output symbols and do not require transferring an explicit code table.

Compression schemes such as gzip are based on Lempel-Ziv encoding. However, for encoding inverted lists it can be beneficial to have a 1:1 correspondence between code words and plaintext characters.

built in dictionary

Lempel-Ziv

- an adaptive dictionary approach to variable length coding.
- Use the text already encountered to build the dictionary.
- If text follows Zipf's laws, a good dictionary is built.
- No need to store dictionary; encoder and decoder each know how to build it on the fly.
- Some variants: LZ77, Gzip, LZ78, LZW, Unix **compress**
- Variants differ on:
  - how dictionary is built,
  - how pointers are represented (encoded), and
  - limitations on what pointers can refer to.

Lempel Ziv: encoding

raw

- 0010111010010111011011

no-loss encoding

Lempel Ziv: encoding

- 0010111010010111011011

analogy with Δencode

1 7 9 13 ...

Δ: 1 6 2 4 ...

- break into known prefixes

block = prefix + 1 bit

new bit

new bit

- 0|01 |011|1 |010|0101|11|0110|11

prefix known

prefix

no prefix

prefix

prefix ∼ go back 3

prefix ∼ go-back-one

idea: encode prefix (known) as a backpointer

Lempel Ziv: encoding

- 00101110100101111011011

- break into known prefixes
- 0|01 |011|1 |010|0101|11|0110|11

- encode references as pointers
- 0|1,1|1,1 |0,1|3,0 |1,1 |3,1|5,0 |2,?

Lempel Ziv: encoding

- 0010111010010111011011
- break into known prefixes
- 0|01 |011|1 |010|0101|11|0110|11
- encode references as pointers
- 0|1,1|1,1 |0,1|3,0 |1,1 |3,1|5,0 |2,?
- encode the pointers with log(?)bits
- 0|1,1|01,1 |00,1|011,0 |001,1 |011,1|101,0 |0010,?

Lempel Ziv: encoding

- 0010111010010111011011
- break into known prefixes: 0|01 |011|1 |010|0101|11|0110|11
- encode references as pointers : 0|1,1|1,1 |0,1|3,0 |1,1 |3,1|5,0 |2,?
- encode the pointers with log(?)bits :

  0|1,1|01,1 |00,1|011,0 |001,1 |011,1|101,0 |0010,?

- final string : 01101100101100011011110100010

Lempel Ziv: decoding

- 01101100101100011011110100010

Lempel Ziv: decoding

- 01101100101100011011110100010

- decode the pointers with log(?)bits

- 0|1,1|01,1 |00,1|011,0 |001,1 |011,1|101,0 |0010,?

Lempel Ziv: decoding

- 01101100101100011011110100010

- decode the pointers with log(?)bits
- 0|1,1|01,1 |00,1|011,0 |001,1 |011,1|101,0 |0010,?

- encode references as pointers
- 0|1,1|1,1 |0,1|3,0 |1,1 |3,1|5,0 |2,?

Lempel Ziv: decoding

- 01101100101100011011110100010

- decode the pointers with log(?)bits

- 0|1,1|01,1 |00,1|011,0 |001,1 |011,1|101,0 |0010,?

- encode references as pointers

- 0|1,1|1,1 |0,1|3,0 |1,1 |3,1|5,0 |2,?

- decode references

- 0|01 |011|1 |010|0101|11|0110|11

Lempel Ziv: decoding

- 01101100101100011011110100010
- decode the pointers with log(?) bits : 0l1,1l01,1 l00,1l011,0 l001,1 l011,1l101,0 l0010,?
- encode references as pointers : 0l1,1l1,1 l0,1l3,0 l1,1 l3,1l5,0 l2,?
- decode references : 0l01 l011l1   l010l0101l11l0110l11
- original string : 0010111010010111011011

Lempel Ziv optimality

- LempelZiv compression rate approaches (asymptotic) entropy
  - When the strings are generated by an ergodic source [CoverThomas91].
  - easier proof : for i.i.d sources
    - that is not a good model for English

LempelZiv optimality –i.i.d source

- let $x = \alpha_1 \alpha_2 ... \alpha_n$ a sequence of length n generated by a iid source and Q(x) = the probability to see such a sequence

- say LempelZiv breaks into c phrases $x = y_1 y_2 ... y_c$ and call $c_l = \#$ of phrases of length l then $-\log Q(x) \geq \sum\limits_l c_l \log c_l$
(proof) $\sum\limits_{|y_i|=l} Q(y_i) < 1$ so $\prod\limits_{|y_i|=l} Q(y_i) < (\frac{1}{c_l})^{c_l}$

- if p_i is the source probab for α_i then by law of large numbers x will have roughly np_i occurrences of α_i and then
$logQ(x) = -\log \prod\limits_i p_i^{np_i} \approx n \sum p_i \log p_i = nH_{source}$

- note that $\sum\limits_l c_l \log c_l$ is roughly the LempelZiv encoding length so th einequality reads
$nH \geq\approx LZencoding$ which is to say $H \approx\geq LZrate$.

Bit-aligned Codes

Bit-aligned codes allow us to minimize the storage used to encode integers.

We can use just a few bits for small integers, and still represent arbitrarily large numbers.

Inverted lists can also be made more compressible by delta-encoding their contents.

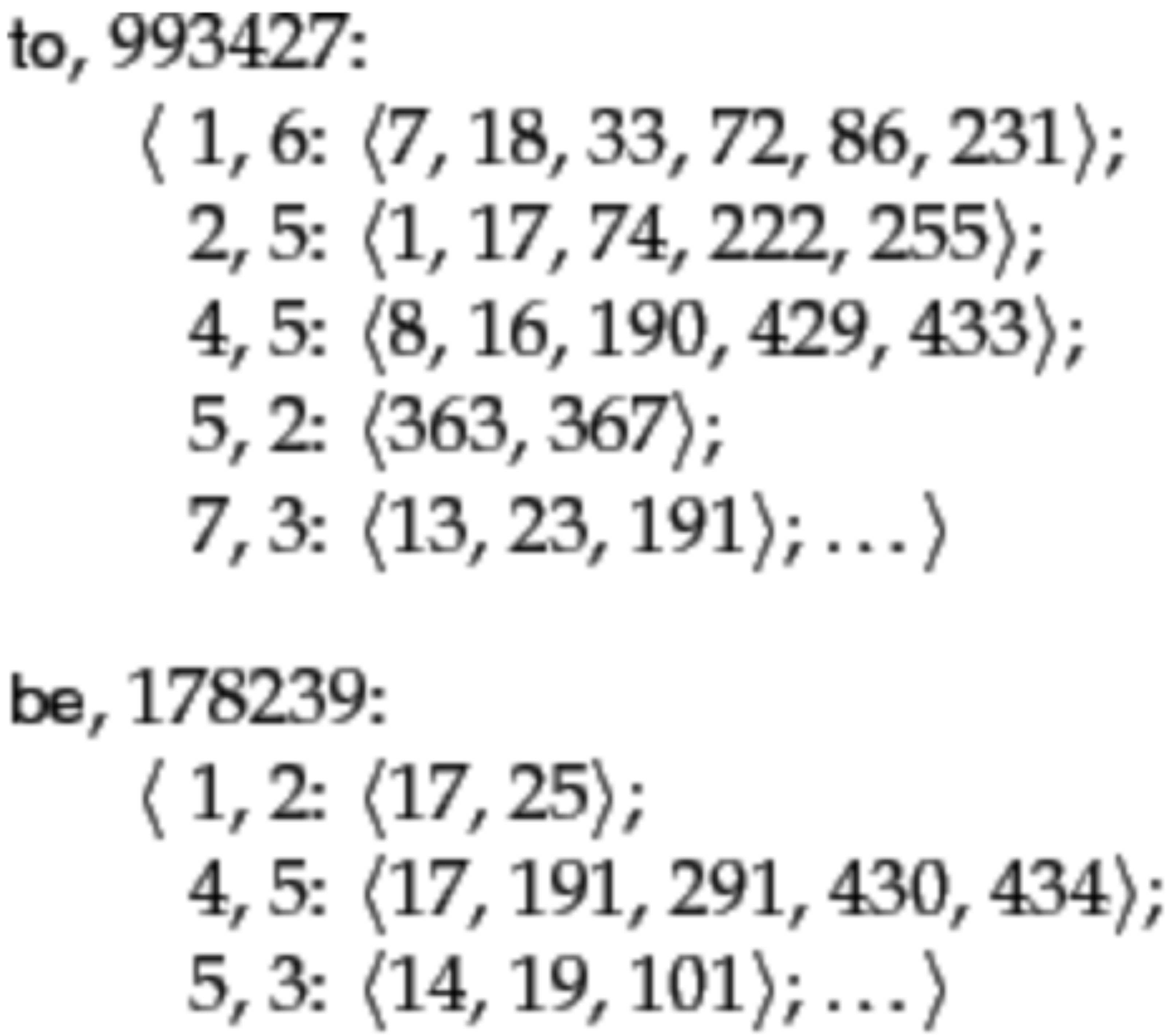
Next, we'll see how to encode integers using a variable byte code, which is more convenient for processing.

Compressing Inverted Lists

An inverted list is generally represented as multiple sequences of integers.

- Term and document IDs are used instead of the literal term or document URL/path/name.
- TF, DF, term position lists and other data in the inverted lists are often integers.

We'd like to efficiently encode this integer data to help minimize disk and memory usage. But how?



to, 993427:
⟨ 1, 6: ⟨7, 18, 33, 72, 86, 231⟩;
2, 5: ⟨1, 17, 74, 222, 255⟩;
4, 5: ⟨8, 16, 190, 429, 433⟩;
5, 2: ⟨363, 367⟩;
7, 3: ⟨13, 23, 191⟩; . . . ⟩

be, 178239:
⟨ 1, 2: ⟨17, 25⟩;
4, 5: ⟨17, 191, 291, 430, 434⟩;
5, 3: ⟨14, 19, 101⟩; . . . ⟩

Postings with DF, TF, and Positions

Unary

The encodings used by processors for integers (e.g., two's complement) use a fixed-width encoding with fixed upper bounds. Any number takes 32 (say) bits, with no ability to encode larger numbers.

Both properties are bad for inverted lists. Smaller numbers tend to be much more common, and should take less space. But very large numbers can happen – consider term positions in very large files, or document IDs in a large web collection.

What if we used a **unary** encoding? This encodes k by k `1`s, followed by a `0`.

decimal	binary	unary
0	00000000	0
1	00000001	10
7	00000111	11111110
13	00001101	11111111111110

Elias-ɣ Codes

Unary is efficient for small numbers, but very inefficient for large numbers. There are better ways to get a variable bit length.

With Elias-ɣ codes, we use unary to encode the bit length and then store the number in binary.

To encode a number k, compute:

$$\begin{cases} k_d = \lfloor \log_2 k \rfloor \\ k_r = k - \boxed{2^{\boxed{\lfloor \log_2 k \rfloor}}} \end{cases}$$

base 2 log

diff to power of 2

K=15

$kd = \lfloor \log_2 15 \rfloor = 3$

$Kr = 15 - 2^3 = 7$

Decimal	k_d	k_r	Code
1	0	0	0
2	1	0	10 0
3	1	1	10 1
6	2	2	d r 110 10
15	$\lfloor \log_2 15 \rfloor$ 3	7	1110 111
16	4	0	11110 0000
255	7	127	11111110 1111111
1023	9	511	1111111110 111111111

→ bits

→ decode

control

binary - how many bits

3 = unary

binary

binary part(kr) uses kd bits (enough ?

$\underbrace{1111},0\,\underbrace{1101}$ | kd=4
kr=13
value=2^{kd}+kr=29

Elias-δ Codes

Elias-ɣ codes take $2\lfloor\log_2 k\rfloor+1$ bits. We can do better, especially for large numbers.

Elias-δ codes encode k_d using an Elias-ɣ code, and take approximately $2\log_2\log_2 k+\log_2 k$ bits.

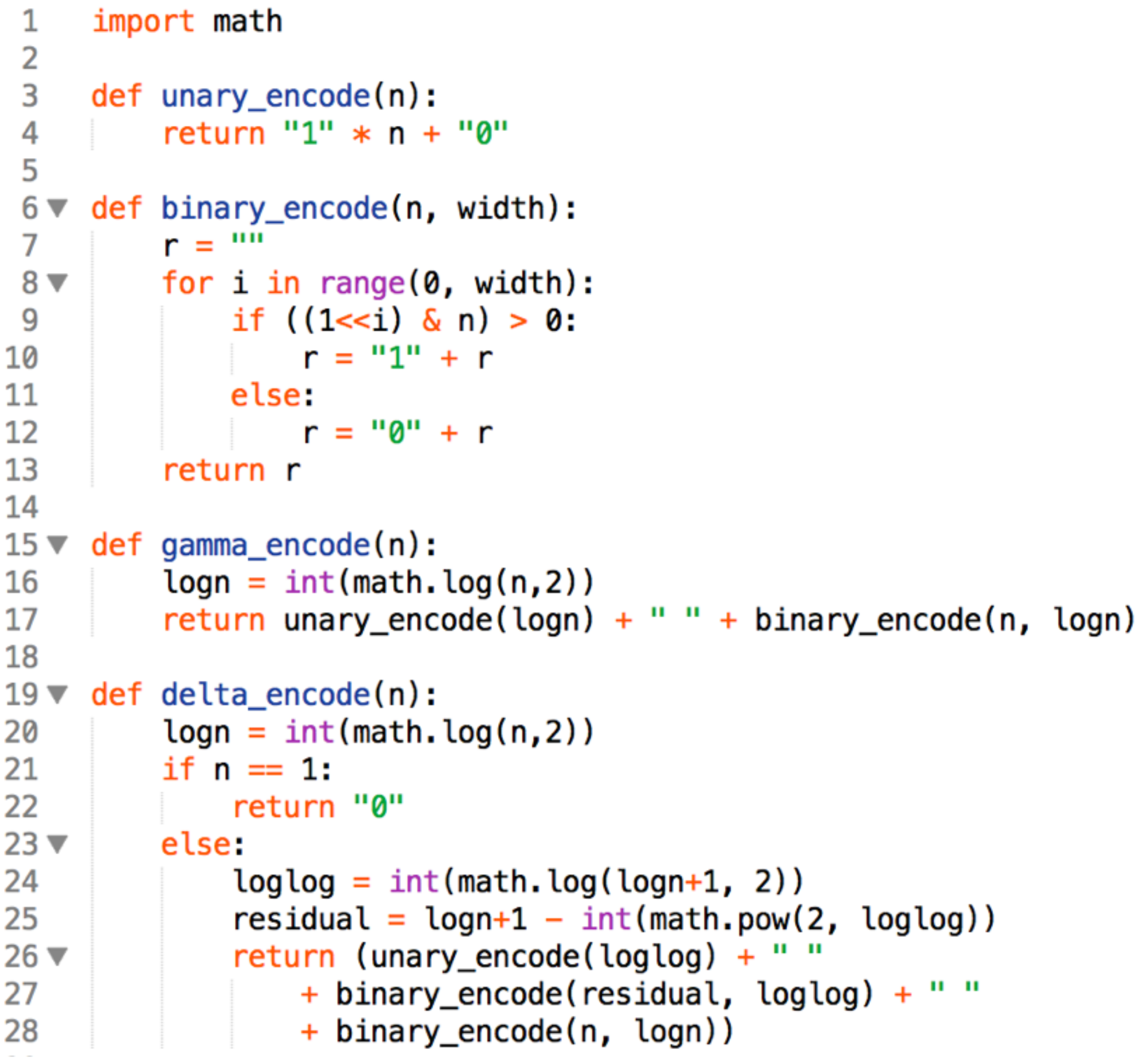
double encode

We split k_d into:

$$k_{dd}=\lfloor\log_2 k_d\rfloor$$
$$k_{dr}=k_d-2^{\lfloor\log_2 k_d\rfloor}$$

Decimal	k	k	k	k	Code
1	0	0	0	0	0
2	1	1	0	0	10 0 0
3	1	1	0	1	10 0 1
6	2	1	1	2	10 1 10
15	3	2	0	7	110 00 111
16	4	2	1	0	110 01 0000
255	7	3	0	127	1110 000 1111111
1023	9	3	2	511	1110 010 111111111

Python Implementation



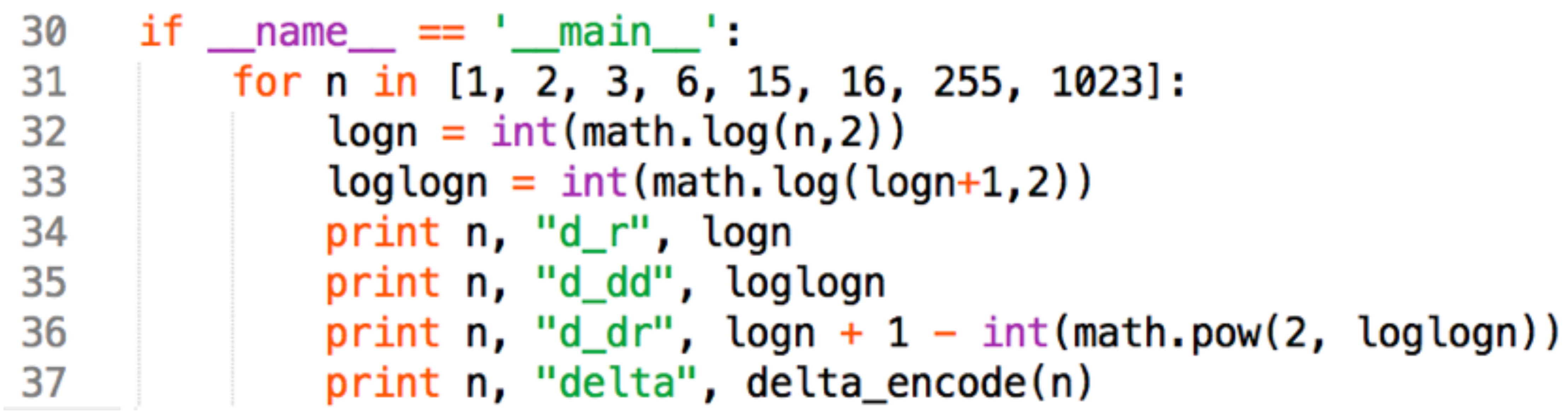
```python
import math

def unary_encode(n):
    return "1" * n + "0"

def binary_encode(n, width):
    r = ""
    for i in range(0, width):
        if ((1<<i) & n) > 0:
            r = "1" + r
        else:
            r = "0" + r
    return r

def gamma_encode(n):
    logn = int(math.log(n,2))
    return unary_encode(logn) + " " + binary_encode(n, logn)

def delta_encode(n):
    logn = int(math.log(n,2))
    if n == 1:
        return "0"
    else:
        loglog = int(math.log(logn+1, 2))
        residual = logn+1 - int(math.pow(2, loglog))
        return (unary_encode(loglog) + " "
            + binary_encode(residual, loglog) + " "
            + binary_encode(n, logn))
```



```python
if __name__ == '__main__':
    for n in [1, 2, 3, 6, 15, 16, 255, 1023]:
        logn = int(math.log(n,2))
        loglogn = int(math.log(logn+1,2))
        print n, "d_r", logn
        print n, "d_dd", loglogn
        print n, "d_dr", logn + 1 - int(math.pow(2, loglogn))
        print n, "delta", delta_encode(n)
```

Delta Encoding

We now have an efficient variable bit length integer encoding scheme which uses just a few bits for small numbers, and can handle arbitrarily large numbers with ease.

To further reduce the index size, we want to ensure that docids, positions, etc. in our lists are small (for smaller encodings) and repetitive (for better compression).

We can do this by sorting the lists and encoding the difference, or delta, between the current number and the last.

list of numbers (ex. positions)

Raw positions: 1, 5, 9, 18, 23, 24, 30, 44, 45, 48

Deltas: =diff 1, 4, 4, 9, 5, 1, 6, 14, 1, 3

6 = 30-24

1 = 45-44

High-frequency words compress more easily:

1, 1, 2, 1, 5, 1, 4, 1, 1, 3, ...

small deltas ⇒ repeating deltas/patterns

Low-frequency words have larger deltas:

109, 3766, 453, 1867, 992, ...

larger deltas ⇒ non repeating

Byte-Aligned Codes

In production systems, inverted lists are stored using byte-aligned codes for delta-encoded integer sequences.

Careful engineering of encoding schemes can help tune this process to minimize processing while reading the inverted lists. This is essential for getting good performance in high-volume commercial systems.

Next, we'll look at how to produce an index from a document collection.

Byte-Aligned Codes

We’ve looked at ways to encode integers with bit-aligned codes. These are very compact, but somewhat inconvenient.

Processors and most I/O routines and hardware are byte-aligned, so it’s more convenient to use byte-aligned integer encodings.

One of the commonly-used encodings is called **vbyte**. This encoding, like UTF-8, simply uses the most significant bit to encode whether the number continues to the next byte.

Vbyte

k	Bytes Used
k	1
2	2
2	3
2	4

k	Binary	Hexadecimal
1	1 0000001	81
6	1 0000110	86
127	1 1111111	FF
128	0 0000001 1 0000000	01 80
130	0 0000001 1 0000010	01 82
20000	0 0000001 0 0011100 1 0100000	01 1C A0

Java Implementation

```
public void encode( int[] input, ByteBuffer output ) {
    for( int i : input ) {
        while( i >= 128 ) {
            output.put( i & 0x7F );
            i >>>= 7;
        }
        output.put( i | 0x80 );
    }
}
```

```
public void decode( byte[] input, IntBuffer output ) {
    for( int i=0; i < input.length; i++ ) {
        int position = 0;
        int result = ((int)input[i] & 0x7F);

        while( (input[i] & 0x80) == 0 ) {
            i += 1;
            position += 1;
            int unsignedByte = ((int)input[i] & 0x7F);
            result |= (unsignedByte << (7*position));
        }

        output.put(result);
    }
}
```

Bringing It Together

Let’s see how to put together a compressed inverted list with delta encoding. We start with the raw inverted list: a sequence of tuples containing `(docid, tf, [pos1, pos2, …])`.

```
(1,2,[1,7]), (2,3,[6,17,197]), (3,1,[1])
```

We delta-encode the docid and position sequences independently.

```
(1,2,[1,6]), (1,3,[6,11,180]), (1,1,[1])
```

Finally, we encode the integers using vbyte.

```
81 82 81 86 81 82 86 8B 01 B4 81 81 81
```

Alternative Codes

Although vbyte is often adequate, we can do better for high-performance decoding.

Vbyte requires a conditional branch at every byte and a lot of bit shifting.

Google's Group VarInt encoding achieves much better decoding performance by storing a two bit continuation sequence for each of the next 4-16 bytes.

Decimal:		1	15		511			131071
Encoded:	00000110	00000001	00001111	11111111	00000001	11111111	11111111	00000001