

$$\begin{aligned} pmi(a, b) &:= \log \left(\frac{p(a, b)}{p(a)p(b)} \right) \\ &= \log \left(\frac{\frac{n_{ab}}{N}}{\frac{n_a}{N} \frac{n_b}{N}} \right) \\ &= \log N + \log \frac{n_{ab}}{n_a n_b} \\ &\stackrel{\text{rank}}{=} \frac{n_{ab}}{n_a n_b} \end{aligned}$$