

$$P(\pi|a) \text{ is } \textit{Dirichlet}(\pi|a_1, \dots, a_V) \propto \prod_{i=1}^V \pi_i^{a_i-1}$$

$$P(d|\pi) \text{ is } \textit{Multinomial}(\pi) \propto \prod_{i=1}^V \pi_i^{tf_{i,d}}$$

$$P(w|d) \propto P(d|\pi)P(\pi|a) = \prod_{i=1}^V \pi_i^{tf_{i,d}+a_i-1}$$

is *Dirichlet*( $\pi|a_1 + tf_{1,d}, \dots, a_V + tf_{V,d}$ )

$$\mathbb{E}[P(w|d)|a = \mathbf{1}] = \frac{tf_{1,d} + 1}{|d| + V}$$