

Office hours 9/14 → ^{digits = 1, 0, 1, 2, 3, 3} Binary representation

Base 16
 A B C D E F
 10 11 12 13 14 15
 Base 16

Base 2

Base 4

Base 10

11011

123

27 +

10001

101

17

230

44

16 + 11 = 1B

16 + 1 = 17

? = 2C

101100 = ?
 power base expansion

$$1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$

$$= 32 + 8 + 4 = 44 \checkmark$$

power base expansion

$$2 \cdot 16^1 + C \cdot 16^0 =$$

$$32 + 12 = 44$$

$$123_{(4)} = 1 \cdot 4^2 + 2 \cdot 4 + 3 =$$

$$16 + 8 + 3 = 27 \checkmark$$

$$230_4 = 2 \cdot 4^2 + 3 \cdot 4 + 0 = 32 + 12 = 44 \checkmark$$

Binary 6 bits



$\left. \begin{array}{l} 000000 \\ 000001 \\ 000010 \\ \vdots \\ 111111 \end{array} \right\} ?$

How many # we can represent?

How many possibilities in binary 6 bits? 64

→ Which 64 numbers do we represent? $[0:63]$

$\{0, 1, 2, \dots, 63\}$

— maybe $[100:163]$

signed range $[-32:31]$ $\{-32, -31, -30, \dots, -1, 0, 1, 2, \dots, 31\}$

rep = two complement

3 shirts S_A, S_B, S_C

2 pants P_1, P_2

4 hats H_A, H_B, H_C, H_D

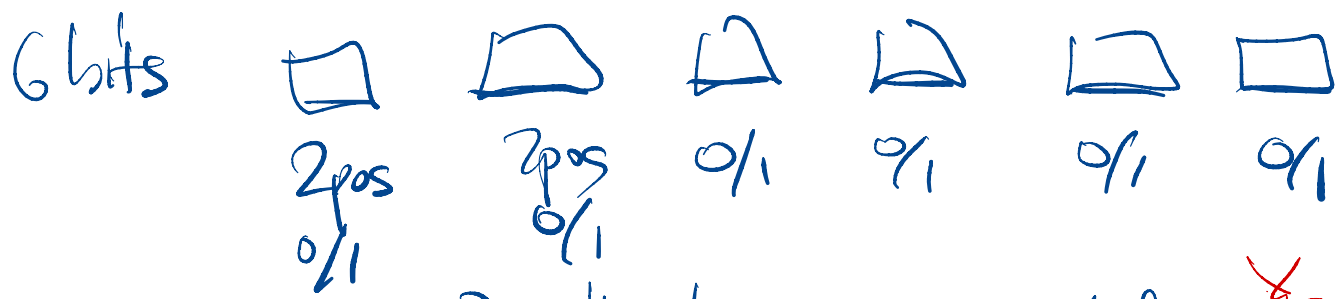
Q: how many ways to dress up?

Product Rule

all choices can be combined
(every choice valid with any other
choice
- no restriction)

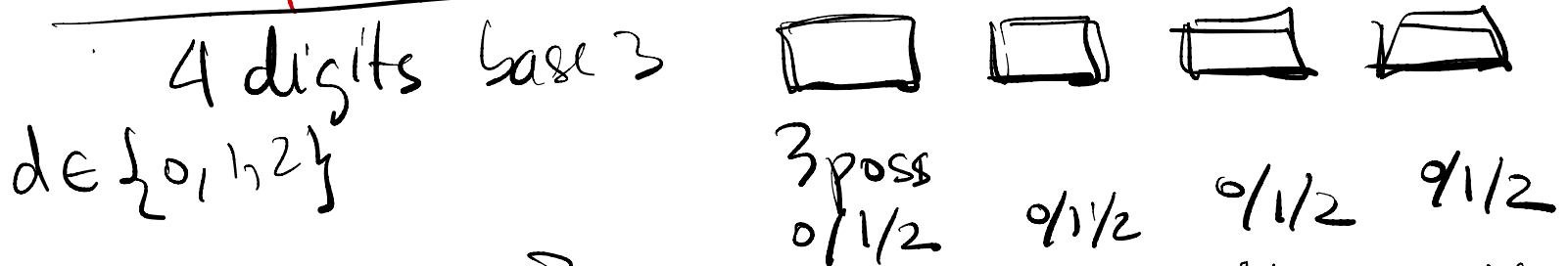
$3 \times 2 \times 4 = 24$ possib.

$(S_A, P_1, H_A), (S_A, P_2, H_B) \dots$
 $(S_A, P_2, H_A), (S_A, P_2, H_B) \dots$



Product Rule? all choices are valid **Yes**

possib = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$.



Product Rule? are all choices valid with any other choices? **YES**

$3 \times 3 \times 3 \times 3 = 3^4 = 81$

How many possible base 3 4 digits

- ending with "2"



$$27 = 3 \times 3 \times 3 \times 1$$

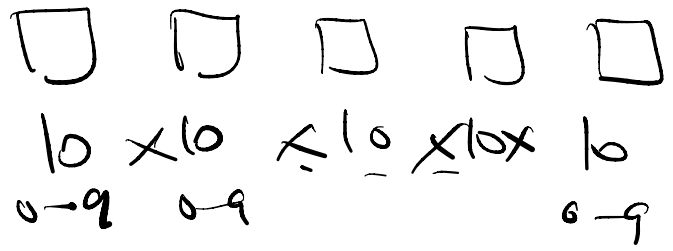
- ending with "1" or "0"



$$3 \times 3 \times 3 \times 2$$

base 10 5 digits

$d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$



10^5 possibilities.

unsigned Range

$\{0, 1, 2, \dots, 99999\}$

size of the range

$[0:99999]$

$$|\text{Range}| = 10^5 - 1$$

why?

$$99999 + 1$$

Range

$$= \underline{\underline{100000}}$$

10^5

What is max in range (unsigned)
base 10 k digits.

$$\underline{\underline{d_{k-1}}} \quad \underline{\underline{d_{k-2}}} \quad \dots \quad \underline{\underline{d_3}} \quad \underline{\underline{d_2}} \quad \underline{\underline{d_1}} \quad \underline{\underline{d_0}}$$

Generic
" $d_{k-1} d_{k-2} \dots d_1 d_0$ "

power expansion: $d_{k-1} \cdot 10^{k-1} + d_{k-2} \cdot 10^{k-2} + \dots + d_1 \cdot 10^1 + d_0$

max possible: $9 \cdot 10^{k-1} + 9 \cdot 10^{k-2} + \dots + 9 \cdot 10^1 + 9$

$= (10-1) (10^{k-1} + 10^{k-2} + 10^{k-3} + \dots + 10^1 + 10^0)$

~~$10^k - 10^{k-1}$~~ +

~~$10^{k-1} - 10^{k-2}$~~

~~$10^{k-2} - 10^{k-3}$~~

~~$10^1 - 10^0$~~

$10^k - 10^0 = 10^k - 1$

telescope

signed binary 6 bits — — — — —

$2^6 = 64$ possibilities
Range = [-32:31]

"E" in a set belongs to

Mechanism/procedure to map each value $\in [-32:31]$ to string of 6 bits Two's complement

• unsigned mechanism = stays. 0:31 ✓
Use the positive ← 0 $\frac{0}{8} + \frac{1}{4} + \frac{0}{1} = 13$ ✓
other 5 bits for unsigned value

decimal value = $(-1) = ?$, negative \Rightarrow first bit = 1
1 1 1 1 1 ?

"Complement of 1" = ?
1 1 1 1 1 0 ? Not sure

add

$$\begin{array}{r}
 -1 = ? \overset{\text{later}}{\boxed{}} \begin{array}{c} \underline{1}^1 \\ \underline{1}^1 \\ \underline{1}^1 \\ \underline{1}^1 \\ \underline{1}^1 \\ \underline{?1} \end{array} \\
 +1 = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \\
 \hline
 0 = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}
 \end{array}$$

$$\begin{array}{r}
 -2 = ? \begin{array}{c} \underline{1} \\ \underline{1} \\ \underline{1} \\ \underline{1}^{(1)} \\ \underline{1} \\ \underline{?0} \end{array} \\
 +2 = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \\
 \hline
 0 = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}
 \end{array}$$

$-32 \text{ (min in range)} = ? \underline{?1} \underline{0} \underline{0} \underline{0} \underline{0} \underline{0}$

$$\begin{array}{r}
 +32 \text{ unsigned} = \begin{array}{c} \underline{1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 \text{unsigned (not sign bit)} \swarrow \\
 \hline
 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

Exercise two's complement works like this

$$\begin{array}{cccccc} \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ \text{power expansion} & \rightarrow & 1 \cdot 2^5 & + 0 \cdot 2^4 & + 1 \cdot 2^3 & + 1 \cdot 2^2 & + 1 \cdot 2^1 & + 0 \cdot 2^0 \end{array}$$

has the first power with negative sign.

$-9 = ?$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{1}$
$+9 =$	0	0	1	0	0	1
	0	0	0	0	0	0

2's comp power exp

$$\begin{aligned} & 110111 = \\ & -1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2 + 1 \\ & -32 + 16 + \quad + 4 + 2 + 1 \\ & = -32 + 23 = -9 \end{aligned}$$