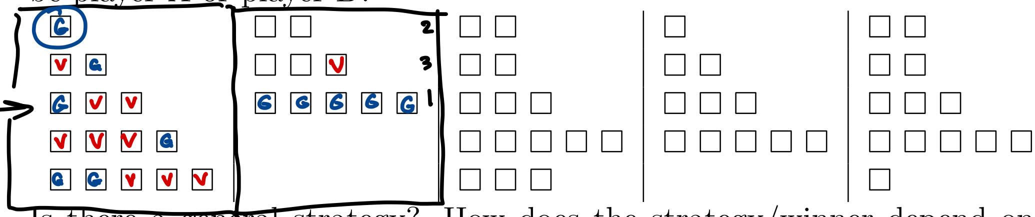
Honors Problem 1 : Square Game. Two players A and B play the following game. Starting with a stack of rows of squares (□), they take turns with player A first in removing squares. In each turn the player

- identifies one row with at least one □
- remove any number of □ from that row (all if so desired), but do not remove them from any other row.

The player who removes the last square wins.

Here are 5 boards to play with a friend. At each one, would you like to to be player A or player B?

V: virgil
G: gabriel



Is there a general strategy? How does the strategy/winner depend on initial configuration of the squares? If you work on this problem, write up the explanation/solution for the general case (any board); 1 page max.

SQUARE GAME

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Binary Numeric base 10 base 16

Representation

I

thousand, hundreds, 10^5, units

$$8792_{(10)} = 8 \cdot \boxed{10^3} + 7 \cdot \boxed{10^2} + 9 \cdot 10^1 + 2 \cdot 10^0$$

base

power-base expansion

biggest power of 10 that fits in 8792 (8 times)

II

visual: positions

8	0	0	0
	7	0	0
		9	0
			2
8	7	9	2

$+$

III

INTEGER DIV

quot, remainder

$$8792 \div 10 = 879 \quad 2$$

$$879 \div 10 = 87 \quad 9$$

$$87 \div 10 = 8 \quad 7$$

$$8 \div 10 = 0 \quad 8$$

Exercise Representation is UNIQUE.

proof: $N = 8792 = d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \dots + d_1 \cdot 10 + d_0$

$k = 3$, $d_3 = 8$, $d_2 = 7$, $d_1 = 9$, $d_0 = 2$

different / another representation $= c_\ell \cdot 10^\ell + c_{\ell-1} \cdot 10^{\ell-1} + \dots + c_1 \cdot 10 + c_0$

Th $\Rightarrow \ell = k$, $c_\ell = d_k$, $c_{\ell-1} = d_{k-1}$, … The same.

Rationals: $0.999\ldots9\ldots = 0.(9) = 1$

2 diff representations of the rational 1

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binary vs base 10 base = 2

$22_{10} = ?_{binary} = \boxed{16} + 6 =$

$= \boxed{16} + \boxed{4} + \boxed{2}$

binary powers

$2^0 \quad 1 = 1$

$2^1 = 2 = 10$

$2^2 = 4 = 100$

$2^3 = 8 = 1000$

$2^4 = 16 = 10000$ 4 zeros $\Rightarrow 2^4$

$2^5 = 32 = 100000$

$2^6 = 64 = 1000000$

$2^7 = 128 = 10000000$

$2^8 = 256 = 1\underbrace{0000\,0000}_{8 = \text{exponent}}$

$2^9 = 512$

$2^{10} = 1024$

$2^{11} = 2048$

$2^{12} = 4096$

$2^{13} = 8192$

1	0	0	0	0
		1	0	0
			1	0

$22_{(10)} = 10110_{(2)}$

$8792_{(10)} = ?$ binary $= \boxed{8192} + 600$

$2^{13} + \boxed{512} + 88 =$

$= 2^{13} + 2^9 + \boxed{64} + 24$

$= 2^{13} + 2^9 + 2^6 + 2^4 + 2^3$

$2^{13} = 1$ _ _ _ _ _ _ _ _ _ _ _ _ _

1 _ _ _ _ _ _ _ _ _

1 _ _ _ _ _ _

1 _ _ _ _

bits $\in \{0,1\}$

1	0	0	0	1	0	0	1	0	1	1	0	0	0

Excercise 8792 → binary by repeated division

base

		q	r
$8792 \div 2$	=	4396	0 → right most digit
$4396 \div 2$	=	2198	0 → 2nd dig from right

⋮

$8792 = $ base 16 ?

hex digit

$$= 2\cdot\boxed{16^3} + 2\cdot\boxed{16^2} +$$

hex digit

8192   512   88

how many times it fits?

3 zeros after

$$= 2\cdot 16^{3} + 2\cdot 16^2 + 5\cdot 16^1 + 8\cdot 16^0$$

3×16^3 = too much . no good

write down

$2\cdot 16^3 \rightarrow 2\,0\,0\,0$

$2\cdot 16^2 \rightarrow 2\,0\,0$

$5\,0$

8

$$\boxed{2258_{(16)}}$$

bases = integers ≥ 2

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base 10 digits $\in \{0, 1, 2, 3, \ldots 9\}$

base 2 bits $\in \{0, 1\}$

base 16 hex $\in \{0, 1, 2, \ldots, 9, \overset{A}{\cancel{10}}, \overset{B}{\cancel{11}}, \overset{C}{\cancel{12}}, \overset{D}{\cancel{13}}, \overset{E}{\cancel{14}}, \overset{F}{\cancel{15}}\}$

$\cancel{A, 9}$

base − 1