Implications | Boolean Algebra

$A \Rightarrow B$ "if A then B"

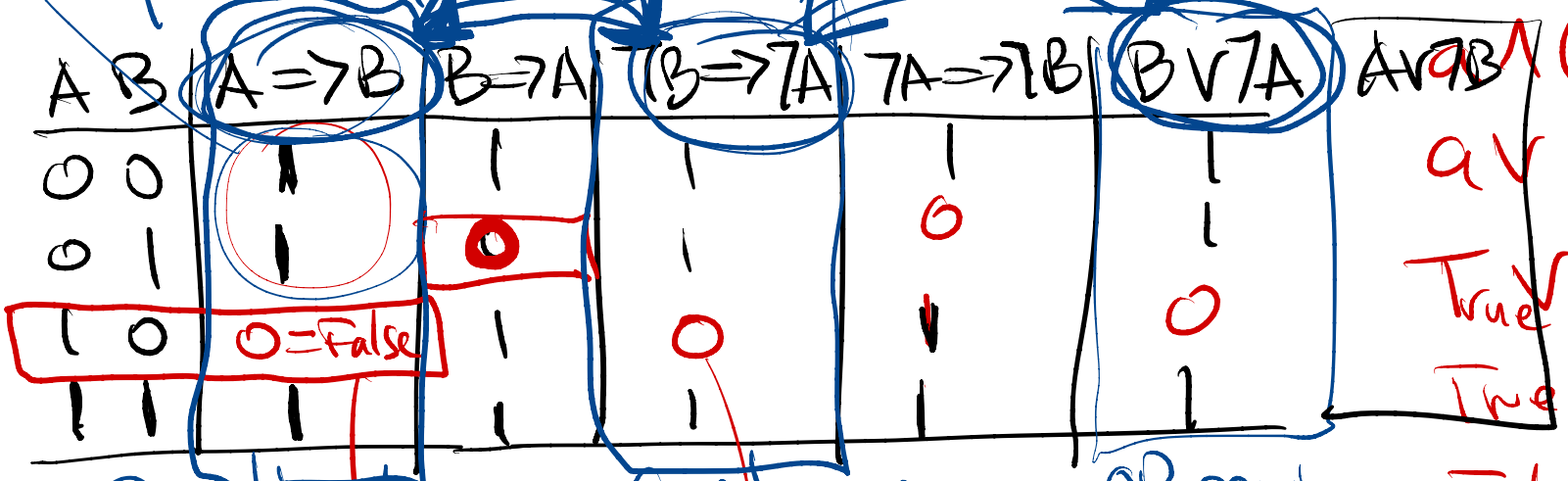
Hyp Concl

"can A happen without B?" NO

"can B happen w/out A" Yes

start/assume False

prove anything (T/F)



2=3 ⇒ direct ⇒ any theorem

False

$A=T \wedge B=F$

$A \wedge \neg B$

$\neg B \wedge \neg(\neg A)$

$\neg B \wedge A$

$A \vee B = B \vee A$

$A \wedge B = B \wedge A$

$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

$\neg(A \wedge B) = \neg A \vee \neg B$

$\neg(A \vee B) = \neg A \wedge \neg B$

$\neg\neg A = A$

$A \wedge (A \vee B) = A$

$A \vee (A \wedge B) = A$

$True \vee A = True$

$True \wedge A = A$

$False \vee A = A$

$False \wedge A = False$

$A \Rightarrow B \equiv \neg B \Rightarrow \neg A \equiv B \vee \neg A$

$A \Rightarrow B$ direct

$\equiv \neg B \Rightarrow \neg A$ contrapositive / contradiction

$\equiv B \vee \neg A$ OR-equivalent

- want B. (concl)
- hypothetically lets assume $\neg B$
- $\neg B \Rightarrow \ldots \Rightarrow \neg A$
- $\neg A$ contradicts A
- therefore B must be True.

$B \Rightarrow A$

$\equiv \neg A \Rightarrow \neg B$ converse (inverse)

$\equiv A \vee \neg B$

Example $P \wedge Q \wedge (P \wedge Q \Rightarrow R) \equiv P \wedge Q \wedge R$

get A in 1800 — get A in 1500 — nice vacation in Hawaii

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$(P \wedge Q) \wedge ((P \wedge Q) \Rightarrow R)$	$P \wedge Q \wedge R$
0	0	0	0	1	0	0
0	0	1	0	1	0	0
0	1	0	0	1	0	0
0	1	1	0	1	0	0
1	0	0	0	1	0	0
1	0	1	0	1	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Algebra

$(A \Rightarrow B) \equiv (B \vee \neg A)$

and $\vee$ not hyp

$P \wedge Q \wedge (\underbrace{P \wedge Q}_{\text{hyp}} \Rightarrow R)$

$P \wedge Q \wedge (R \vee \neg(P \wedge Q))$

$[(P \wedge Q) \wedge R] \vee [(P \wedge Q) \wedge \neg(P \wedge Q)]$

False

$P \wedge Q \wedge R$ ✓

$(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ trasitivity

A B C	A⇒B	B⇒C	(A⇒B) ∧ (B⇒C)	A⇒C
0 0 0			1	1
0 0 1			1	1
0 1 0		0	0	1
0 1 1			1	1
1 0 0	0		0	0
1 0 1	0		0	1
1 1 0		0	0	0
1 1 1			1	1

look for 1⇒0

?

NONE FOUND!

⇓

Correct!

$(A \to B) \wedge (B \to C) \Rightarrow (A \to C)$

$(B \vee \neg A) \wedge (C \vee \neg B) \Rightarrow (C \vee \neg A)$ → concl

$[C \wedge (B \vee \neg A)] \vee [\neg B \wedge (B \vee \neg A)]$

$[C \wedge (B \vee \neg A)] \vee [(\neg B \wedge B) \vee (\neg B \wedge \neg A)]$

False

$[C \wedge (B \vee \neg A)] \vee [\neg B \wedge \neg A]$

$[C \vee (\neg B \wedge \neg A)] \wedge [(B \vee \neg A) \vee (\neg B \wedge \neg A)]$

OR

Right side doesn't help the left side for the OR

exercise (absorption)

$[(C \vee \neg B) \wedge (C \vee \neg A)] \wedge (B \vee \neg A)$

$(B \vee \neg A) \wedge (C \vee \neg B) \wedge (C \vee \neg A)$ → cond.

as result of first two.

Predicates & Quantifiers

Who studies math gets A in 1800

P ⇒ Q

P	Q	P⇒Q	Q⇒P
0	0		
0	1		
1	0	0	
1	1		

In set A in 1800, ∃ st study math

Q ⇒ P

False case does not happen

¬(P⇒Q)

P ∧ ¬Q

"study math AND do not get A 1800"

Q ∧ R

got A in 1800 AND did not study math

get A in 1800 AND get A in 1500 then vacation in Hawaii

$Q \wedge R \Rightarrow Z$

assume true

Then $Q \Rightarrow Z$ False

$Z \Rightarrow Q$ False

$Z \Rightarrow Q \wedge R$ False

Negation: $\neg(Q \wedge R \Rightarrow Z)$

$\neg(Z \vee \neg(Q \wedge R))$

$\neg Z \wedge Q \wedge R$

$x \in$ Students

predicate = boolean function: Student(x), getA_math(x), getA_tandes(x)

values 0/1

$\forall x$ (every) student(x) $\wedge$ getAmath(x) $\wedge$ getAfundic(x) $\Rightarrow$ hawaii(x)

$\forall_{c}$ course $\exists_{s}$ student such that grade of student to course is A

boolean (predicate) grade_A(c,s) → T / F

non-boolean grade(c,s) = {A, A-, B, B+, B-, ...}

- $\forall c$ course(c) $\exists s$ student(s) grade(s,c) = A
- $\exists c \; \forall s$ gradeA(s,c) —NEGATION→ $\neg[\exists c \; \forall s$ gradeA(s,c)$]$
  - ↳ c fixed, all students
  - $\equiv \forall c \; \exists s \; \neg$gradeA(s,c)
  - for every course there's a student who does not get A grade
- $\exists s \; \forall c$ gradeA(s,c)
  - ↳ s = fixed, all courses
- $\forall s \; \exists c$ gradeA(s,c) —NEGATION→ $\neg[\forall s \; \exists c$ gradeA(s,c)$]$
  - $\equiv \exists s \; \forall c \; \neg$gradeA(s,c)
  - there's a student who doesn't get A in any course
  - for each s, $\exists c$ depends on s