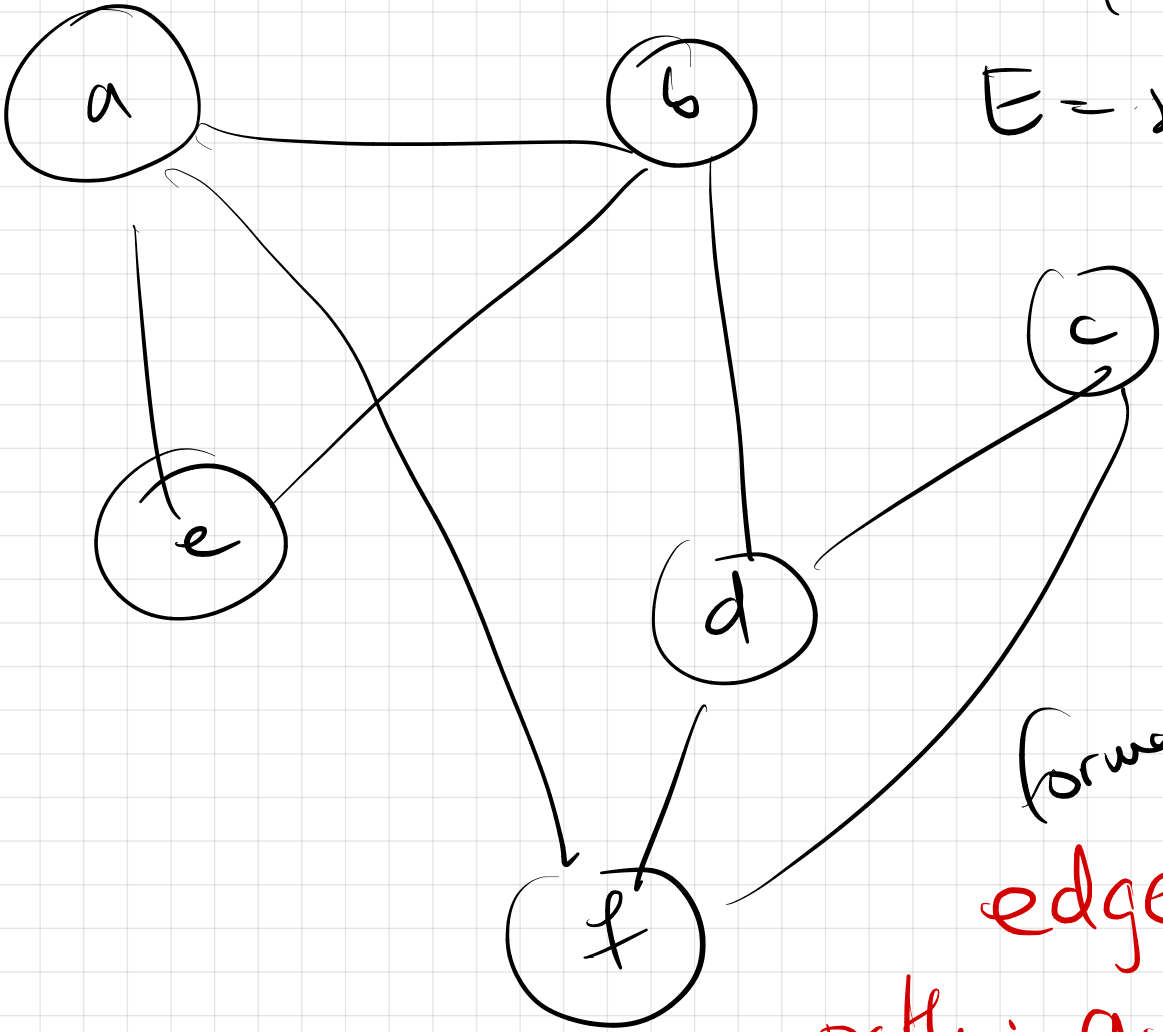
Lecture 20: Graphs part 1

- project proposal due Thu
- no recitations next week
- class next Mon 11/22 but not Wed 11/24
- hw PB 4 (mainly after Thxgv)

Graphs: vertices / nodes / circles

edges / connections / lines / pairs-of-vertic



$V = \{a, b, c, \ldots f\}$

$E =$ set of edges

informal

$\{ ab, ae, bd, be, dc, df, cf \}$

formal

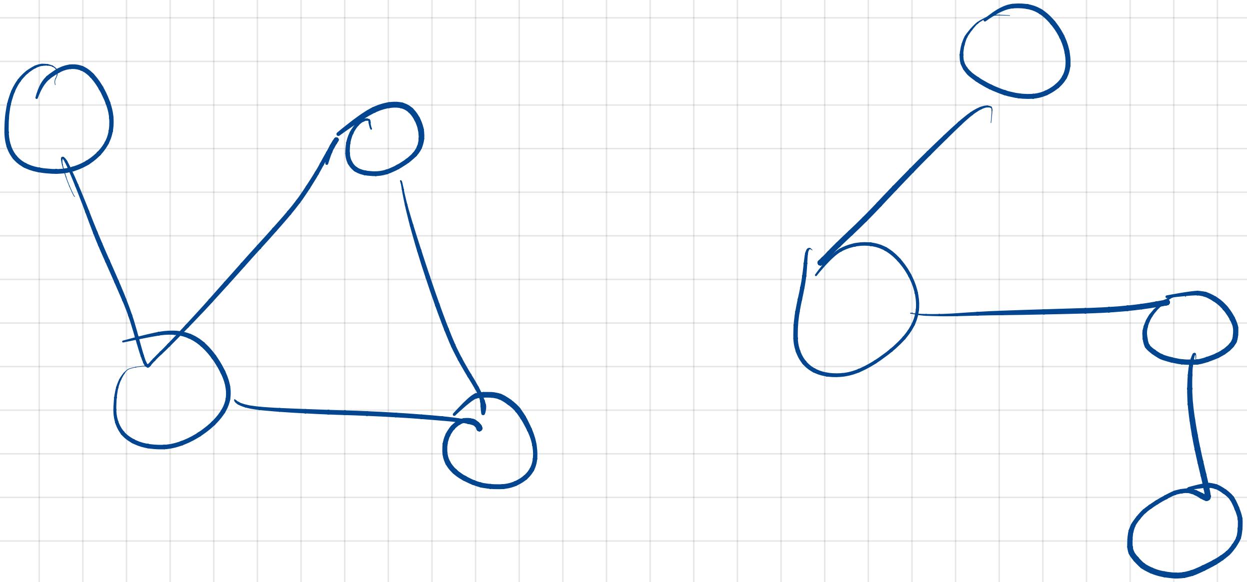
$= \{ (a,b), (a,c), (b,d) \ldots \}$

edge $a \to b$, $a - b$

path: $a - b - d - f$: $a \rightsquigarrow f$

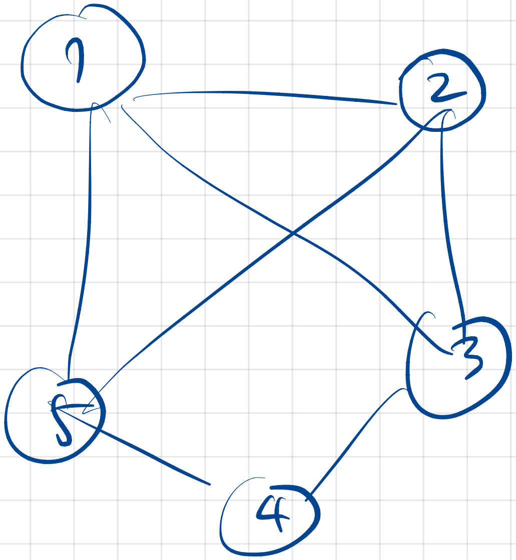
Connected : "path" from any vertex $u \rightsquigarrow$ vertex v

disconnected: 2 connected components



Subgraph: $V' \subset V$ and all corresponding edges

$V = \{1, 2, 3, 4, 5\}$

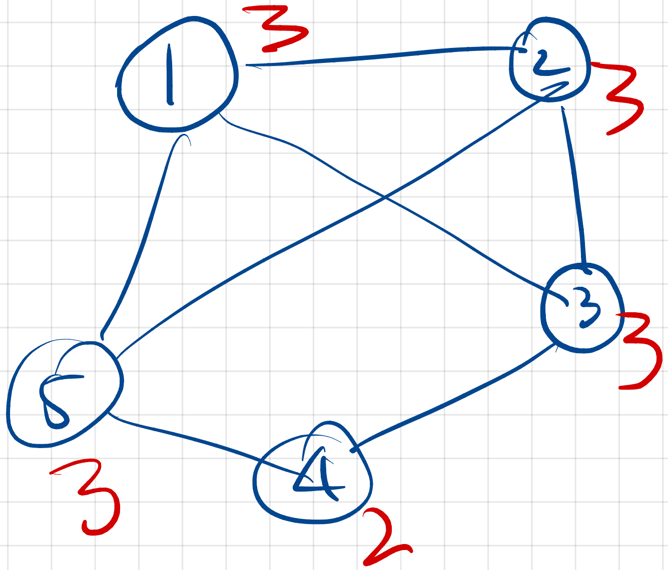


subgraph: $V' = \{1, 4, 5\}$

edges $E' = \{15, 45\}$

degree (vertex) deg(a) d(a) deg = red

= # edges incident at that vertex



deg(1) = 3

deg(4) = 2

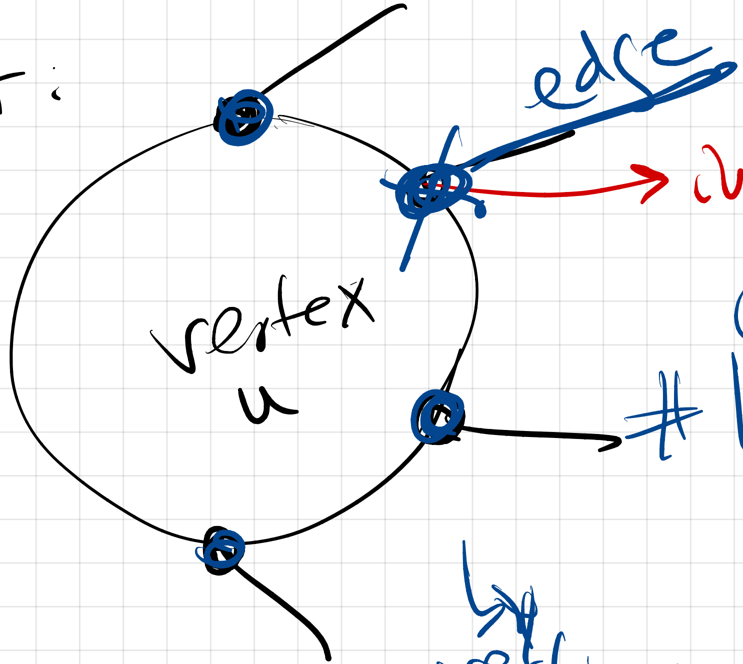
Theorem (hand-shake lemma):

Graph $G = (V, E) = (\text{vertex set}, \text{edge set})$

$$\sum_{u \in V} \deg(u) = 2|E|$$

sum of vertex deg = twice # of edges

proof:



incident point $\left(\overset{\overset{\text{vertex}}{\downarrow}}{u}, \overset{\downarrow}{\text{edge}}\right)$

Count

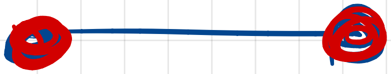
|incident points in two ways.

by vertices

by edges

$$\sum_{u} \deg(u) = \sum_{e \in E} 2$$

deg(u) = # incident in u

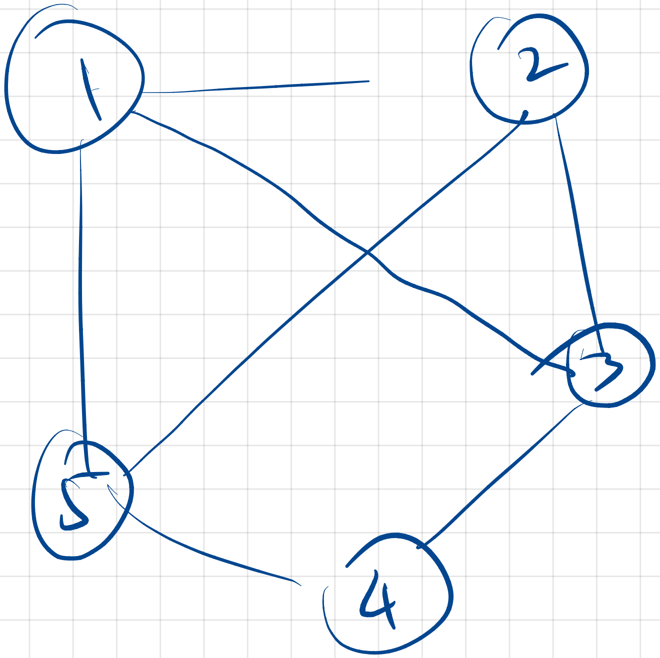


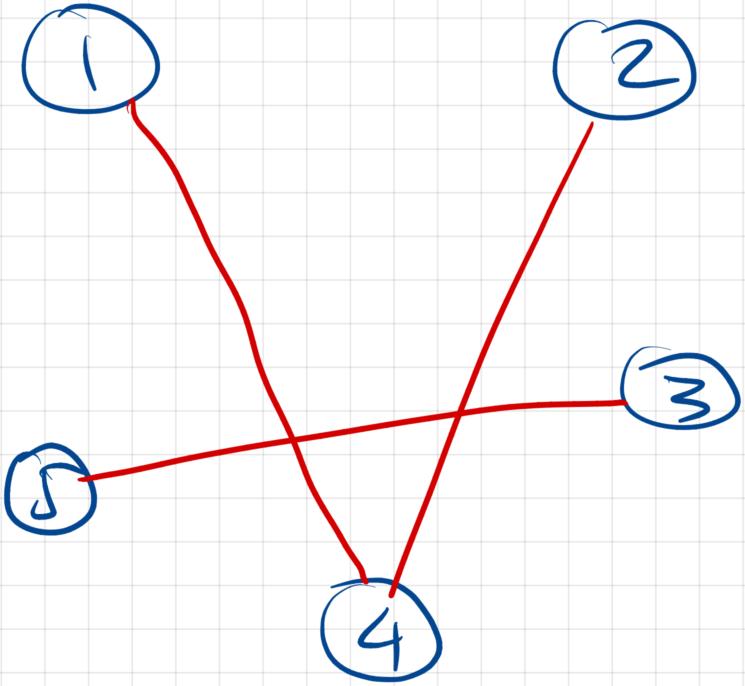
Corolary: # vertices with odd degree = even

deg(u) = odd = 2k+1

Complement (G) = $\overline{G}$ = (V , $\overline{E}$ = missing edges in E)

same





{3,4} clique

△ = clique of 3 : 125
123

max clique size = 3

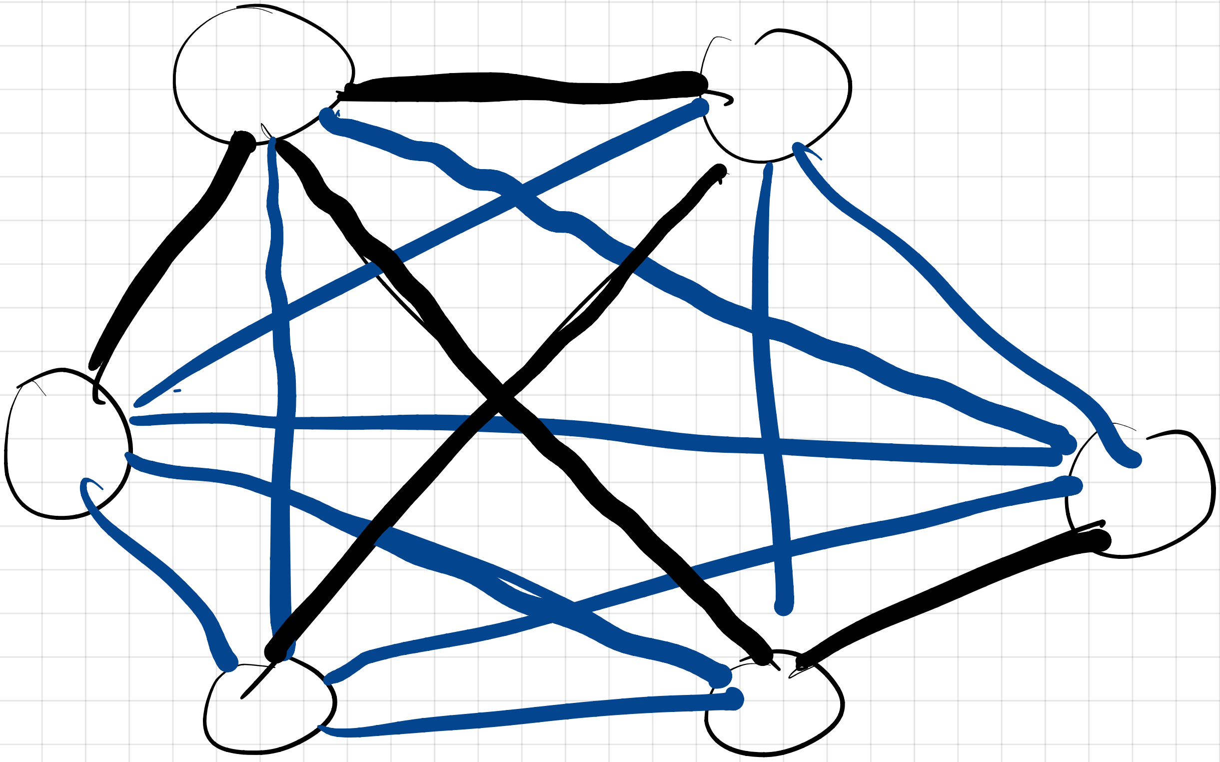
$\overline{E}$ = {14, 24, 35}

$E \cup \overline{E}$ = all edges (pairs)

clique : subset (or whole set) of vertices with all edges present

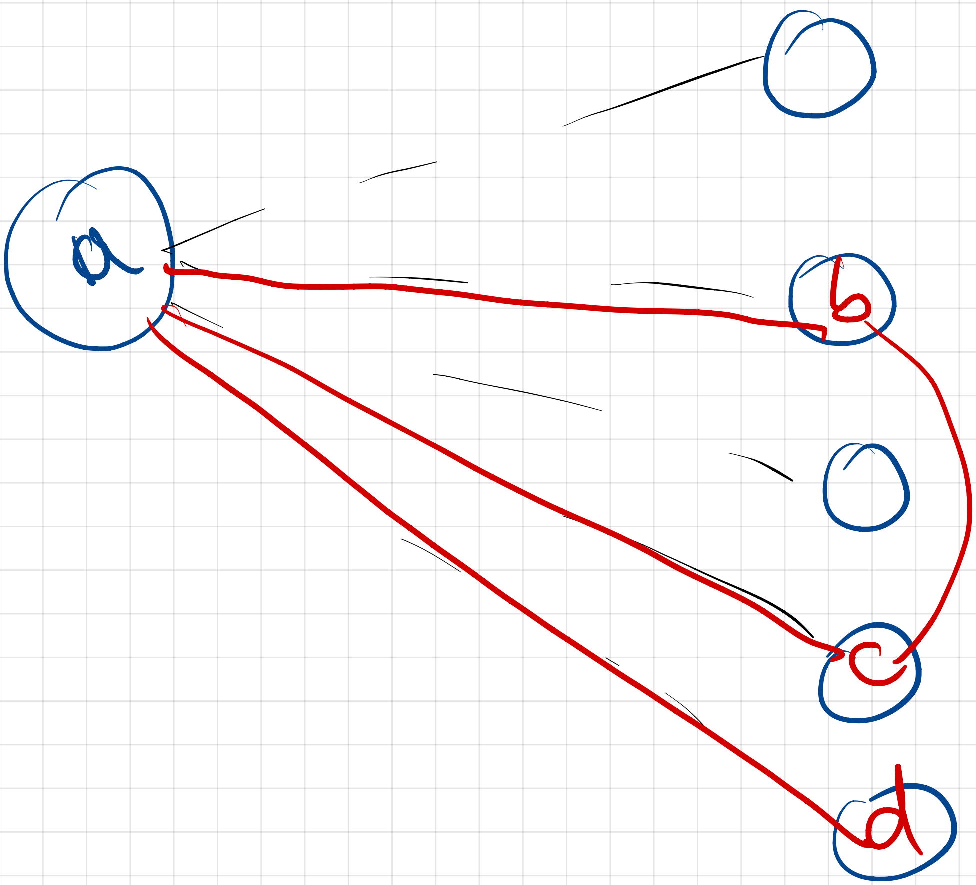
Pb ⊗ $\boxed{G = (V, E)}$ $\boxed{\bar{G} = (V, \bar{E})}$ $|V| = 6$

prove that one of G or $\bar{G}$ has a $|\text{clique}| = 3$ (a triangle)



Proof $a \in V$

look at all possible $\boxed{a\text{-edges } (5)}$



- some are in G
- the other are in $\bar{G}$

PHP ⇒ one of the $G/\bar{G}$ has 3 edges ab, ac, ad

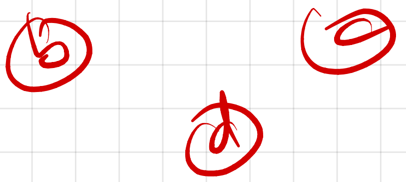
"red" graph is either $G/\bar{G}$

b, c, d:

- either they have an edge in same "red" graph

  say bc ⇒ Δabc

- or no red edge between (b, c, d)

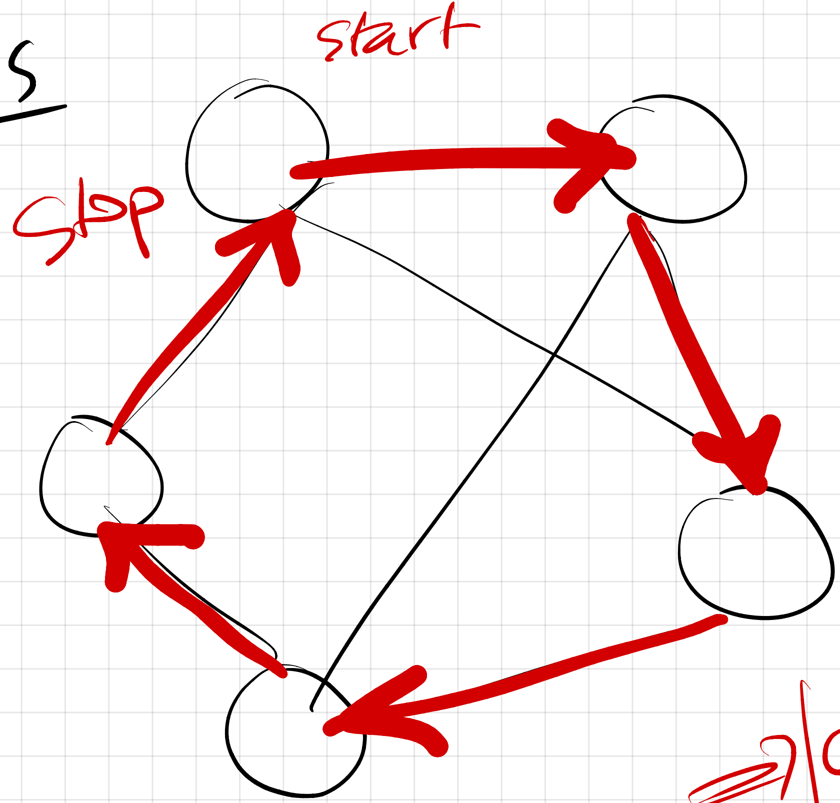


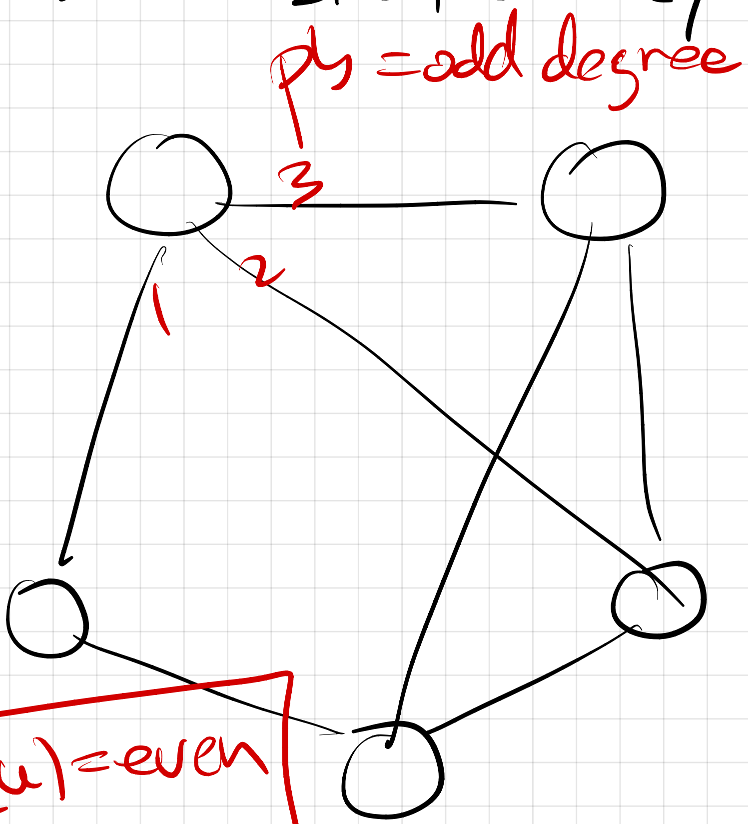
⇒ all 3 edges bc, cd, bd must be in red

Δ bcd

Tours : path that ends where it started = cycle

cycles

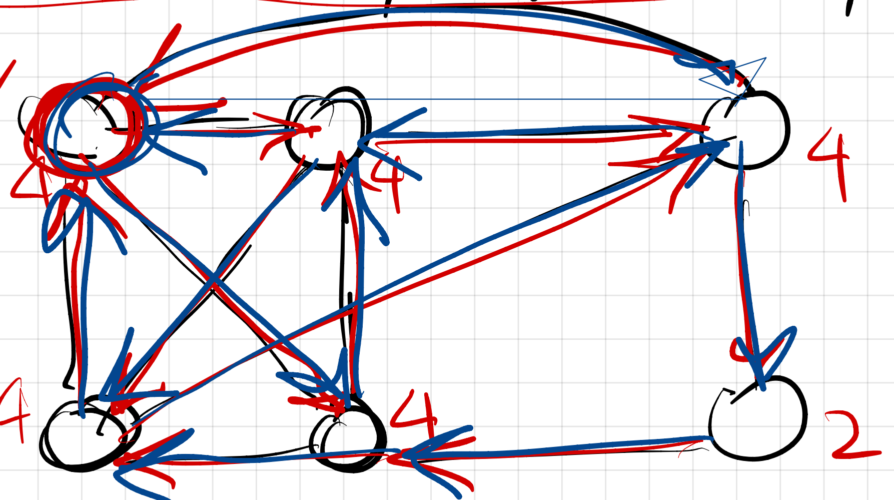




$\Rightarrow \boxed{deg(u)=even}$

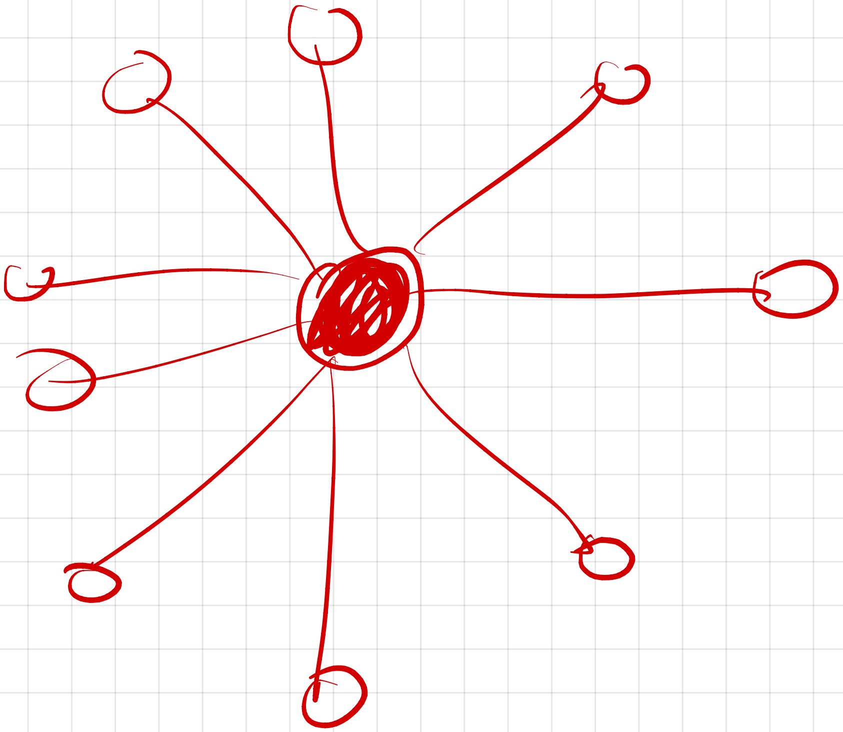
Euler Tour: Tour that visits every edge exactly once

all even degrees



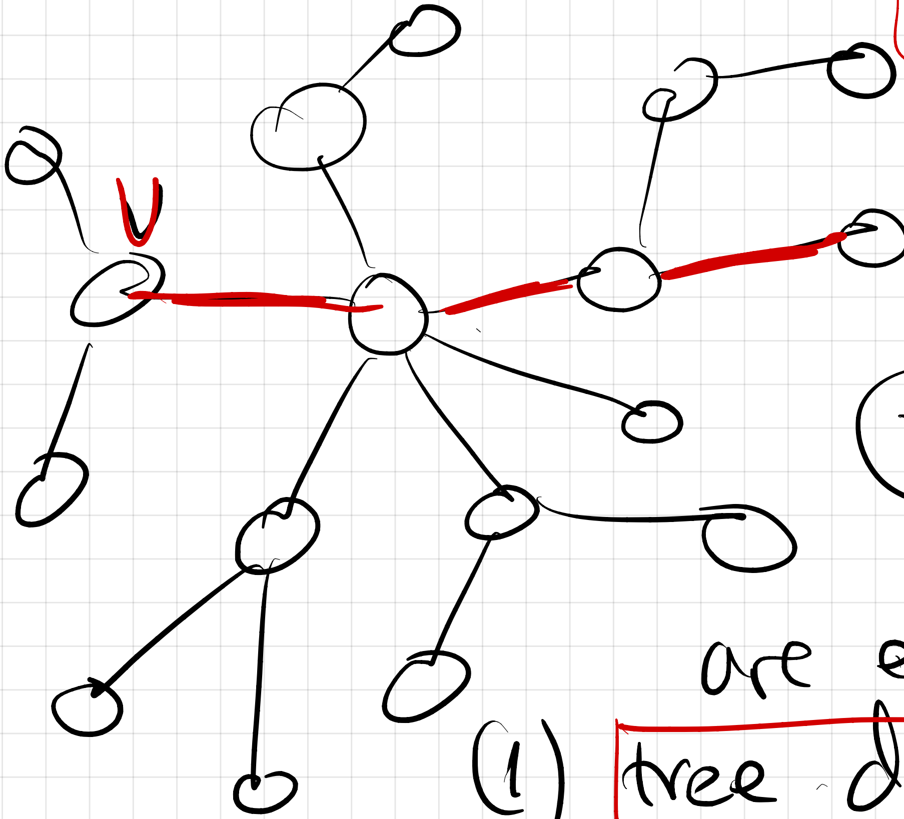
Vertex Cover: Find <u>smallest</u> set of vertices incident in all edges.

extreme: $|vc|=1$



Tree

$T = G(V, E)$ = tree $\begin{cases} \text{connected} \\ \text{no cycles} \end{cases}$ &



exercise 1: $|E|$ in a tree is precisely $|V| - 1$

Th exercise 2 following statements are equivalent:

(1) tree def

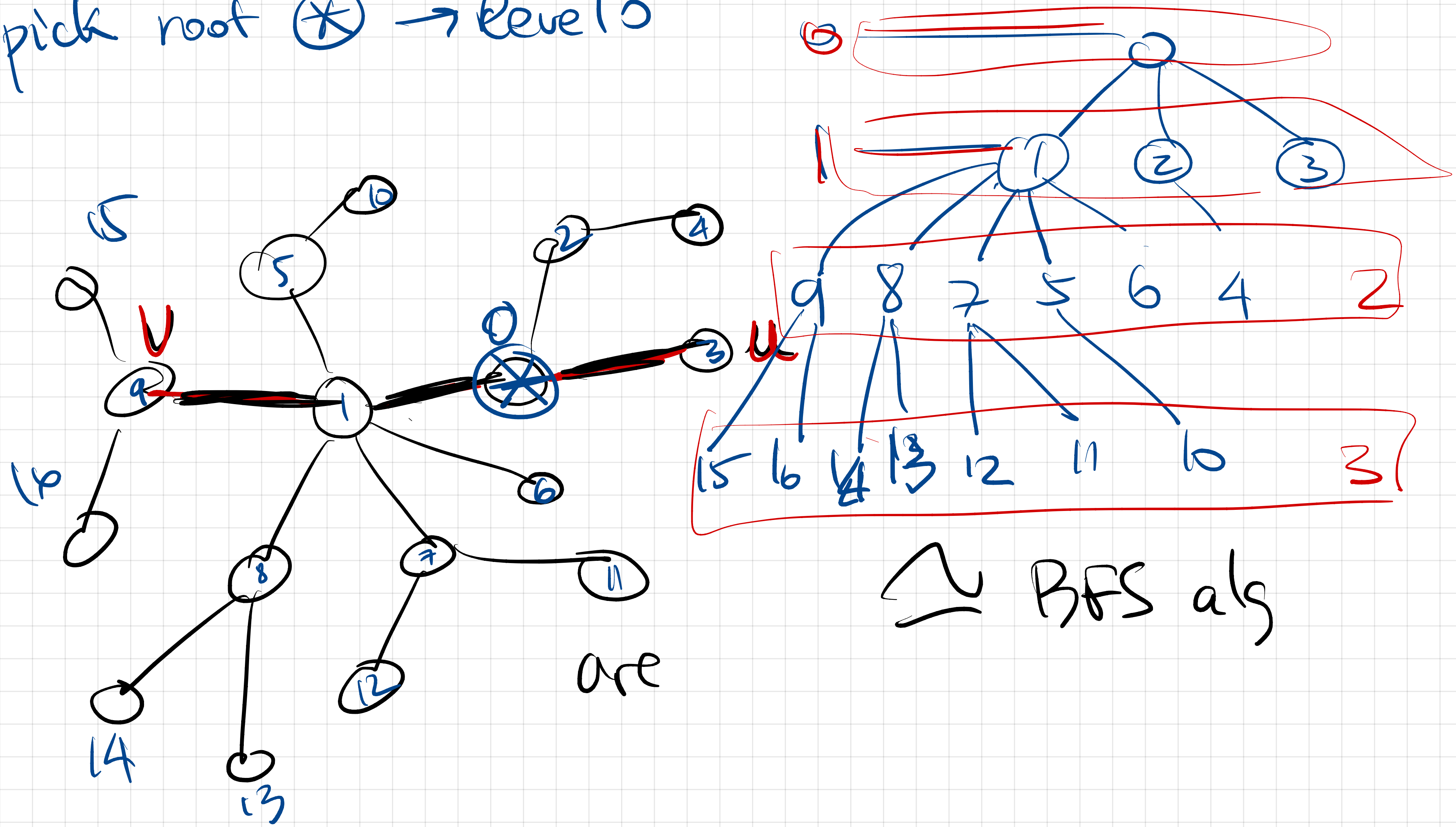
(2) any two vertices (u, v) connected with unique path

(3) T minimal connected (remove any edge $\Rightarrow$ disconnect)

(4) T max acyclic (add any missing edge $\Rightarrow$ cycle)

(5) connected & $|E| = |V| - 1$ || ⑥ acyclic & $(|E| = |V| - 1$

pick root ⊛ → level 0



are

≃ BFS alg