Recitation 9: Advanced Counting

Problem 1 Permutation cycles Three permutations are given for $n = 6$ as values in each position
$a = [425163]; b = [426135]; c = [351624]$.

i. Decompose each permutation into cycles

ii. Write a 3x3 table that computes all possible products of 2 of these. Each cell is the product of the row-permutation $\times$ col-permutation

	a	b	c
a			
b			
c			

Problem 2. Non-decreasing sequences How many non-decreasing sequences of length 8 are there if the values are integers in range [11:20] ? For example such sequence can be (12,12,14,16,16,19,20,20).

Problem 3 Fruit Share

In how many ways can 5 people divide 4 apples, 3 oranges, 6 bananas and 2 pears? People are distinguishable, but fruits of the same kind are not. All divisions are possible: a person can end up with no fruit, or can end up with all.

Problem 4 Sequence to Generative Functions

For each sequence below state the generative function (recap: a polynomial in compact form that has the given sequence as coefficients). a, b, c etc are constants; n is the largest degree; k indices run from 0 to ∞ unless otherwise indicated. For some its easier to compute; for others you can enumerate the terms and use Taylor Series, or look it up online.

i. $< a^k >=< a^0, a^1, a^2, a^4... >$

ii. $< \binom{n}{k} \cdot a^k >$

iii. ★ Prove using a combinatorial argument the following
$< \binom{n+k-1}{k} >\equiv \frac{1}{(1-x)^n}$
Use the fact that the LHS is the balls-into-bins count, and that RHS is $(1 + x + x^2 + ...)^n$

iv. $< \binom{n+k-1}{k} a^k >$

v. $< \frac{1}{k!} >$

vi. $< (-1)^{k+1}/k >$

Problem 5 Counting with Generative Functions

Use generating functions to determine the number of ways to insert tokens worth \$1, \$2, and \$5 into a vending machine to pay for an item that costs 17 dollars in these cases below.
You can use an online calculator for your GF coefficient such as `https://www.wolframalpha.com/input?` "SeriesCoefficient [GF, x,0,deg]"

i. The order in which the tokens are inserted does not matter

ii. ★ The order in which the tokens are inserted matters (inserting \$1 followed by \$2 is different from inserting \$2 followed by \$1.)

Problem 6 Check on Project 3: Valid Dates

i. Write a bullet plan for part A. How do you generate all possible dates? How to check validity condition on each ?

ii. Write a bullet plan for part B.
- What is the R() recurrence? What is R() close form?
- What is the decomposition of T(n) into R(k) and T(n-k) ?

Problem 7 (optional, no credit)

A certain computer room has 5 computers and 9 printers (all distinguishable). Computers have many ethernet ports; printers have one ethernet port.

i. Each printer must be connected to a computer. In how many ways can the connections be made?

ii. ★ Each printer must be connected to a computer and each computer must be connected to a printer. In how many ways can the connections be made?

iii. ★ Each printer must be connected to at least one computer and each computer must be connected to at least one printer. Each computer has 4 ports, and each printer 3 ports. In how many ways can the connections be made?