Recitation 7: Induction, Series (2)

Problem 1 Multiple of 4

Prove by induction that for all n odd positive integers $1 + 3^n$ is divisible by 4.

Problem 2 Fibonacci numbers properties by induction

i. $F_1 - F_2 + F_3 - F_4 + ... + (-1)^n F_{n+1} = (-1)^n F_n + 1$

ii. $F_1 F_2 + F_2 F_3 + F_3 F_4 + ... + F_{2n-1} F_{2n} = F_{2n}^2$

Problem 3 Approximation

i. Let $x > -1$ a real value. Prove by induction over $n \geq 0$ that $(1+x)^n \geq 1 + nx$

ii. Prove that $(\frac{n}{n+1})^n \geq \frac{1}{n+1}$ by using a particular x in the previous inequality.

Problem 4 Square Game Project check.

A) describe a specific strategy for your move. Include how to determine the row, and the number of tiles to delete.

B) Show a (partial) code setup.

PB 5 recurrences

i. If mergesort divided its input array into five pieces instead of two, calling mergesort on each piece and combining with a linear-time 5-way merge, what would Run-Time $T(n)$ its recurrence be?

ii. What is the asymptotic of the 5-way mergesort Run Time from previous exercise? Use the iterative method to drive recursion into a pattern than can be stated in terms of $k =$ number of iterations, then solve it by using "last-k" that makes recursive term be $T(1)$

iii. Show that the following recursion has asymptotic upper and lower bound $\Theta(n)$.
$T(n) = T(n/2) + T(n/4) + T(n/8) + n$

PB 6 ★★★ (optional, no credit)

Prove that the inverse-$n \log n$ series diverges : $\sum_{k=2}^{\infty} \frac{1}{n \log n} = \infty$