Recitation 6: Series, Induction, Square Game

Problem 1 Sequences For each of the following sequences, (1) indicate whether the sequence is "arithmetic," "geometric," "quadratic," or "none", and (2) give a close form formula that generates the terms. For example $3, 5, 7, 9, 11, \ldots$ is *arithmetic* with formula $x_n = 2n + 1$ starting at $n = 1$. For summations use a sequence formula to sum by indices; after some algebra you will need the indices sumations done in class.

i. $-5, -1, 3, 7, 11, 15, \ldots$

ii. $0, 5, 16, 33, 56, 85, \ldots$

iii. $6, 12, 24, 48, 96, 192, \ldots$

iv. $-9 - 4 + 1 + 6 + 11 + \cdots + 66$

v. $2 + 6 + 18 + 54 + \cdots + 1458$

Problem 2 Series by induction

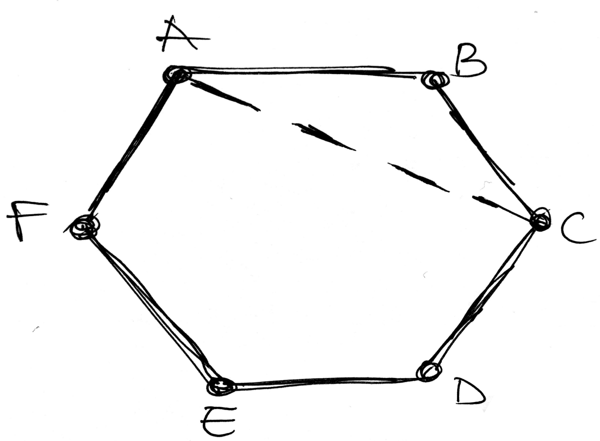
Prove the following series formulas by induction over n

i. $S_n = (1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16})....(1 - \frac{1}{n^2}) = \frac{n+1}{2n}$

ii. $S_n = \sum_{k=1}^{n} (3k-2)^2 = n(6n^2 - 3n - 1)/2$

iii. $S_n = \sum_{k=1}^{n} k * k! = (n+1)! - 1$

PB 3 Polygon sum of angles Prove that the sum of the interior angles of a convex polygon with n sides is $(n-2)\pi$. You can assume known that the sum of angles of any triangle is π



Problem 4 Square Game These are the same boards as last time. This time: as game progresses, write the count of each row in both decimal and binary. Is there a pattern of these counts that leads to winning? If yes, briefly describe it in bullet points.

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PB 5 Will everyone get the same grade? Jack Sparrow found a proof that every student in CS1800 will get the same exact grade, by induction over the number n of students in the class:

- Define the statement P_n="in a class of size n, all students get the same grade"
- Base step $n = 1, P_n$ is true, since there is only one student
- Inductive step: If a the class has size n, consider all n subsets of size $n - 1$. Since P_{n-1} is true, then all these subsets of students will get the same grade (per subset). But the subsets intersect, so it means all students will get the same grade, or P_n is true.

Where is this proof wrong?

PB 6 Josephine's Infidel Shooting ★ In Josephine's Kingdom every woman has to pass a logic exam before being allowed to marry. Every married woman knows about the fidelity of every man in the Kingdom except for her own husband, and etiquette demands that no woman should be told about the fidelity of her husband. Also, a gunshot fired in any house in the Kingdom will be heard in any other house. Queen Josephine announced that at least one unfaithful man had been discovered in the Kingdom, and that any woman knowing her husband to be unfaithful was required to shoot him at midnight following the day after she discovered his infidelity. How did the wives manage this? Explain your reasoning by using a formalism similar to an induction step.