Recitation 10: Probabilities, Bayes, Poker

Problem 1

Suppose that a permutation of the alphabet is chosen at random. What is the probability that

(a) The first 13 letters of the permutation are in alphabetical order?

(b) a is the first letter of the permutation and z is the last?

(c) a and z are next to each other?

(d) a and b are not next to each other?

Problem 2

A group of students grades 6-12 were surveyed about this favorite winter sports. The table is listed below but only shows grades 6-8.

Grade	Snowboarding	Skiing	Ice Skating	TOTAL
6th	68	41	46	155
7th	84	56	70	210
8th	59	74	47	180
TOTAL	711	471	363	1545

Using these 1545 students as the sample space, a student from this study is randomly selected.

1. What is the probability of selecting a student whose favorite sport is skiing

2. What is the probability of selecting a 6th grade student

3. If the student selected is a 7th grade student, what is the probability that the student prefers ice-skating?

4. If the student selected prefers snowboarding, what is the probability that the student is a 6th grade student?

Problem 3 Dragon Battle

Bob plays a game with his friends where they battle a dragon. A dragon has an "armor class", or AC, which is a non-negative integer. Bob has a to-hit modifier, an integer. To hit the dragon, Bob rolls a twenty-sided die and adds his to-hit modifier to the result. If Bob's number meets or exceeds the AC of the dragon, Bob "hits". To defend itself, the dragon can impose "disadvantage" on Bob. As a result, Bob must roll his twenty-sided die twice, each roll independent, and take the lower number before adding his modifier. This new number must beat the AC.

i. The dragon imposes "disadvantage" on Bob. The dragon has an AC of 13 and Bob's to-hit modifier is -2. You hear later that Bob hit the dragon. In this scenario, what was the probability Bob rolled a 15?

ii. You later find out that Bob actually *missed* the dragon in that scenario. What is the probability Bob rolled a 9 in that scenario?

Problem 4 Blind Sampling without Repetition An urn contains 100 balls. 60 of the balls are white and 40 of them are black.

(A) We take out one ball and put it aside without looking at its color. We then take out a second ball. What is the probability that the second ball is black?

(B) We put aside first two balls without looking at them. Then take out a third ball. What is the probability that the third ball is black?

Problem 5 Play Poker: Texas Hold'em

Form 3-5 tables of 4-6 students each. Purpose: obtain the highest value 5-card combination using any of the 2 "hole" cards and any of the "community" 5 cards. See picture for combinations ranked from high to low.

Chips value
$1=White $5=Red $25=Green $100=Black

One of you can act as the dealer. Game rules:

• everyone is dealt 2 "hole" cards. After looking at the cards, each player in clockwise order can decide to play ($1 fee) or to fold. The remaining players can do the first round of betting: in clockwise order, either
- "check" : don't change the pot, assuming no raise happened
- "call" : match highest bet
- "raise" : increase the the highest bet
- "fold" : give up and lose the current pot.

There can be at most 2 raises of $1 each for a total pot of $3/player.

• The dealer reveals face up the "flop" 3 community cards. Players update their chances of wining, and another round of betting follows up. At most three raises of at most $3 each to a pot of max $12/player.

• The dealer reveals face-up the "turn" a forth community card. Another round of betting of at most 3 raises of up to $3 each can take the pot to max $21/player

• The dealer reveals face-up the "river" a fifth and final community card. Last round of betting can be at most 3 raises of at most $5 each take the pot to max $36 /player.

Problem 6 (optional no credit) We chose a random (integer) number between 1 to 6, and then we (randomly) choose a second number that is either 0 or 1. What is the probability that the second number is 1 if we know that the sum of the numbers is greater than or equal to 5?

Problem 7 Small BlackJack (optional, no credit)

A game designer wants to propose a new game, "Small Blackjack", played with a dealer and a player. The dealer has a stack of 10 cards, each labeled with a unique integer from 1 to 10. The dealer shuffles the cards and draws one. The player then draws their own from the remaining. If the player's card is higher than the dealer's card, the player wins. Otherwise, the player loses.

- What is the probability the player draws a 7 and then the dealer loses?

- What is the probability the player draws a 3 and then the dealer wins?

Problem 8 (optional, no credit) You can play the following game 100 times:
You roll a die. If the result is x and x is even, you get x extra points to your grade in CS 1802. If the result is odd you lose 4.25 points.
Should you play this game?