Homework 4: Bases, Two's complement

Instructions:

- We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. **Cheating will not be tolerated**.
- To get full credit, **show INTERMEDIATE steps** leading to your answers, throughout.

Problem 1 Bases, Negative numbers, Two's complement

i. Convert the decimal integer 2011 to binary, octal, and hexadecimal.

ii. Give the 8-bit two's complement representations of the following integers: 27, 83, -75, -69.

iii. Compute the following sums and differences using 8-bit two's complement representations, as shown in class and described in the text: $-69 + 27$, $83 - 75$, and $-75 - 69$. In each case, indicate whether the calculation results in an overflow. If it does not result in an overflow, then verify that your answer is correct by converting the result back to standard base-10 notation.

iv. Give the integer (in standard base-10 notation) which is represented by each of the following 8-bit two's complement numbers: 10110011, 01100011, 01101111, 10001111.

v. What range of numbers can be represented in (a) 9-bit two's complement and (b) 13-bit two's complement? What are the minimum number of bits necessary to represent (c) 1994 and (d) -61 in two's complement?

vi. Convert the decimal numbers -54 and 29 to two's complement, assuming an eight bit two's complement representation (show details). Then add the two numbers in binary, and transform the result in decimal.

vii. ★ repeat the previous exercise in base 4 (4's complement for integers -54 and 29) with enough digits.

viii. multiply in unsigned-binary 71×15

ix. multiply in binary (convert first into two's complement 5 bits) integers : $(-10) * 3$

Problem 2 IP address formats

IP (version 4) addresses are 32-bit binary numbers, most commonly expressed in the dotted-decimal format in which the 32 bits are grouped into four bytes of 8 bits each, separated by the dot symbol, and each byte is written out in decimal form. Thus, the following IP address, written in binary,

10000001000010100111010011001000

is written as follows in the dotted decimal notation:

129.10.116.200

The first eight bits 10000001 form the number 129 in decimal notation, the next eight bits 00001010 form the number 10, the next eight bits 01110100 form the number 116, and finally the last eight bits 11001000 form the number 200.

IP addresses can also be represented in other formats, including hexadecimal and decimal. In fact, most browsers will accept IP addresses in these representations as well. These formats are used sometimes for purposes of obfuscation and identity-hiding.

- **iv.** Convert the IP address 205.203.140.65 from dotted decimal format to the hexadecimal and the decimal formats.
- **v.** Convert the IP address AA4B8EA1 from hexadecimal format to the dotted decimal and decimal formats.
- **vi.** Your organization has been allocated a block of IP addresses, that allows all addresses whose most significant 8 bits are 11001010. How many different IP addresses have been assigned to your organization? Use the approximation $2^{10} \approx 1,000$ and express your answer as an integral number of millions, billions, trillions, or whatever is necessary; for example, an answer of the form "12 billion" would be acceptable (though incorrect, in this case). Explain your answer, e.g., Why billion? Why 12?

Problem 3 Book Conversion two's complement formula

Assume the following definition of two's complement represenation on k bits of value $N = \overline{b_{k-1}b_{k-2}...b_1b_0}$: the power expansion works as $N = -b_{k-1}2^{k-1} + \sum_{t=0}^{k-2} b_t * 2^t$. That is :
· for positives, the first bit is $b_{k-1} = 0$ and the rest of the bits make the unsigned binary
· for negatives the first bit is $b_{k-1} = 1$, and the rest of the bits make the unsigned positive "complement" $C = N + 2^{k-1}$.

With this definition, prove that the positive-unsigned-flip-bits-add-1 rule from the book is correct for computing two's complement representation of negative numbers.

Problem 4 Divisibility by 3 in binary

It turns out if an unsigned binary number's 1's are all arranged in pairs, the number in decimal is divisible by 3. For instance, 110011 and 111100 are both divisible by 3 in decimal.

i. Suppose the unsigned binary number only has a single pair of 1's located at bits k and $k+1$.

- What is the decimal conversion of the binary number?
- Explain why this number is divisible by 3 in decimal in a couple of sentences.

ii. Now suppose the unsigned binary number has n pairs of 1's. Assume any unsigned binary number with $n-1$ pairs of 1's is divisible by 3 in decimal. Briefly explain why a number with n pairs of 1's is divisible by 3 in decimal. (Hint: Rewrite the binary number as a sum of a binary number with a single pair of 1's and a number with $n-1$ pairs of 1's)

iii. Is it true that all unsigned binary numbers divisible by 3 are arranged in pairs of 1's? If so, briefly explain why. If not, give an example.

iv. State and prove a general criteria of divisibility with 3, in base 2.

Problem 5 ★ Balanced Ternary

Balanced ternary is a non-standard positional numeral system (a balanced form), useful for comparison logic. While it is a ternary (base 3) number system, in the standard (unbalanced) ternary system, digits have values 0, 1 and 2. The digits in the balanced ternary system have values -1, 0, and 1. Different sources use different glyphs used to represent the three digits in balanced ternary. In this article, T (which resembles a ligature of the minus sign and 1) represents -1, while 0 and 1 represent themselves.

(`http://en.wikipedia.org/wiki/Balanced_ternary`)

So the 2-trit balanced-ternary numbers are

balanced-ternary	TT	$T0$	$T1$	$0T$	00	01	$1T$	10	11
decimal	-4	-3	-2	-1	0	1	2	3	4

i. Express each of the following decimal numbers as a 4-trit balanced-ternary number.

8, 15, 23, 32, -32

ii. The best-known application of balanced ternary notation is in mathematical puzzles that have to do with weighing. Given a two-pan balance, you are asked to weigh a coin known to have some integral weight between 1 gram and 40 grams. How many measuring weights do you need? A hasty answer would be six weights of 1, 2, 4, 8, 16 and 32 grams. If the coin must go in one pan and all the measuring weights in the other, you can't do better than such a powers-of-2 solution. If the weights can go in either pan, however, there's a ternary trick that works with just four weights: 1, 3, 9 and 27 grams. For instance, a coin of 35 grams will balance on the scale when weights of 27 grams and 9 grams are placed in the pan opposite the coin and a weight of 1 gram lies in the same pan as the coin.

Prove that every coin up to 40 grams can be weighed in this way by describing a procedure to find the power-of-3 weights on each side. This is in effect the balanced-ternary representaion.

Problem 6 ★ N is an integer whose representation in base b is 777. Find the smallest positive integer b for which N is the fourth power of an integer.