1 Summary: Sets, Counting, Permutations & Combinations

• Set builder notation
A = {positive integers smaller than 100 divisible with 7} = $\{x|x \in Z; 0 < x < 100; 7|x\}$

• count A via indexing
$A = \{x|x \in Z; 0 < x < 100; 7|x\} = \{7 * i|i \in Z; 1 \leq i \leq 14\} \Rightarrow |A| = 14$

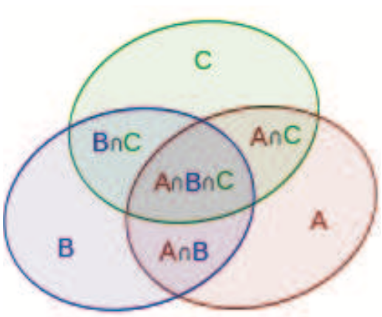
• union, intersection, set difference, set symmetric difference
$A \cup B = \{x|x \in A \text{ OR } x \in B\}$
$A \cap B = \{x|x \in A \text{ AND } x \in B\}$
$A - B = \{x|x \in A \text{ AND } x \notin B\}$
$A \Delta B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

• Cartesian Product $A \times B = \{(x, y)|x \in A; y \in B\}$

• Product Rule: if any element from A can be combined with any element from B, then the number of combinations is $|A \times B| = |A| * |B|$

• power set of A is $\mathbf{P}(A)$ = the set of all subsets of A, including A and $\emptyset$; $|\mathbf{P}(A)| = 2^{|A|}$. Example: $A = \{x, y, z\}$. Then $\mathbf{P}(A) = \{\emptyset; \{x\}; \{y\}; \{z\}; \{x, y\}; \{x, z\}; \{y, z\}; \{x, y, z\}\}$

• inclusion-exclusion principle : $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$



• Sum (Partition) Rule : if $\emptyset = |A \cap B| = |A \cap C| = |B \cap C|$ then $|A \cup B \cup C| = |A| + |B| + |C|$

• Pigeonhole Principle: if n items are put into k boxes, then at least one box contains at least $\lceil \frac{n}{k} \rceil$ items

• Permutations $P(n, k)$ ways to choose a sequence of k items out of n. ORDER MATTERS.
$P(n, k) = n * (n - 1) * (n - 1) * ... * (n - k + 1) = \frac{n!}{(n-k)!}$

• Combinations $C(n, k) = \binom{n}{k}$ ways to choose a set of k items out of n. ORDER DOESNT MATTER.
$\binom{n}{k} = n * (n - 1) * (n - 1) * ... * (n - k + 1)/(1 * 2 * ... * k) = \frac{n!}{k!(n-k)!}$

• $P(n, k)$ can be thought of as choosing a set of k, then permute the elements chosen in all $k!$ ways. Thus $P(n, k) = \binom{n}{k} * k! = \frac{n!}{k!(n-k)!} * k! = \frac{n!}{(n-k)!}$

• $\binom{n}{k} = \binom{n}{n-k}$ since choosing k items to "take" is equivalent to choosing $n - k$ to "remain"

• Binomial Theorem
$(x + y)^n = \binom{n}{0} x^0 y^n + \binom{n}{1} x^1 y^{n-1} + ... \binom{n}{n} x^n y^0 = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

$x=1, y=1: 2^n=(1+1)^n=\sum_{k=0}^{n}\binom{n}{k}=\binom{n}{0}+\binom{n}{1}+...+\binom{n}{n}$

$x=-1, y=1: 0=(-1+1)^n=\sum_{k=0}^{n}\binom{n}{k}(-1)^k=\binom{n}{0}-\binom{n}{1}+...+\binom{n}{n}(-1)^n$

• Pascal Formula $\binom{n+1}{k}=\binom{n}{k}+\binom{n}{k-1}$

• Pascal Triangle applies his formula at every row:

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	$\binom{n}{5}$ ...
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5	10	10	5	1

• Balls in Bins: The number of ways to throw indistinguishable n balls into m distinguishable bins $B_1, B_2, ..B_m$ is counted by choosing $m-1$ bin-separator locations out of $n+m-1$ spots for balls and separators.
For example for $n=10, m=6$ the throw with bin counts 1-2-4-1-0-2 corresponds to the following choices of $m-1=5$ separator locations among $m+n-1=15$ balls+separators: •| • •| • • • •| • || • •
Balls in Bins count is $\binom{n+m-1}{m-1}$

• circular permutation : when sitting n people at a round table the number of ways to sit them (not including rotations) is by keeping one chair-fixed and permute the rest; thus $(n-1)!$ ways.

• counting with bijective functions: if a bijection (pairing) can be established between set A and set B then they have the same size, $|A|=|B|$. A bijection function f has two properties
- injectivity (different arguments produce different values): $\forall x, y \in A; x \neq y \Rightarrow f(x) \neq f(y)$
- surjectivity (all B are possible function values): $\forall v \in B, \exists x \in A, f(x)=v$