CS1800-Midterm practice super-problems ★

These problems are definitely harder than the midterm (even the ones without ★), so if you solve them you should have no problem at all with the exam. However be aware that not all concepts are covered in these problems, so make sure to recap:

- binary/hex/octal representations and tranformations, powers of 2, fast multiplication

- logic operators, truth tables, dnf/cnf and logic rules, logic gates, circuits

- sets operations, power set, cartesian products, inclusion-exclusion

- product rule, permutations, combinations, balls in bins, binomial theorem, Pascal triangle

EC 5 (difficulty ★). We have 100 red balls and 150 blue balls, and 20 bins; each bin must contain at least as many blue balls as red ones. In how many ways we can arrange all 250 balls into these 20 bins?

EC 6. In how many ways 20 husband-wife couples can sit at a round table with 40 seats unnumbered, such that every husband sits next to his wife?

EC 7 (difficulty ★★★). What is the probability of 20 husband-wife couples sitting randomly at a round table with 40 seats unnumbered, that no husband sits next to his wife?

EC 8 (difficulty ★★). Show that picking a random positive integer (say int 4 bytes), the chance of getting a prime is smaller than 5%.

EC 10 . A is a set with 21 elements. How many subsets of A have size multiple of 4?

EC 11 (difficulty ★). Given any 7 integers, show that there are 2 of them with either sum or difference or product = multiple of 15.

EC 12 . Given any 5 integers a,b,c,d,e show that there is a nonempty subset of them that with +,- operands gives a multiple of 31 (i.e. at least one of these is multiple of 31: +a; +a+b ; -a+c-d ; -b-d; -c ; c-b-d+e, etc)

EC 13 (difficulty ★★). At bridge card game, the 52 cards in the deck (4 suits each with numerals 2-10 and JQKA) are evaluated as numerals=0points; J=1 point; Q=2points; K=3points; A=4points. A "bridge hand" is a subset of any 13 cards. How many different hands have at least 12 points?

EC 14.(★) NeverWin Casino uses Red dice on Mon-Fri which are uniform on all faces; and Blue dice on Sat+Sun, which are 50% likely to come out "6" and then equaly likely 10% for the other faces "1"-"5". You remember that you rolled a particular faboulos "6" to get the big win, but dont remember the day, since you played every day during that time.
- What is the chance that the big win came on Saturday?
- What is the chance that the "6" roll for the big win came from the Red die?

EC 17 (difficulty ★★★). In how many ways can we parenthesize (orders of operations) the multiplication of 10 values $abcdefghij$? To be clear, there

are 5 ways to multiply 4 numbers *abcd*:
$((ab)c)d$; $(a(bc))d$; $(ab)(cd)$; $a((bc)d)$; $a(b(cd))$

EC 18 (difficulty ★). Show that $\binom{22}{11}$ is a multiple of 12

EC 19 . Show that the number obtained by concatenating "337" with itself 5 times 337337337337337 is not prime.

EC 21. A group of 11 students is such that everyone has exactly 10 friends in the group (including himself/herself; friends are reciprocal). Thats impossible.

EC 22. Show that $\sqrt{12}$ is not a ratio of two integers

EC 23. Two positive integers a, b have both the sum and the difference a power of 2. Show that neither of them is a power of two, but their gcd is.

EC 24 (difficulty ★★★). Evil Search Engine finds for a query 15 relevant URL results, and while keeping them ranked in a given score-relevance order, it is mixing them together with other 15 random ads; then serves them as a ranked list of 30. How many arrangements of 30 are there (order among ads doesnt matter) so that when served as 3 pages each with 10 links, each page contains at least 3 relevant and at least 3 ads ?

EC 26 (difficulty ★). A = {1,2,3,4,5,6,7}. How many sequences of length 20 with elements from A are never decreasing ?