Consider two independent Random variables A, and B, now I know that, **E[A+B] = E[A] + E[B]**, **E[AB] = E[A] * E[B]**.

I am looking for a prove of these properties, I am successful in proving the first one, but I am unable to prove the 2nd property.

Can anyone throw some guideline, or a starting point for the second proof?

Screen Shot 2017-11-07 at 12.28.58 AM.png Open with Preview

$$\begin{aligned}
\mathrm{E}(XY) &= \sum_{\omega\in\Omega} X(\omega)Y(\omega)\mathrm{Pr}(\omega) \\
&= \sum_x \sum_y xy \cdot \mathrm{Pr}(X = x \text{ and } Y = y) \\
&= \sum_x \sum_y xy \cdot \mathrm{Pr}(X = x)\mathrm{Pr}(Y = y) \\
&= \left(\sum_x x \cdot \mathrm{Pr}(X = x)\right)\left(\sum_y y \cdot \mathrm{Pr}(Y = y)\right) \\
&= \mathrm{E}(X)\mathrm{E}(Y),
\end{aligned}$$

asked Oct 7 '15 at 9:19

Johnny

36 ▲5

active oldest votes

In the finite case with $E[A] = \sum_j p_j a_j$ (where $p_j = P(A = a_j)$) and $E[B] = \sum_k q_k b_k$ (where $q_k = P(B = b_k)$), we have

$$E[AB] = \sum_{j,k} p_j q_k a_j b_k = \sum_j p_j a_j \cdot \sum_k q_k b_k = E[A]B[K]$$

(where by independece $p_j q_k = P(A = a_j)P(B = b_k) = P(A = a_j, B = b_k)$).

$$\mathsf{Var}(X) = \mathsf{E}(X^2) - (\mathsf{E}(X))^2.$$

Consequently, $\mathsf{Var}(X) \leq \mathsf{E}(X^2)$.

Proof. Using the linearity of expectation and the fact that the expectation of a constant is itself, we have

$$\begin{aligned}
\mathsf{Var}(X) &= \mathsf{E}(X - \mathsf{E}(X))^2 \\
&= \mathsf{E}(X^2 - 2X\mathsf{E}(X) + (\mathsf{E}(X))^2) \\
&= \mathsf{E}(X^2) - 2\mathsf{E}(X)\mathsf{E}(X) + (\mathsf{E}(X))^2 \\
&= \mathsf{E}(X^2) - (\mathsf{E}(X))^2
\end{aligned}$$

Proposition 6.11. *Given a discrete probability space* (Ω, Pr)*, for any random variable* X *and* Y*, if* X *and* Y *are independent, then*

$$\mathsf{Var}(X + Y) = \mathsf{Var}(X) + \mathsf{Var}(Y).$$

Proof. Recall from Proposition 6.9 that if X and Y are independent, then $\mathsf{E}(XY) = \mathsf{E}(X)\mathsf{E}(Y)$. Then, we have

$$\begin{aligned}
\mathsf{E}((X + Y)^2) &= \mathsf{E}(X^2 + 2XY + Y^2) \\
&= \mathsf{E}(X^2) + 2\mathsf{E}(XY) + \mathsf{E}(Y^2) \\
&= \mathsf{E}(X^2) + 2\mathsf{E}(X)\mathsf{E}(Y) + \mathsf{E}(Y^2).
\end{aligned}$$

Using this, we get

$$\begin{aligned}
\mathsf{Var}(X + Y) &= \mathsf{E}((X + Y)^2) - (\mathsf{E}(X + Y))^2 \\
&= \mathsf{E}(X^2) + 2\mathsf{E}(X)\mathsf{E}(Y) + \mathsf{E}(Y^2) - ((\mathsf{E}(X))^2 + 2\mathsf{E}(X)\mathsf{E}(Y) + (\mathsf{E}(Y))^2) \\
&= \mathsf{E}(X^2) - (\mathsf{E}(X))^2 + \mathsf{E}(Y^2) - (\mathsf{E}(Y))^2 \\
&= \mathsf{Var}(X) + \mathsf{Var}(Y),
\end{aligned}$$