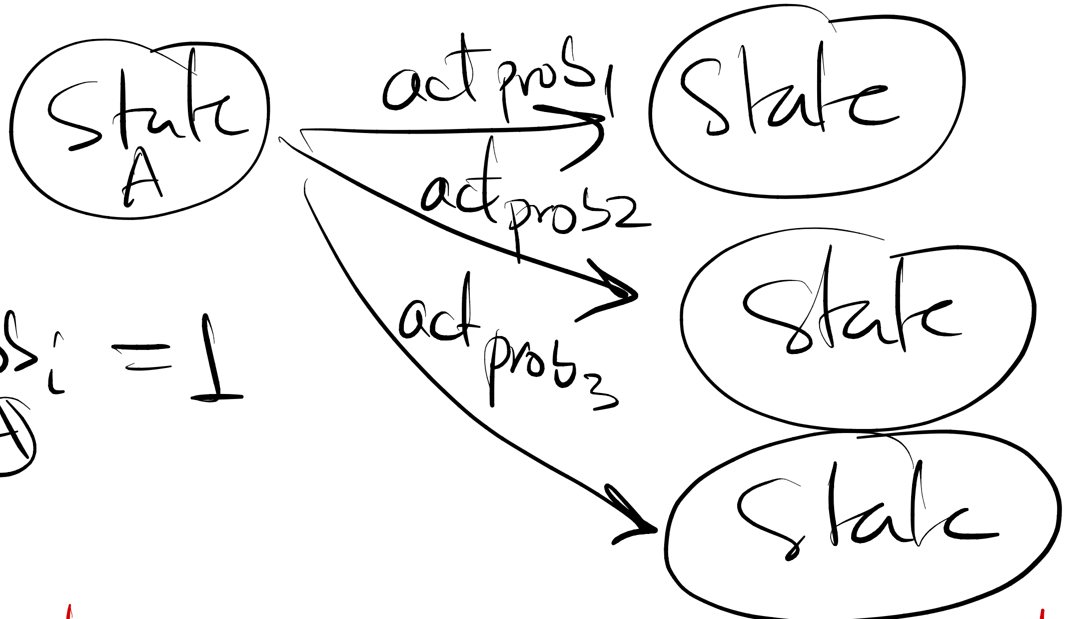
Markov Chains : transition state $\xrightarrow{\text{action}}$ state

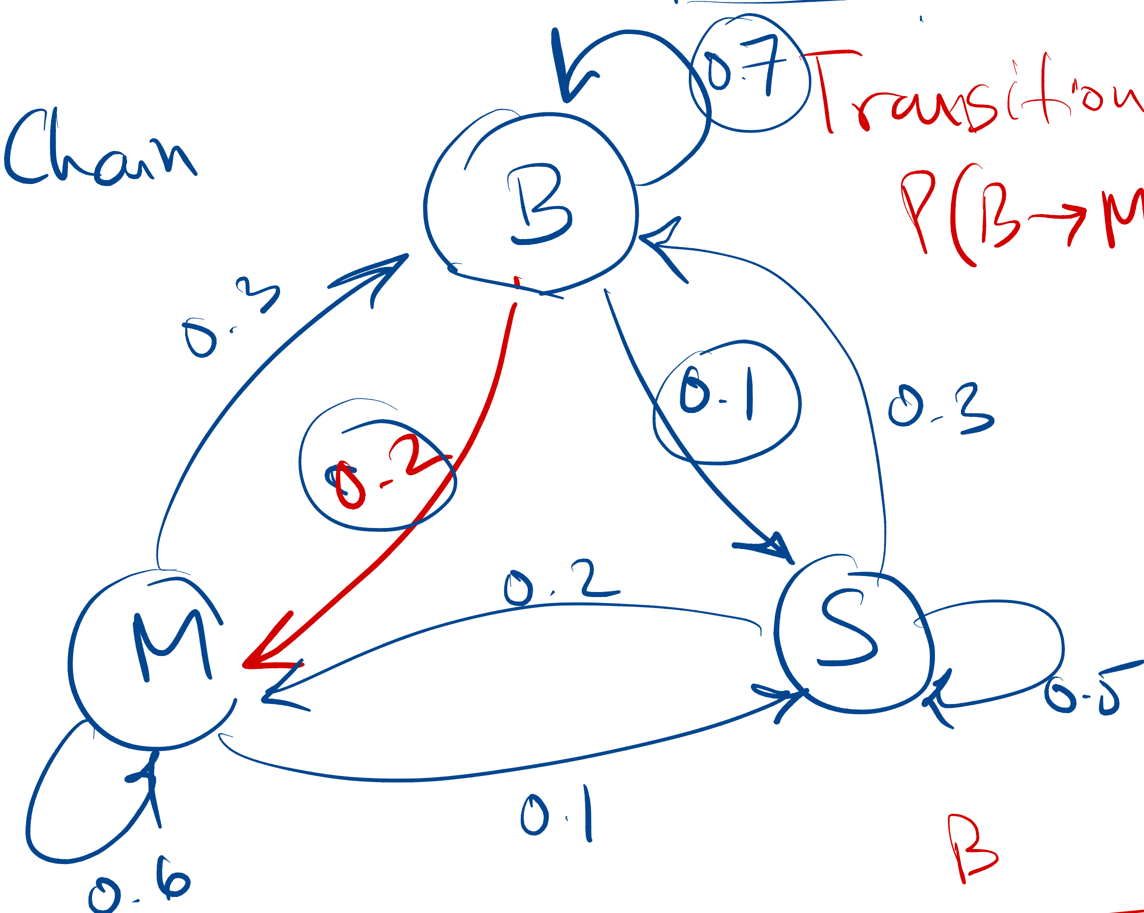


$$\sum_i \text{act prob}_i = 1 \quad \text{for } A$$

memoryless : the prob (destination state) depends only on current state (no past)

Markov Example: Boston population going to eat out every day ; restaurants B, M, S.

Markov Chain



Transition prbs $P(B \to M) = 0.2$

Trans prbs Matrix

$$P = \begin{array}{c|ccc} & B & M & S \\ \hline B & .7 & .2 & .1 \\ M & .3 & .6 & .1 \\ S & .3 & .2 & .5 \end{array}$$

distribution of agents/population in day i

at restaurants B, M, S

$i=0$

$$\overline{\pi}_0 = \begin{array}{ccc} B & M & S \\ 1/3 & 1/3 & 1/3 \end{array}$$

$$\overline{\pi}_{i+1} = \pi_{i+1}^{B} \qquad \pi_{i+1}^{S}$$

$$\pi_i^B \cdot P(B \to B) + \pi_i^M \cdot P(M \to B) + \pi_i^S \cdot P(S \to B)$$

$$\pi_{i+1}^S = \pi_i^B \cdot P(B \to S) + \pi_i^M \cdot P(M \to S) + \pi_i^S \cdot P(S \to S)$$

$$\pi^B_{next} = \pi^B_{current} \cdot .7 + \pi^M_{curren} \cdot .3 + \pi^S_{curre} \cdot .3$$

$$\underset{1 \times n}{\overline{\pi}_{i+1}} = \underset{1 \times n}{\overline{\pi}_i} \cdot \underset{n \times n}{P}$$

Math $\Rightarrow$ $\pi^* \cdot 1 = \pi^* \cdot P$ $\Rightarrow$ π^* = eigen vect (P)

convergence dist

STATIONARY

eigen val = 1

π^* = " importance / popularity " of each restaurant B, M, S

• unless rare/weird situation (periodicity) $\Rightarrow \exists\ \pi^*$

dist $\pi^* = \pi^* \cdot P$

B, M, S

$\pi^*_B = B \; ; \; \pi^*_M = M \; ; \; \pi^*_S = S$

$B = .7B + .3M + .3S$

$M = .2B + .6M + .2S$

$S = .1B + .1M + .5S$

incomple rank

$-.2B + .4M - .2S = 0$

$.1B - .1M + .5S = 0$

$B + M + S = 1$

$\times 5$

$-B + 2M - S = 0$

$\times 10$

$-B - M + 5S = 0$

$3M = 1 \Rightarrow M = 1/3$

$6S = 1 \quad\;\; S = 1/6 \Rightarrow B = 1/2$

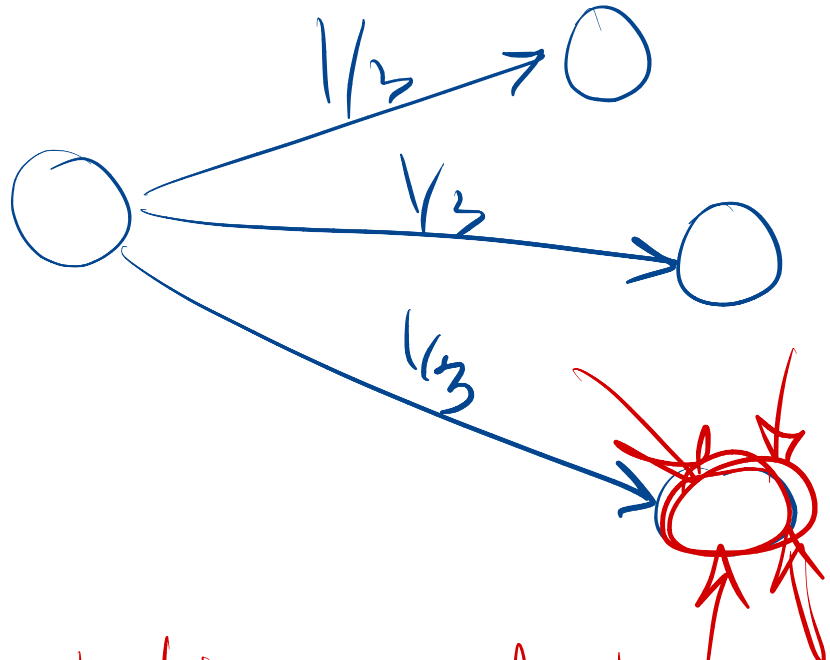
Page Rank

states = webpages.

actions/edges = links.

probs = uniform (default)

no-unif (by design)



not realistic for web browsing

PageRank = stationary dist (same Markov Chain)

- stuck $\Rightarrow$ teleportation (still easy math): $\left\{ \begin{array}{ll} \text{transition} & 85\% \\ \text{teleport random} & 15\% \end{array} \right.$

- at scale different math method
  (not eigenvalue)

instead manual/iterative convergence

$$\pi_0 = \text{init}$$

$\pi_{i+1} = \pi_i \cdot P$ untill convergence.