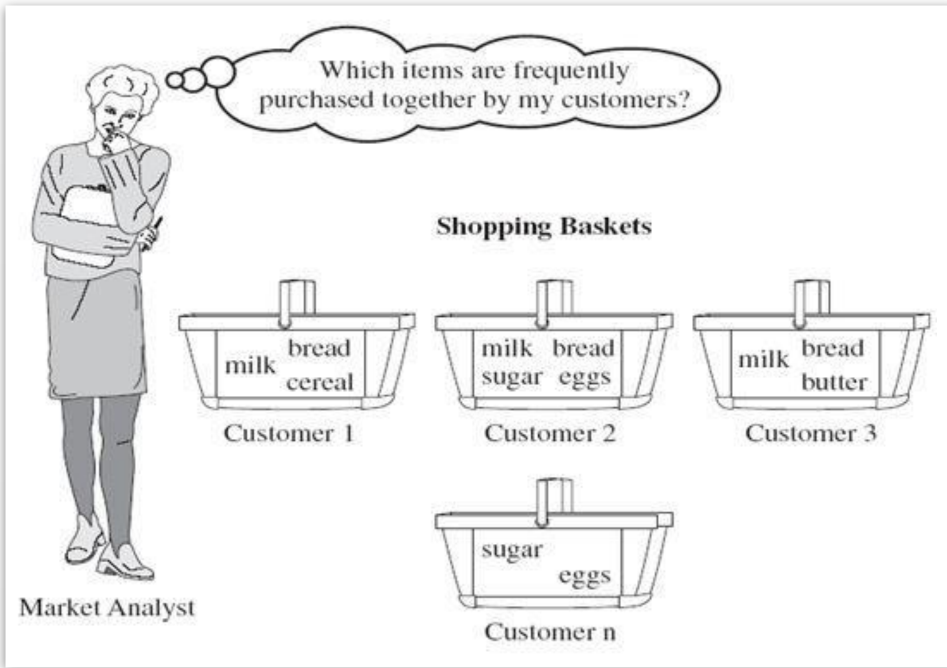
Mining Frequent Itemsets with A-Priori

Market Basket Analysis



Baskets of items

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

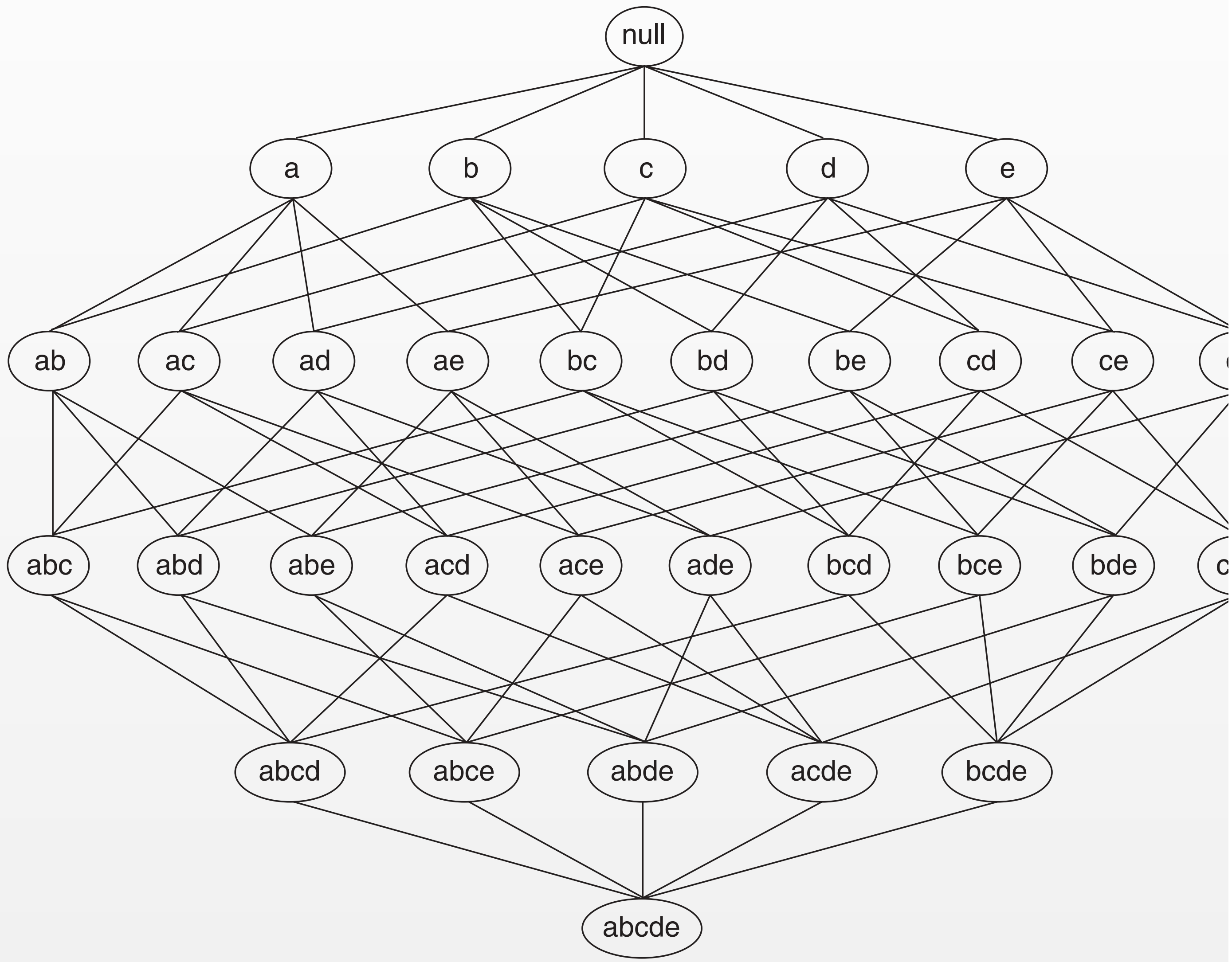
Association Rules

{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

Finding Frequent Item Sets

e the set of all items

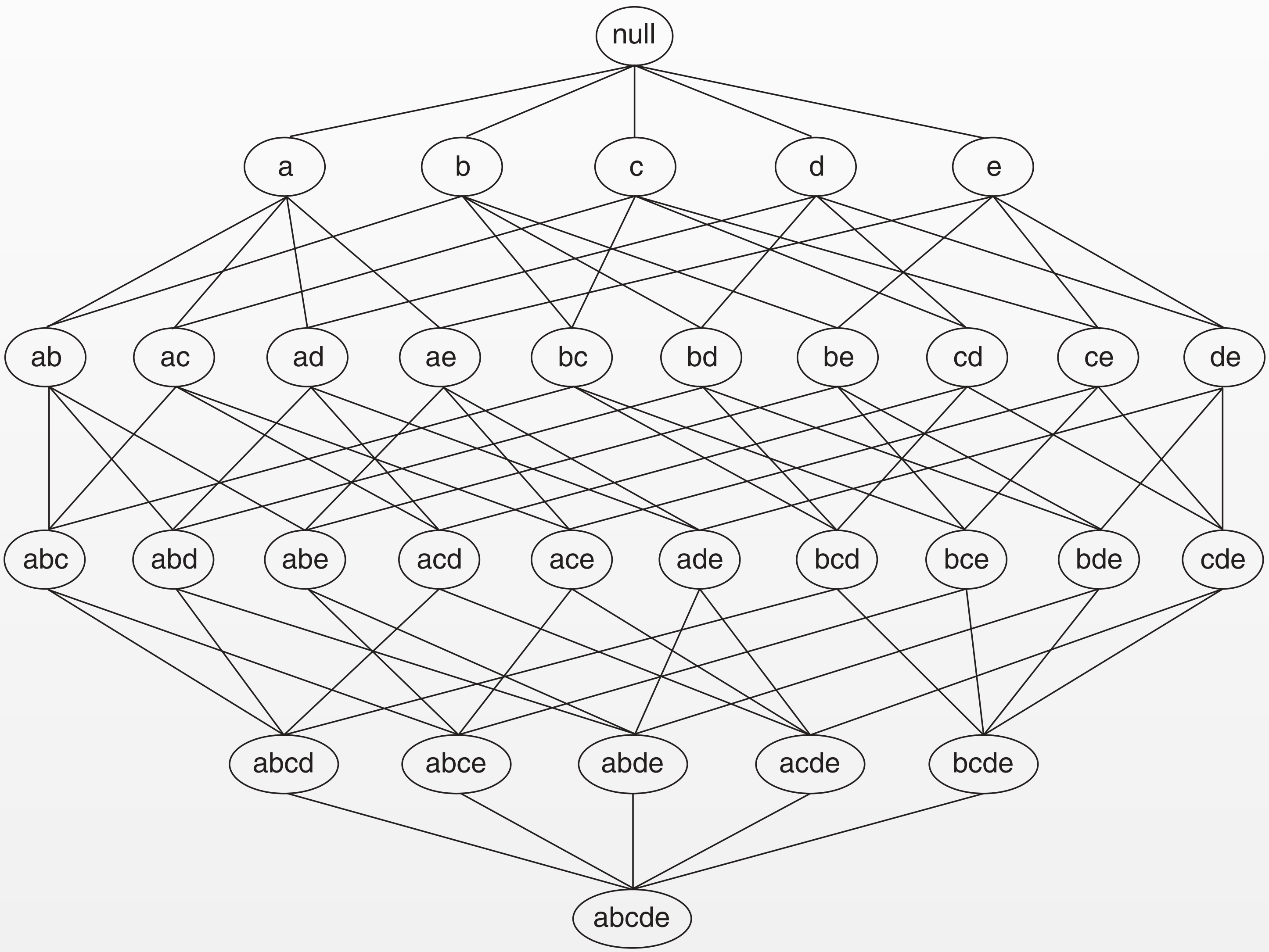
$M = |I|$, how
any possible
temsets are
there?



adapted from: Tan, Steinbach & Kumar, “Introduction to Data Mining”, http://www-users.cs.umn.edu/~kumar/dmbook/ch6.pdf

Finding Frequent Item Sets

Answer: $2^M - 1$; Cannot enumerate all possible sets



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Anti-monotone Property

A function f (defined on sets) is said to follow the anti-monotone property if

$$\forall A, B \in 2^I : \quad A \subseteq B \Rightarrow f(A) \geq f(B)$$

I is the set of all items
2^I denotes the power set of I

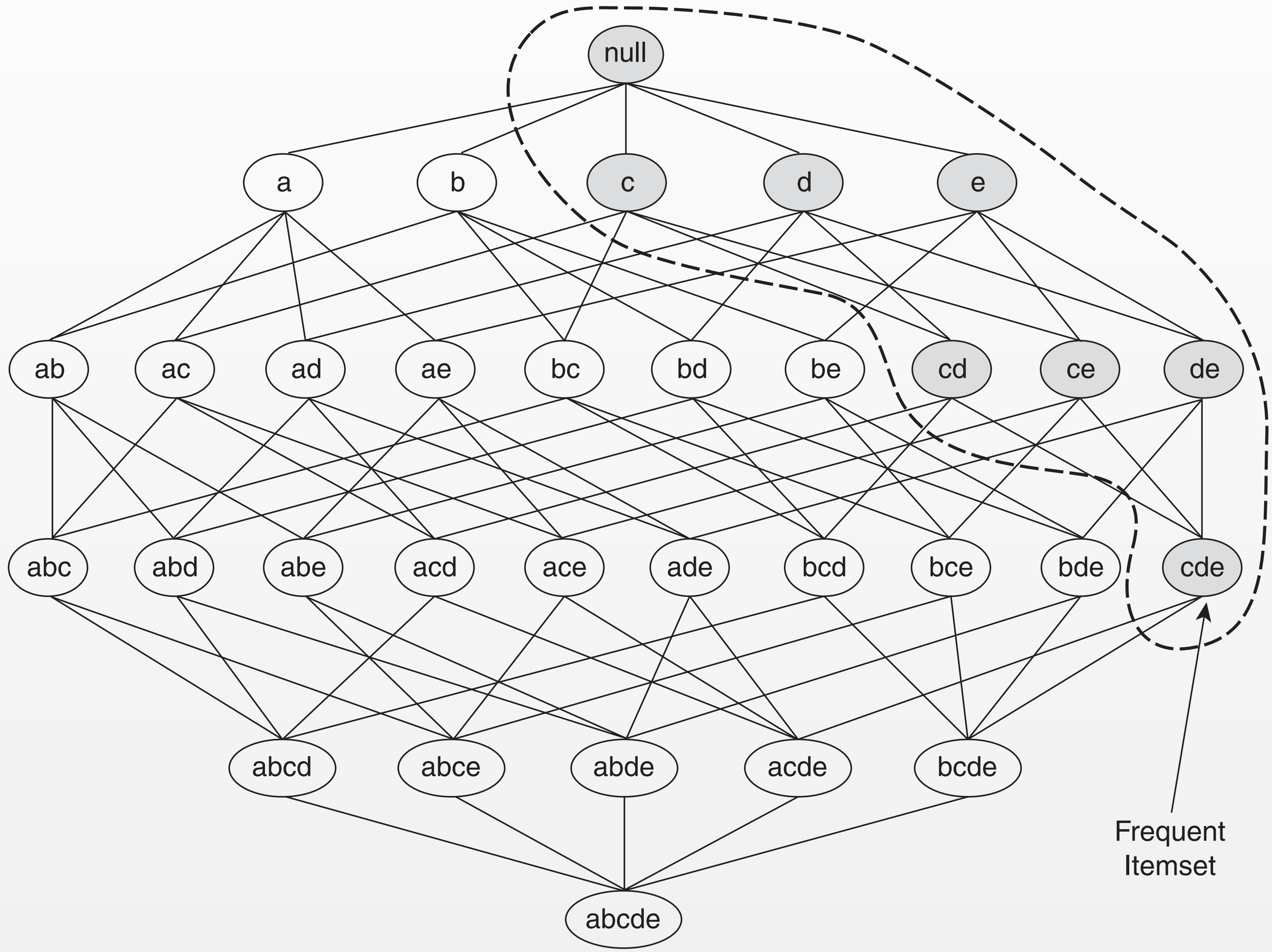
Support follows the anti-monotone property

$$\sigma(A) = |\{t \in T : A \subset t\}|$$

$\sigma : 2^I \rightarrow \mathbf{N}$
$N = \{0, 1, \ldots, \infty$
set of natural nur
T: the set of all tr

Intuition: A-priori Principle

Observation: Subsets of a frequent item set are **also** frequent



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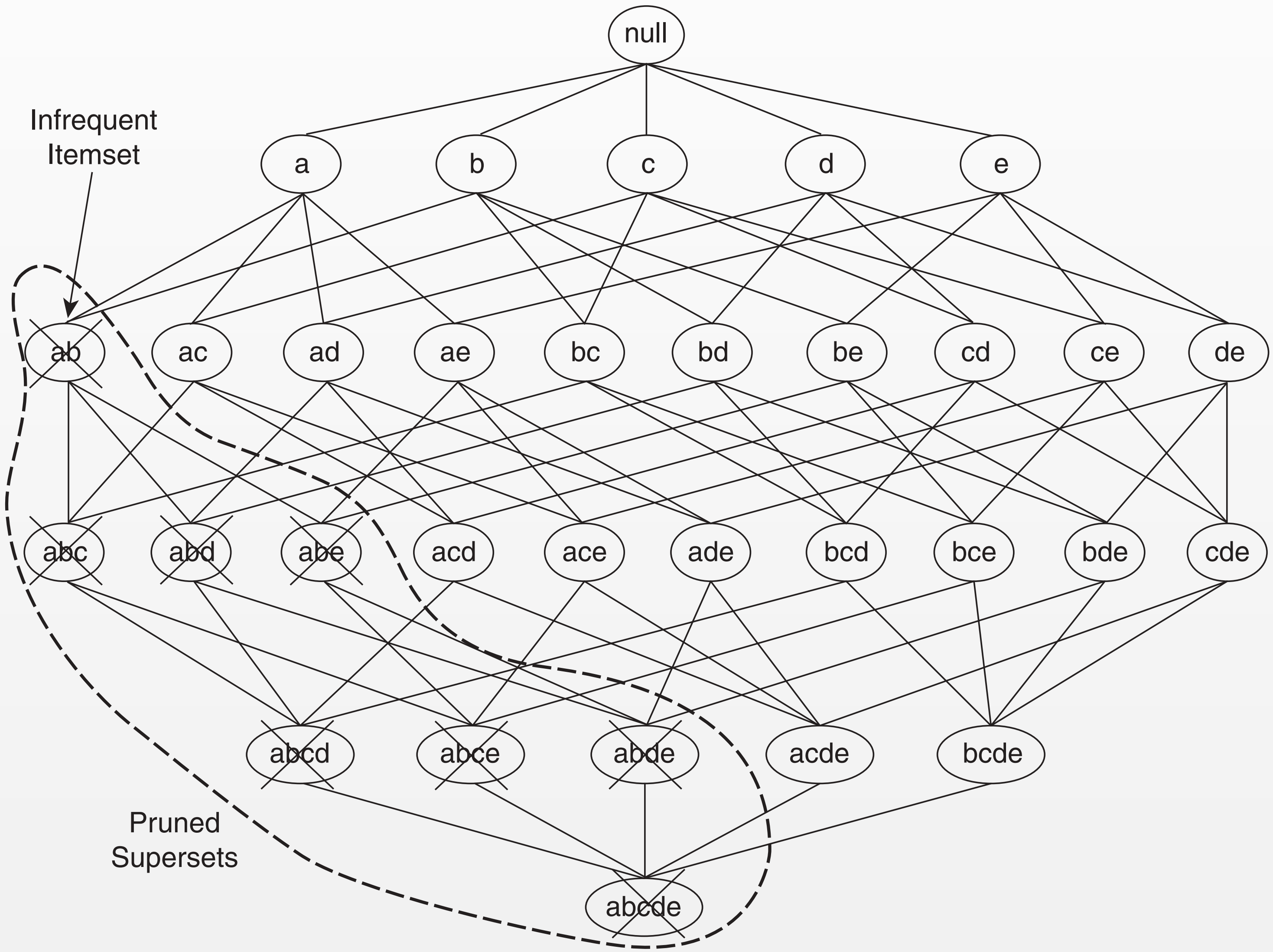
Intuition: A-priori Principle

ollary: If a set is **not** frequent, then its supersets are **also** not freq



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A-priori Algorithm

1. Find all frequent itemsets of size 1
   (*only have to check* $M = |I|$ *possible sets*)
2. For $k = 1,2,\ldots M$
   - Extend frequent itemsets of size $k-1$ to create *candidate* itemsets of size k
   - Find candidate sets that are frequent

A-priori Algorithm

Algorithm 6.1 Frequent itemset generation of the *Apriori* algorithm.

$k = 1$.
$F_k = \{\ i \mid i \in I \wedge \sigma(\{i\}) \geq N \times minsup\}$. {Find all frequent 1-itemsets}
repeat
  $k = k + 1$.
  $C_k =$ apriori-gen(F_{k-1}). {Generate candidate itemsets}
  for each transaction $t \in T$ **do**
    $C_t =$ subset(C_k, t). {Identify all candidates that belong to t}
    for each candidate itemset $c \in C_t$ **do**
      $\sigma(c) = \sigma(c) + 1$. {Increment support count}
    end for
  end for
  $F_k = \{\ c \mid c \in C_k \wedge \sigma(c) \geq N \times minsup\}$. {Extract the frequent k-itemsets}
until $F_k = \emptyset$
Result $= \bigcup F_k$.

Interpret minsup as a fraction here

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tching Transactions to Candidate S

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```
k = 1.
F_k = { i | i ∈ I ∧ σ({i}) ≥ N × minsup}.    {Find all frequent 1-itemsets}
repeat
  k = k + 1.
  C_k = apriori-gen(F_{k−1}).    {Generate candidate itemsets}
  for each transaction t ∈ T do
    C_t = subset(C_k, t).    {Identify all candidates that belong to t}
    for each candidate itemset c ∈ C_t do
      σ(c) = σ(c) + 1.    {Increment support count}
    end for
  end for
  F_k = { c | c ∈ C_k ∧ σ(c) ≥ N × minsup}.    {Extract the frequent k-itemsets}
until F_k = ∅
Result = ⋃ F_k.
```

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Generating Candidates C_k

Objectives

1. ***No Duplicates:*** A candidate itemsets must be unique.

2. **Completeness:** At least, all frequent k-itemsets should be included.

3. **No infrequent subsets**: A candidate should not have any infrequent subset.

Generating Candidates C_k

stions

ɔw many k-itemsets are there?

ɔw to reduce number of candidates using the computations ready performed?

$F_{k-1} \times F_1$*: combine frequent (k-1)-itemsets with frequent 1-itemsets to get candidates of size k.*

$$\{a, b, c\} \cup \{d\} = \{a, b, c, d\}$$

$F_{k-1} \times F_{k-1}$*: combine frequent (k-1)-itemsets with other freque (k-1)-itemsets that differs in only 1 item to get candidates of siz k.*

$$\{a, b, c\} \cup \{a, b, d\} = \{a, b, c, d\}$$

Generating Candidates C_k

uplicates

Combining sets arbitrary pairs of sets from F_{k-1} and F_1 will lead to duplicate candidates

$$\{a,b,c\} \cup \{d\} = \{a,b,c,d\}$$
$$\{a,b,d\} \cup \{c\} = \{a,b,c,d\}$$

Each candidate of size could be generated k times

Combining sets arbitrary pairs of sets from F_{k-1} will lead to duplicate candidates

$$\{a,b,c\} \cup \{a,b,d\} = \{a,b,c,d\}$$
$$\{a,b,c\} \cup \{a,c,d\} = \{a,b,c,d\}$$

Each candidate of siz could be generate $\binom{k}{k-2}$ times

Generating Candidates C_k

tion to duplicates

Sort the items

- **Item ordering:** Define an ordering on all items
  - Either by assigning a unique id to each item. The items are ordered based on their ID.
  - Or by a lexicographic ordering on the item string; e.g. 'coke' < 'cookie'
- Assume that the items in the itemsets in F_{k-1} are sorted. $a_1 < a_2 < \ldots < a_k$ in $A = \{a_1, a_2, \ldots, a_k\}$

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Generating Candidates C_k

tion to duplicates

$\times F_1$: Combine $A \in F_{k-1}$ and $B = \{b\} \in F_1$ only if $\forall a \in A \quad a < b$.

ombine $\{a, c, e\}$ with $\{f\}$ to give candidate $\{a, c, e, f\}$.

o not combine $\{a, c, e\}$ with $\{d\}$.

o duplicates: Each candidate has only one way of being enerated. $\{a, c, e, f\}$ can only be generated by combining $\{a, c, e$ nd $\{f\}$.

ompleteness: If $\{a, c, e, f\}$ is indeed frequent, $\{a, c, e\}$ and $\{f\}$ hav be present in F_3 and F_1. And they would get combined to enerate $\{a, c, e, f\}$.

ubsets of generated candidates might still be infrequent

Cor
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Generating Candidates C_k

tion to duplicates

Combine
$k-2$ items
are the san
last element
than tha

- $\times F_{k-1}$: Combine $A \in F_{k-1}$ and $B \in F_{k-1}$ only if
  $_i$, for $i = 1,2\ldots k-2$ and $a_{k-1} < b_{k-1}$.
  - ombine $\{a, c, e\}$ with $\{a, c, f\}$ to give candidate $\{a, c, e, f\}$.
  - o not combine $\{a, c, e\}$ with $\{a, b, e\}$.
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- ubsets of generated candidates might still be infrequent

Generating Candidates C_k

to efficiently find itemsets that could be combined

set Ordering: Use the ordering on items to define an ordering
sets

sume that the items in each itemset are presorted.

$< a_2 < \ldots < a_k$ in $A = \{a_1, a_2, \ldots, a_k\}$

$< B$ if $a_i < b_i$, where i is the index of the first item differing in A an

ple, bread, coke, sauce$\}$ < $\{$apple, bread, cookie, milk$\}$ or $\{4,7,21,50\} < \{4,7$

ements of F_{k-1} sorted with the itemset ordering

$\{a, b, c\}, \{a, b, e\}, \{a, b, g\}, \{a, c, d\}, \{a, c, g\} \ldots$

$_{k-1}$

$b, c\}$ can't be combined with any
nset beyond $\{a, b, g\}$. So no need
to compare beyond $\{a, c, d\}$

Without exploiting the itemset
$|F_{k-1}|(|F_{k-1}| - 1)/2$ comp
need to be made

Generating Candidates C_k

ng candidates with infrequent subsets

or a candidate of size k, one only needs to check
ubsets of size $k-1$.

numerate subsets of size $k-1$ by removing one
ement at a time from the candidate.

earch for the subsets one after the other in F_{k-1} until
subset is not found or the list of subsets is exhausted

Binary search could be performed if F_{k-1} is sorted
under the itemset ordering for an efficient search.

Alternatively a hash tree could be build to store the
itemsets of F_{k-1} for an efficient search.

a subset wasn't found the candidate should be
iscarded.

If all size $k-$
are frequent s
subsets of sm

Each candida
will give k sub
size $k-1$

For each
$O(k|F_{k-1}|)$
might be ne
worst case,
naiv
This can be
binary
$O(k\log(|$
constructing

Generating Candidates C_k

Self-joining: Find pairs of sets in F_{k-1} that have first $k-2$ items in common and differ by **one** element.

Pruning: Remove all candidates with infrequent subsets

Example: Generating Candidates C_k

equent itemsets of size 2:
,c}:5, {b,m}:4,{c,j}:3 {c,m}:3

elf-joining:
,c,m}, {c,j,m}

uning:
~~j,m}~~ since ~~{j,m}~~ not frequent

equent items of size 3:

o,c,m}

B_1 = {b, c, m} B_2 = {j,

B_3 = {b, m} B_4= {c,

B_5 = {b, c, m} B_6 = {b

B_7 = {b, c, j} B_8 = {b

tching Transactions to Candidate S

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$k = 1$.
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repeat
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      $\sigma(c) = \sigma(c) + 1$. {Increment support count}
    end for
  end for
  $F_k = \{ c \mid c \in C_k \wedge \sigma(c) \geq N \times minsup \}$. {Extract the frequent k-itemsets}
until $F_k = \emptyset$
Result $= \bigcup F_k$.

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Interpret minsup as a fraction here
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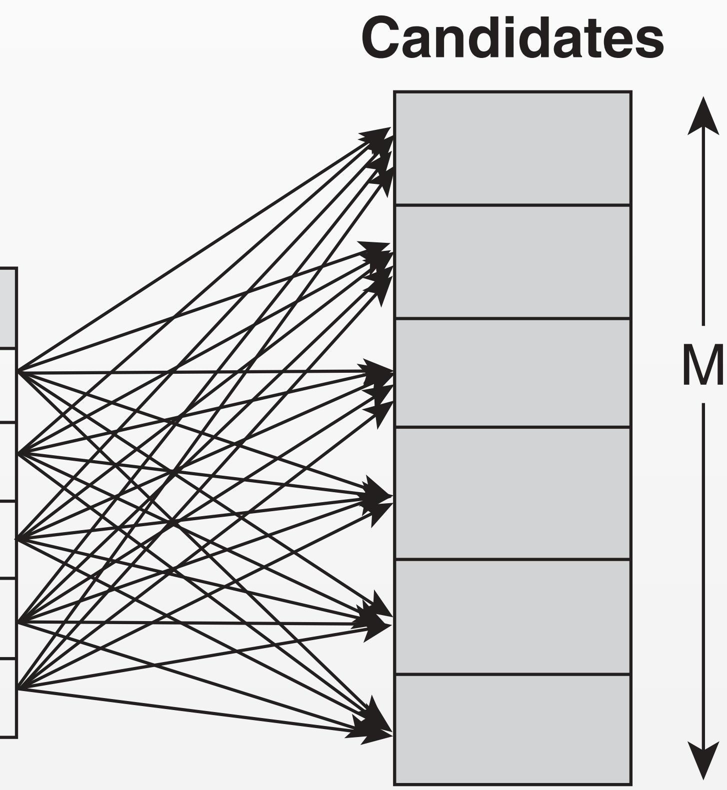
roblem: Naive Matching is Expensiv

)(N M), where N is number of baskets and M is number of canc

Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diapers, Beer, Eggs
3	Milk, Diapers, Beer, Coke
4	Bread, Milk, Diapers, Beer
5	Bread, Milk, Diapers, Coke

N



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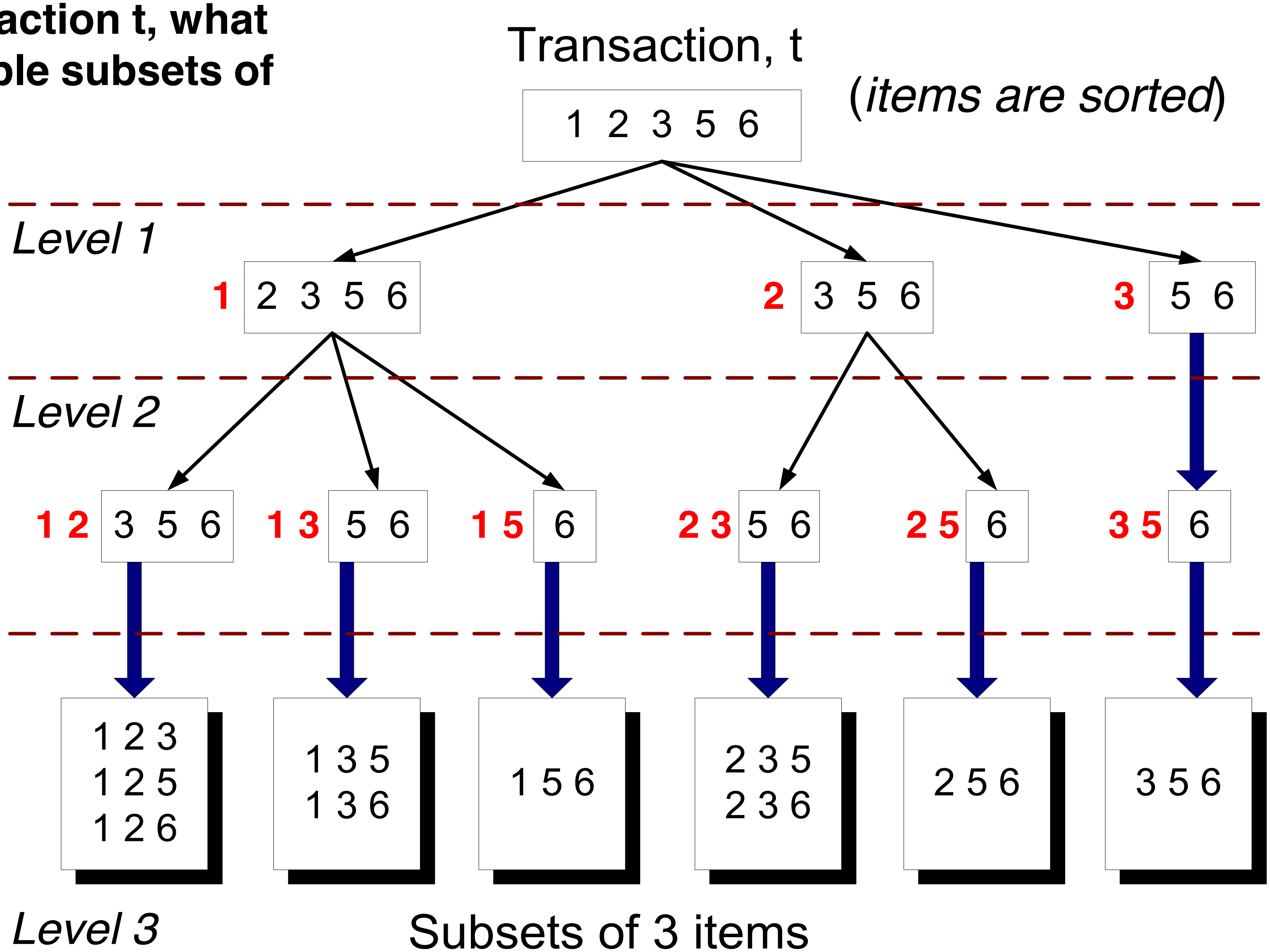
:egy 1: Enumerating Transaction Sub

Given a transaction t, what are the possible subsets of size 3?



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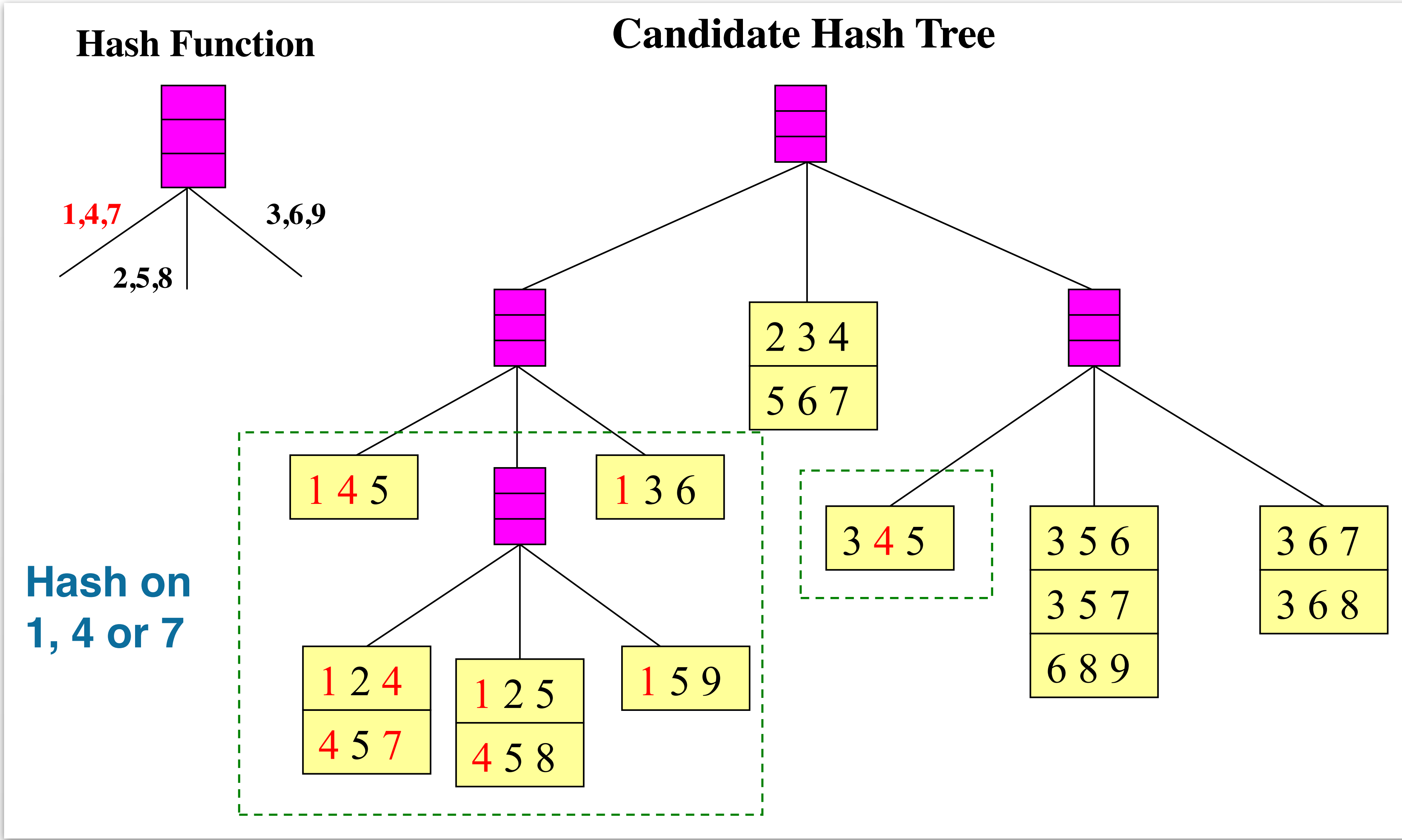
Hash Tree for Itemsets



15 candidate 3-itemsets, distributed across 9 leaf nodes

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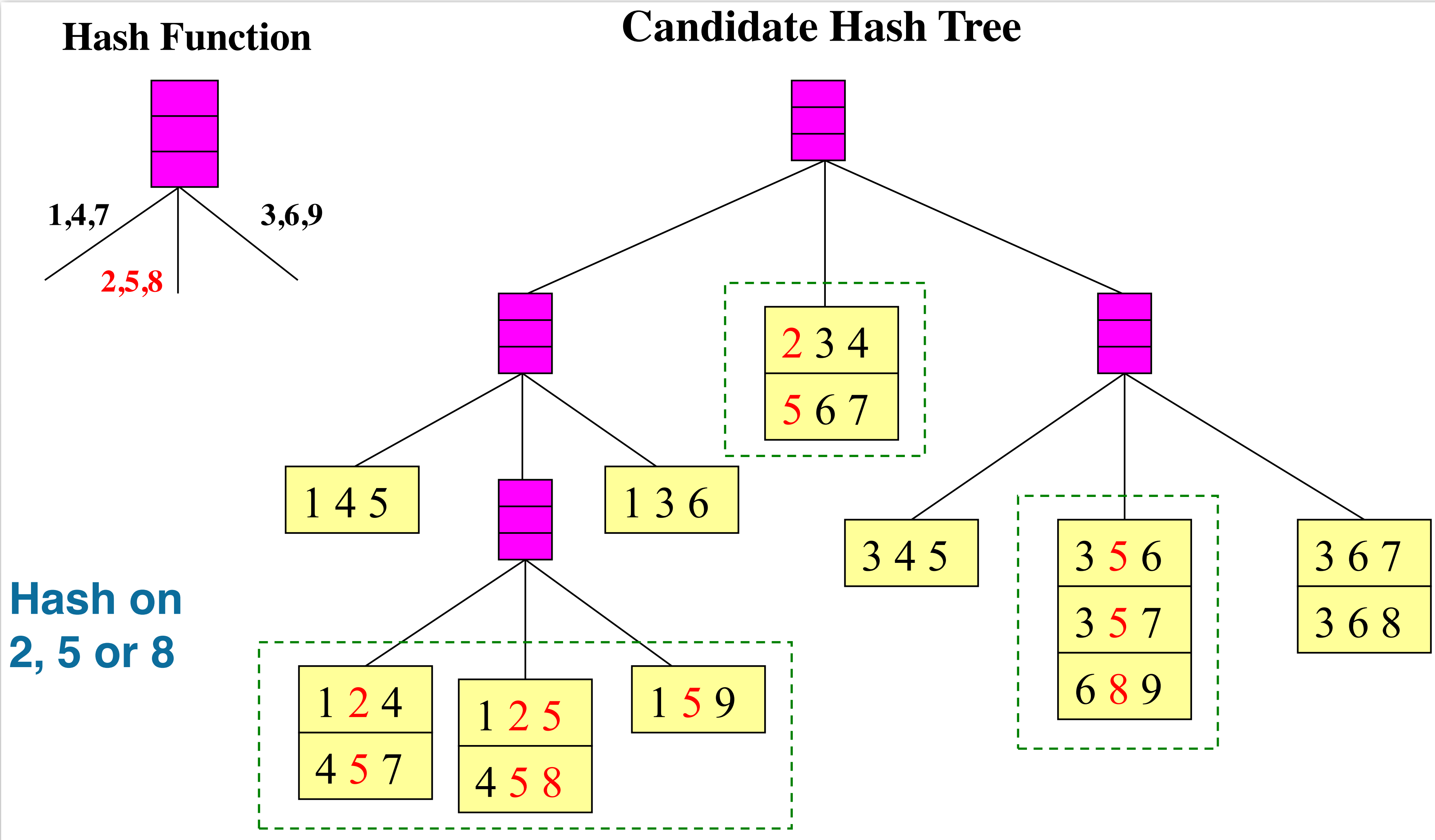
Hash Tree for Itemsets



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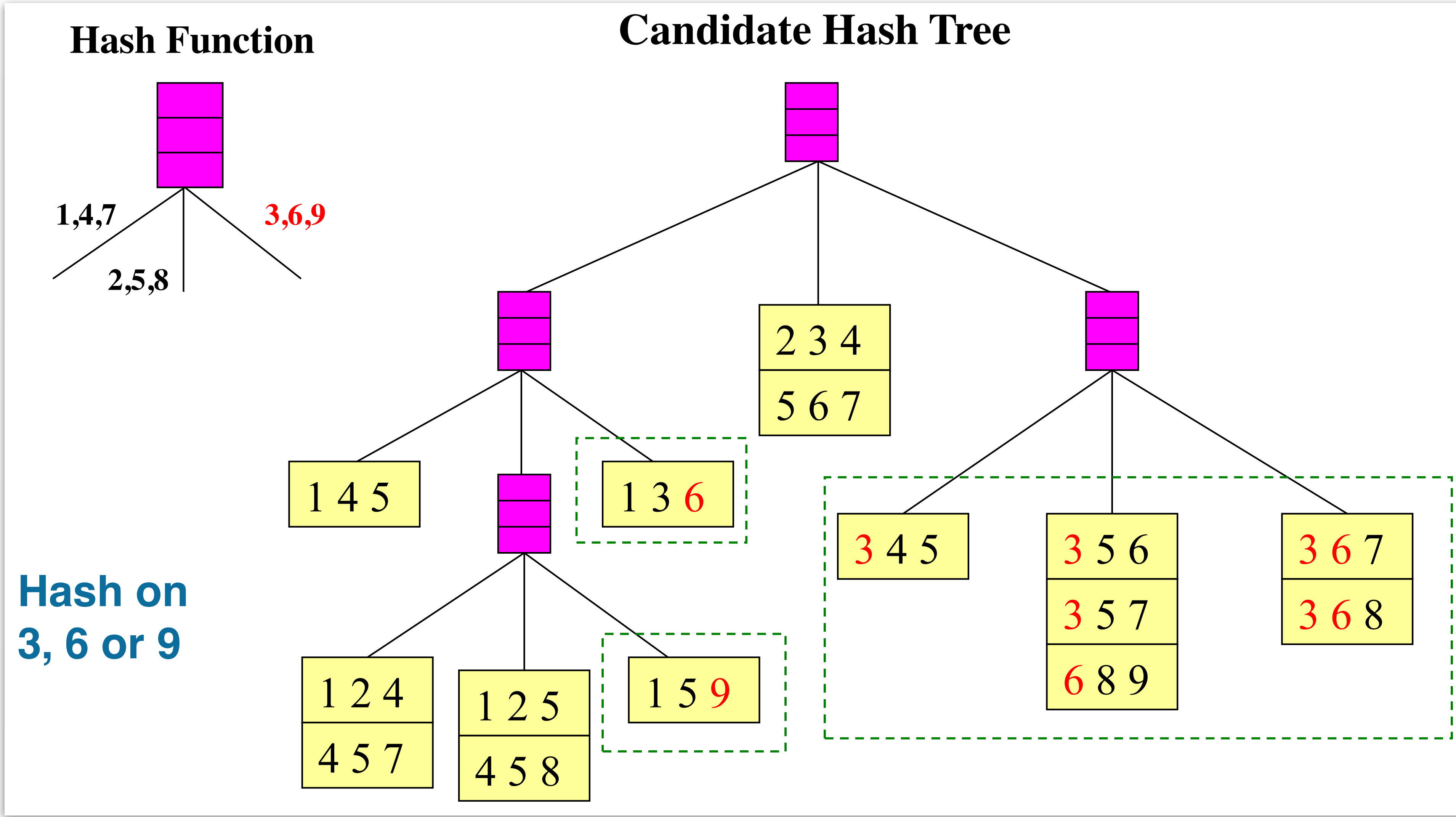
Strategy 2: Hashing Itemsets



15 candidate 3-itemsets, distributed across 9 leaf nodes

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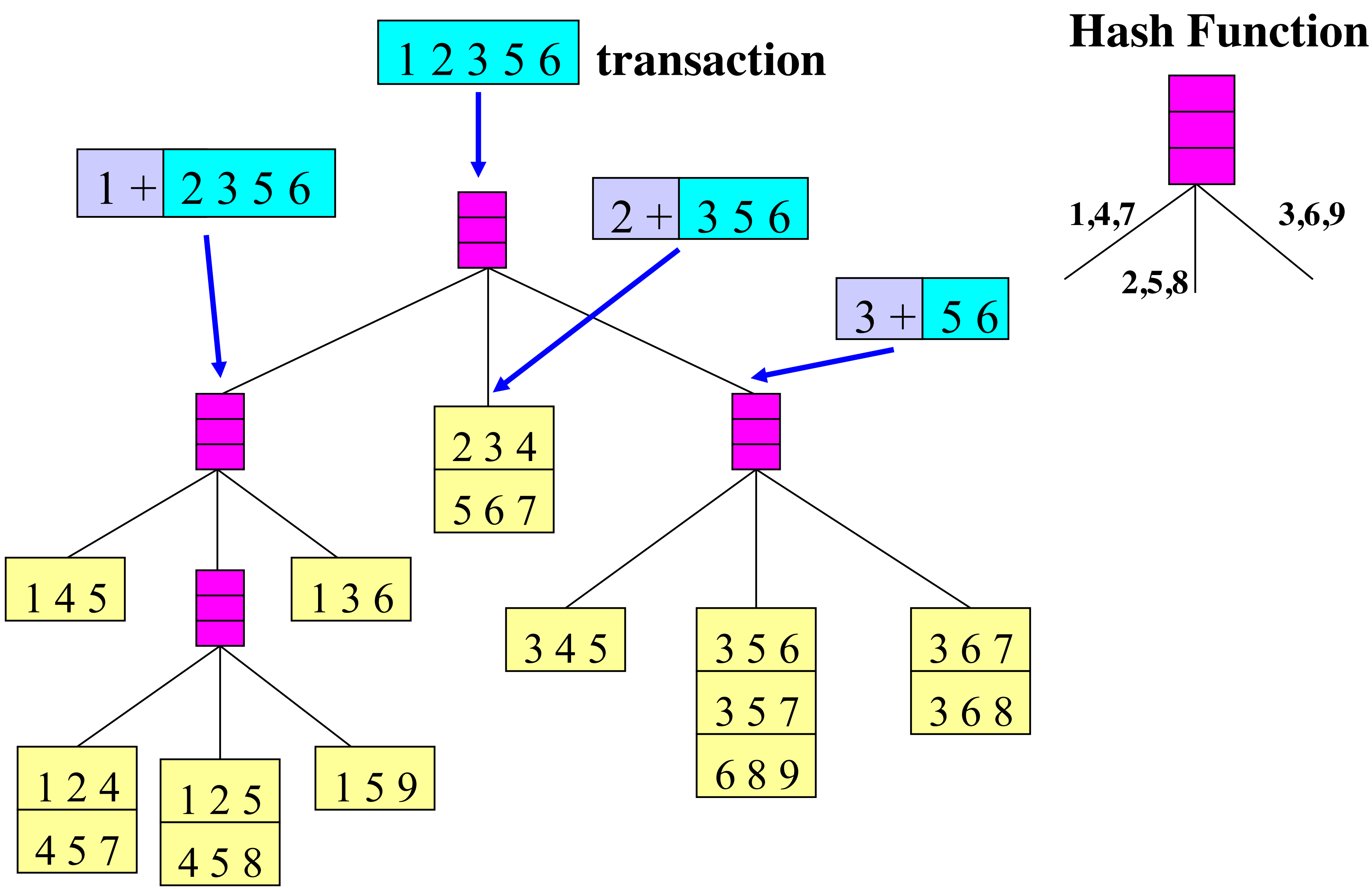
Strategy 2: Hash Tree for Candidates



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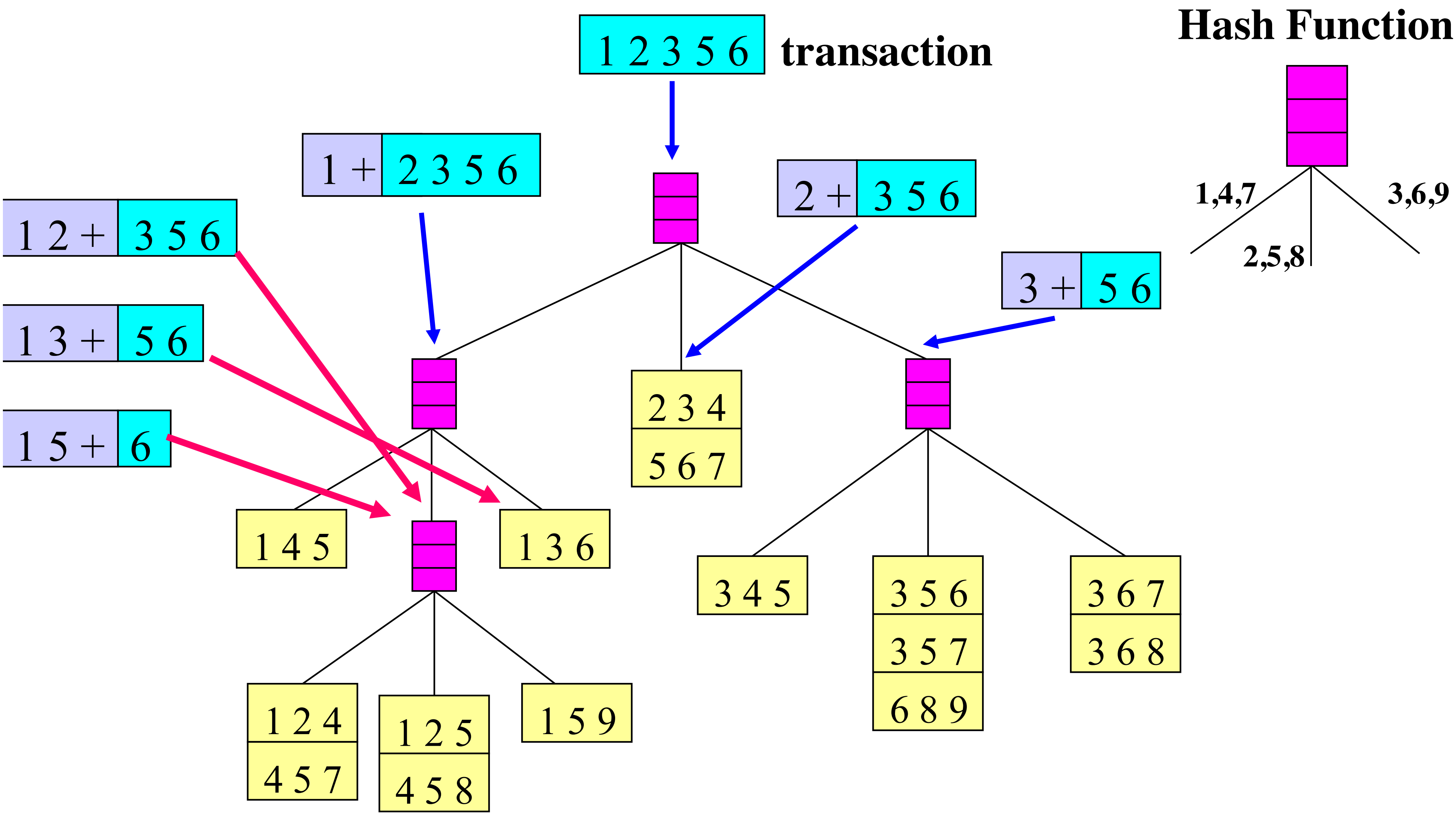

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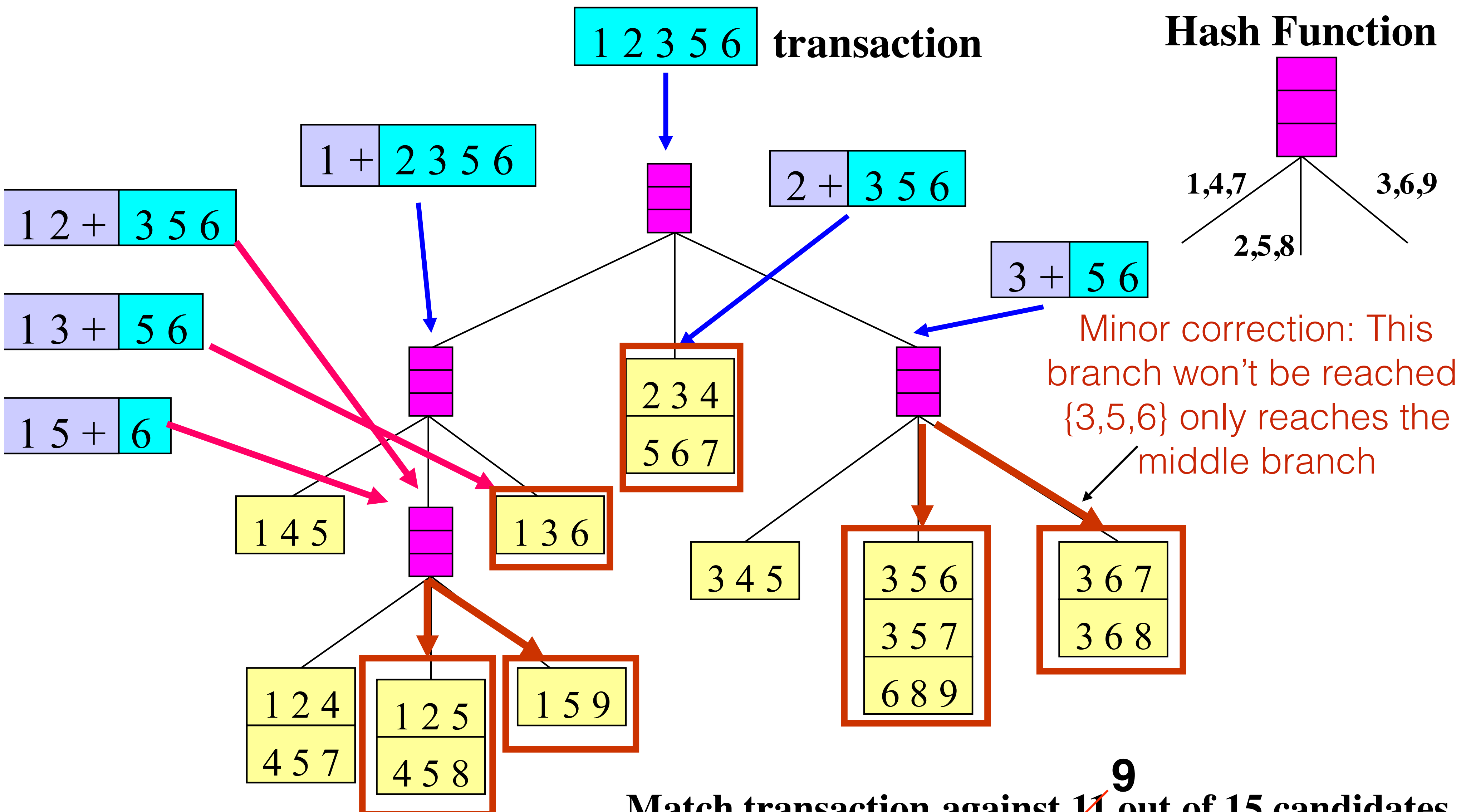
Strategy 2: Hash Tree for Candidates



Match transaction against ~~11~~ 9 out of 15 candidates

A-priori Algorithm

Algorithm 6.1 Frequent itemset generation of the *Apriori* algorithm.

$k = 1$.
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repeat
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  for each transaction $t \in T$ **do**
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      $\sigma(c) = \sigma(c) + 1$. {Increment support count}
    end for
  end for
  $F_k = \{\ c \mid c \in C_k \wedge \sigma(c) \geq N \times minsup\}$. {Extract the frequent k-itemsets}
until $F_k = \emptyset$
Result $= \bigcup F_k$.

Interpret minsup as a fraction here

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Rule Generation

ems of each frequent itemset Y can be partitioned into the
onsequent and the the antecedent to give a rule. For an $X \subset Y$

$$X \rightarrow Y - X$$

$= \{a, b, c\}$ could give the six rule $\{a, b\} \rightarrow \{c\}, \{a, c\} \rightarrow \{b\},$
$b, c\} \rightarrow \{a\}, \{a\} \rightarrow \{b, c\}, \{b\} \rightarrow \{a, c\}, \{c\} \rightarrow \{a, b\}$.

frequent k-itmeset can potentially give to $2^k - 2$ rules.

ot all rules are confident

$$C(X \rightarrow Y - X) = \sigma(Y)/\sigma(X) < \text{minconf}$$

X is also frequent by anti-monotonicity. However, the rule might not meet the minimum confidence threshold.

Pick a
k it
conse
rema
bec
antece
$Y \rightarrow \emptyset$

ow to find confident association rule without enumerating them all?

Rule Generation

e Pruning

Theorem 6.2. *If a rule $X \longrightarrow Y - X$ does not satisfy the confidence threshold, then any rule $X' \longrightarrow Y - X'$, where X' is a subset of X, must not satisfy the confidence threshold as well.*

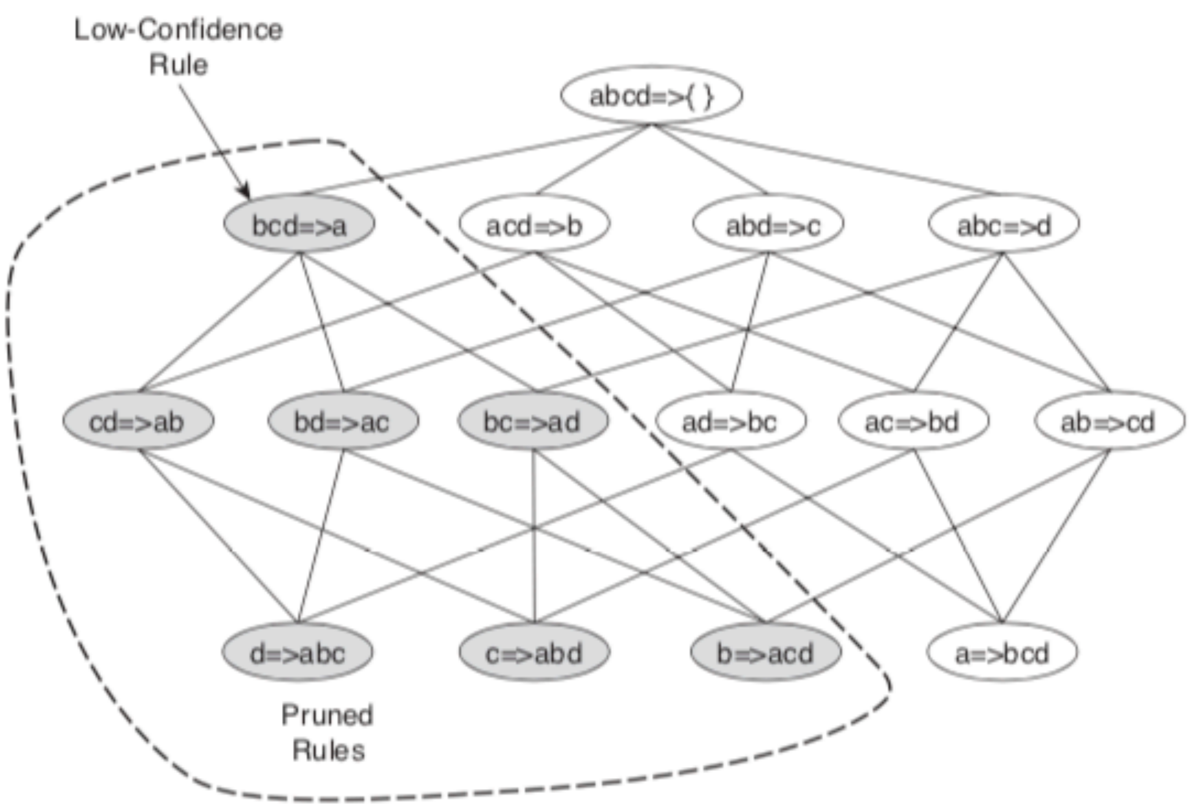


Figure 6.15. Pruning of association rules using the confidence measure.

Rule Generation

Algorithm 6.2 Rule generation of the *Apriori* algorithm.

for each frequent k-itemset f_k, $k \geq 2$ **do**
    $H_1 = \{i \mid i \in f_k\}$ {1-item consequents of the rule.}
    call `ap-genrules`$(f_k, H_1.)$
end for

Algorithm 6.3 Procedure `ap-genrules`(f_k, H_m).

$k = |f_k|$ {size of frequent itemset.}
$m = |H_m|$ {size of rule consequent.}
if $k > m + 1$ **then**
    $H_{m+1} = \text{apriori-gen}(H_m)$.
    for each $h_{m+1} \in H_{m+1}$ **do**
        $conf = \sigma(f_k)/\sigma(f_k - h_{m+1})$.
        if $conf \geq minconf$ **then**
            output the rule $(f_k - h_{m+1}) \longrightarrow h_{m+1}$.
        else
            delete h_{m+1} from H_{m+1}.
        end if
    end for
    call ap-genrules$(f_k, H_{m+1}.)$
end if

Compacting the Output

umber of frequent item
can be exponential in
ıumber of items.

ıt be useful to work with
pact representations

imal frequent itemsets:
mmediate superset is
ɹent

es more pruning

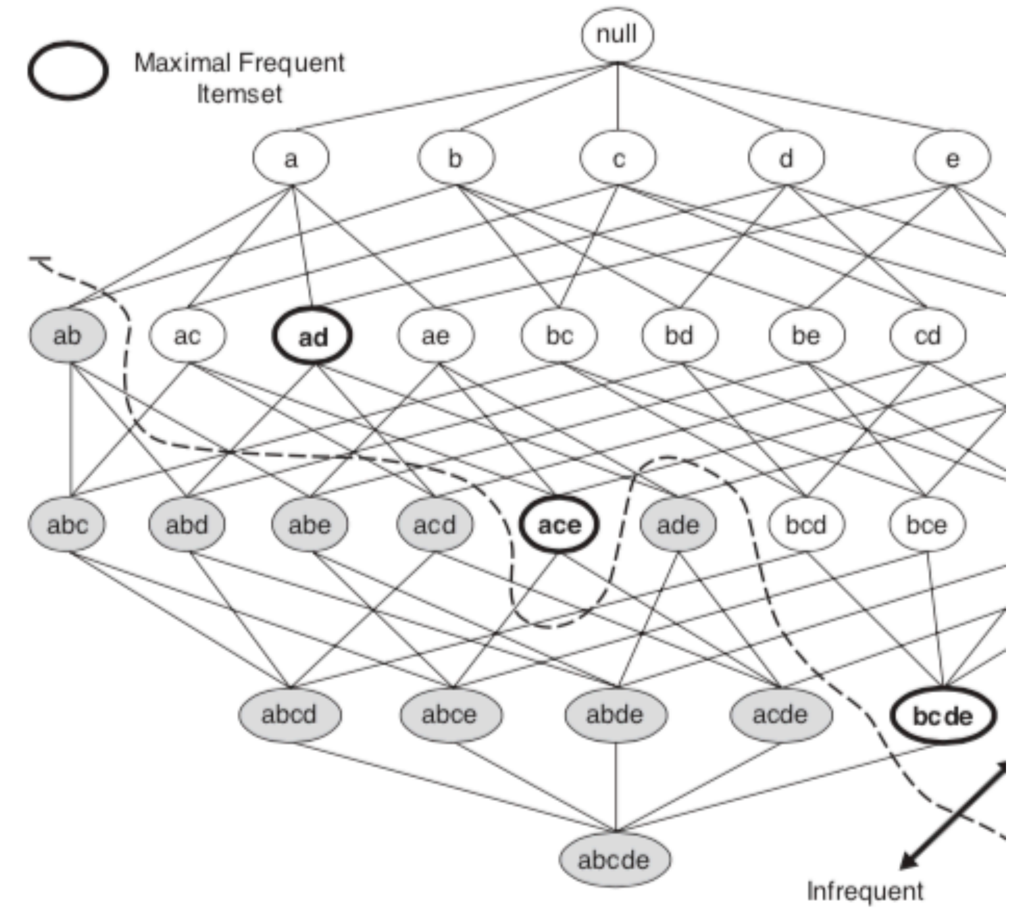


Figure 6.16. Maximal frequent itemset.

adapted from: J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Compacting the Output

requent itemsets:

nediate superset has same

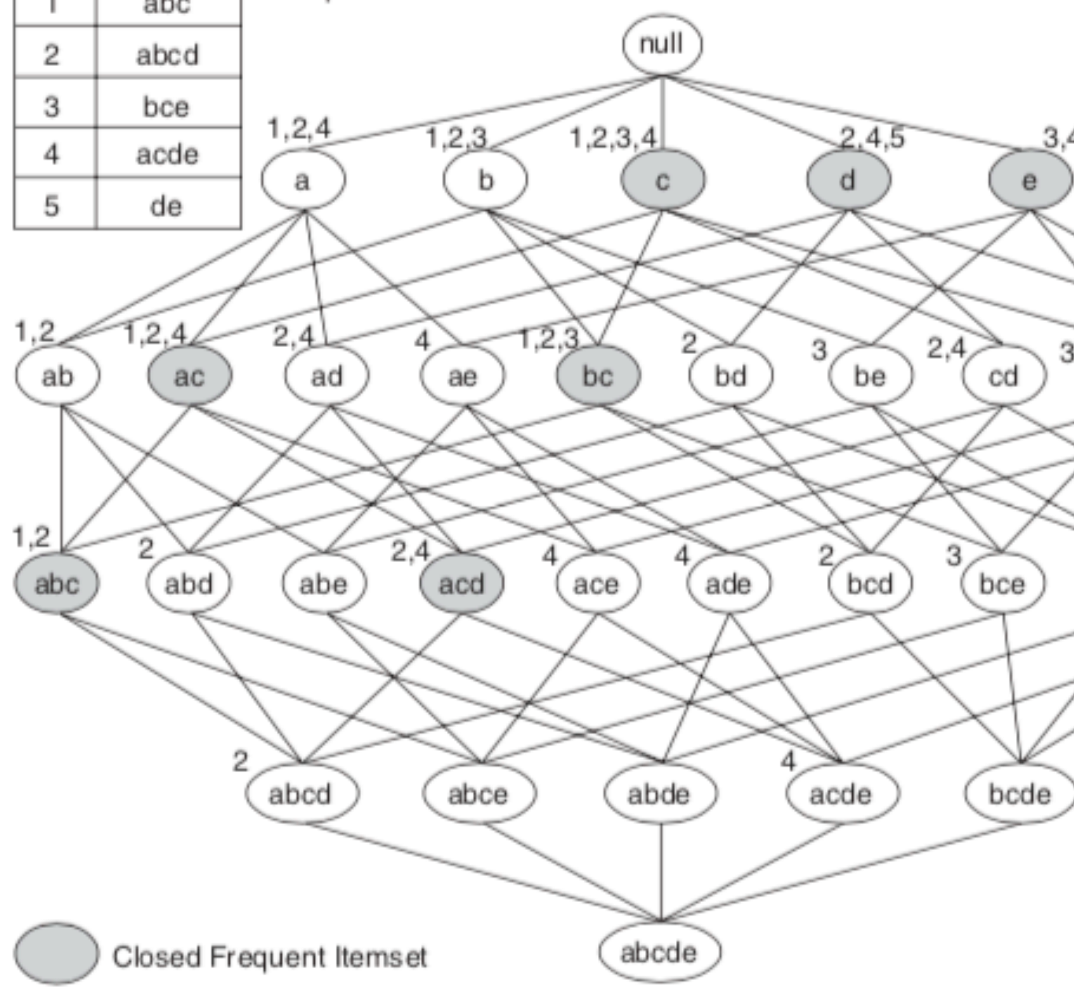
not only frequent
ation, but exact counts

ounts of non-closed frequent
can be obtained as the
um of its closed frequent
set

dant association rules are
nerated if using closed
nt itemsets.

$\to \{a\}$ and $\{b, c\} \to \{a\}$ will have the
same support and confidence
ause $\{b\}$ is not closed, but $\{b, c\}$ is

TID	Items
1	abc
2	abcd
3	bce
4	acde
5	de



adapted from: J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Example: Maximal vs Closed

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$

$B_3 = \{m, b\}$ $B_4 = \{c, j\}$

$B_5 = \{m, c, b\}$ $B_6 = \{m, c, b, j\}$

$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

Frequent itemsets:

{m}:5, **{c}:6**, **{b}:6**, **{j}:4**,
{m,c}:3, **{m,b}:4**, **{c,b}:5**, **{c,j}:3**,
{m,c,b}:3

Closed

Maximal

Example: Maximal vs Closed