Functions
Recursion

C++ functions

- Declare/prototype

```
int myfunction (int );
```

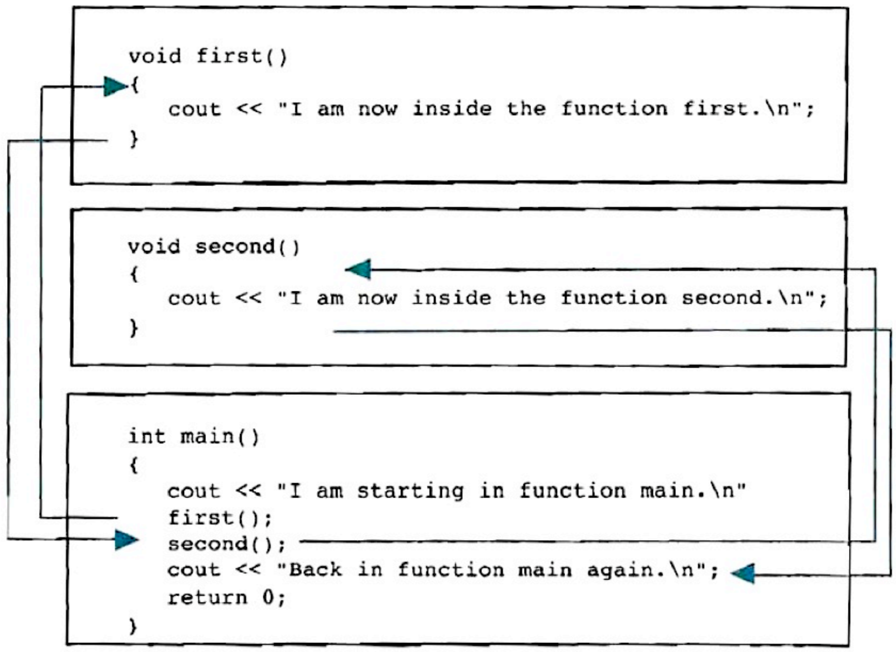
- Define

```
int myfunction (int x){
   int y = x*x;
   return y;
}
```

- Call

```
int a;
a = myfunction (7);
```

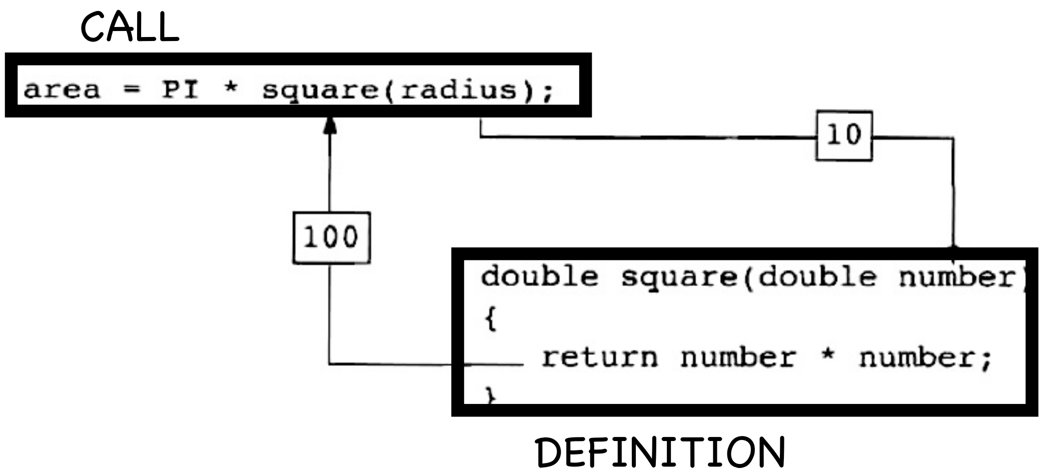
function call flow



types

- type of function (of the return value)
  - `double myfunction (....)`
- type of arguments
  - `double myfunction (int a, double b, char c)`
- the types have to be consistent between declaration, definition and call

function arguments



arguments by value

- value is copied to parameter/argument
- parameters have the scope the function
  - same as a local variable

return

- returns the function output value to the call instruction
  - has to match function output type

```
int myfunction (int x){
  int y = x*x;
  return y;
}
```

- terminates the function
  - even if there are more statements to exectute

Argument default value

- if no argument is given at the call, use a default value
  - default value given in function definition

```
double log5(double x=125){...
    ...
}
```

Scope: local and global

- global : define outside any function
  - visible everywhere (preserve value)
- local : define inside a function (or block)
  - invisible outside the definition block

Static variables

- static local variables do not get erased when function/ block terminates
- the next time the function is called, a static variable still has the previous value
  - initialized only one time

```
int function (int param){
   static double myvar=0;//initialization
happens only at the first function call
   ... do something ...
}
```

Overload function names

- myfunction does `y = 2*x`$_1$ `- 3*x`$_2$
  - I want it to work for doubles and int types
- `int myfunction (int, int)`
- `double myfunction (double, double)`
- `double myfunction (int, double)`
- `double myfunction (double, int)`

Arguments by reference

- usually (call by value), if the argument passed to the function changes value inside the function, the variable used as argument does not.

```
//call
int a=0,b=0;
b = f1(a);//now a=0, b=1

//function definition
int f1 (int x){
  x = x +1;
  return x;
}
```

- to modify the variable used as argument at the call, pass the argument by reference

```
//call
int a=0,b=0;
b = f2(a); //now a=1, b=1

//function definition
int f2 (int& x){
  x = x +1;
  return x;
}
```

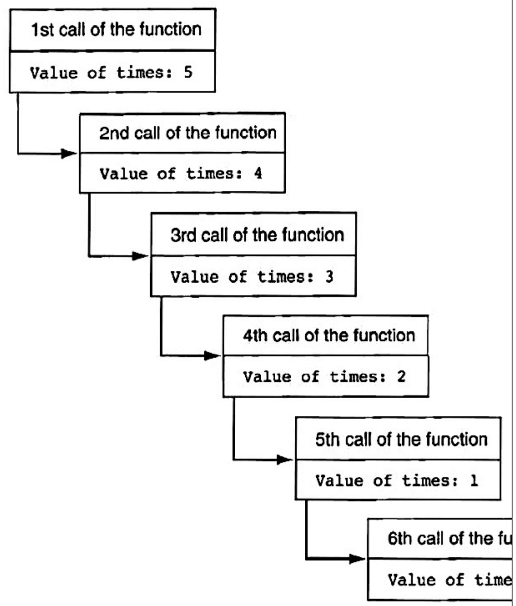
Recursive calls

Recursion of a function

- A function that calls itself
  - OR cyclic: function f calls function g; function g calls function f
- Creates a stack of calls
- Calls terminate in the reverse order of calling
- Local variables are defined independently for each call

Recursion: flow

```
void message(int times){
 if (times>0){
  cout<<"call t="<<times<<"\n";
  message(times-1);
 }
}
```



Solving a problem recursively

- recognize recursive/inductive nature
  - many problems easier to solve with a loop
- build up the recursion mechanism
- follow the principle of mathematical induction
- most often, find an "invariant" operation
  - can be a math formula
  - can be an inductive form
  - do one step of it, then call the recursion (or the other way)
- look carefully at the base cases

Sum of first n integers

- $S(n) = 1 + 2 + 3 + 4 + .. + n = n(n+1)/2$
- induction : $S(n) = S(n-1) + n = (n-1)*n/2 + n = n(n+1)/2$

Sum of first n integers

- $S(n) = 1 + 2 + 3 + 4 + .. + n = n(n+1)/2$
- induction : $S(n) = S(n-1) + n = (n-1)*n/2 + n = n(n+1)/2$
- recursion

```
int sum (int n){
   if (n<0) {
      cout<<"ERROR, negative";
      return -1;
   }
   if (n==0) return 0;
   return n + sum(n-1);
}
```

Factorial

- n! = 1 * 2 * 3 * … * n
- induction: n! = n* (n-1)!
  - 1!=0!=1
- can be very very large
  - 10! = 3628800
  - 50! ≈ 3.0414* 10^64

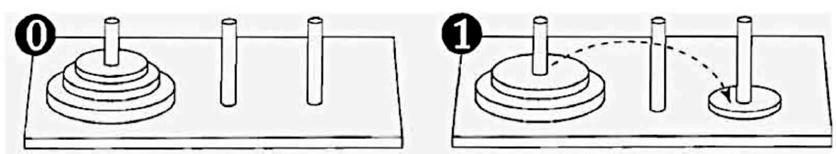
Factorial

```
long factorial (long n){
    cout<< “call: factorial(“<<n<“)\n”;
    int out;
    if(n<=1) out=1;
    else out = n*factorial(n-1);
    cout<<“return: factorial(“<<n<“)\n”;
    return out;
}
```

Tower of Hanoi

- three towers/rods A, B, C
- A contains pegs 1 to n, in order, n at the bottom
- B, C empty
- TASK: move all pegs to A such that
  - a peg at a time
  - only top peg of a tower can move
  - peg can "sit" only on higher value pegs

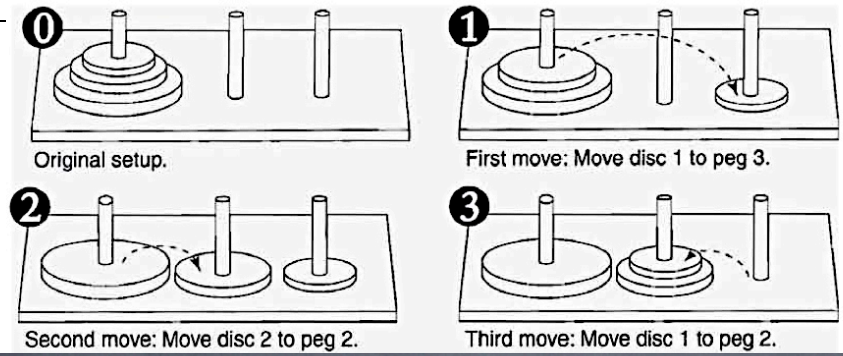
Tower of Hanoi



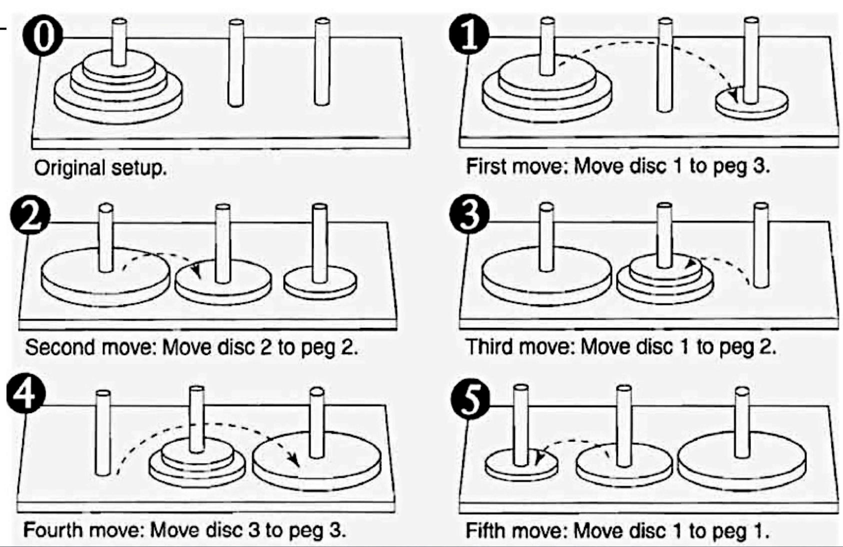
Original setup.

First move: Move disc 1 to peg 3.

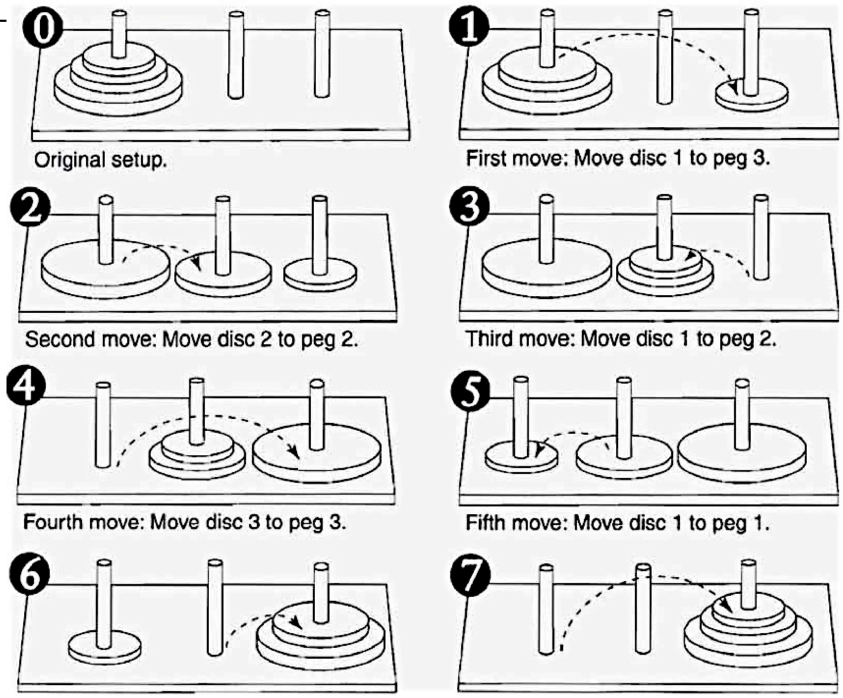
Tower of Hanoi



Tower of Hanoi



Tower of Hanoi



Tower of Hanoi

- function f: moves top k pegs from tower X to tower Y
  - leaves all pegs existing on Z and Y unmoved
  - leaves all pegs on tower X below top k unmoved
- function f is recursive
  - moves top k-1 pegs from X to Z (recursive call)
  - moves k peg from X to Y
  - moves top k-1 pegs from Z to Y (recursive call)

Euclid GCD

- given positive integers a and b
  - find d=GCD(a,b)
  - find integers m,n such that a*m + b*n = d. Do they always exist?
- recursion: if a>b and a=q*b+r then
  - `GCD(a,b) = GCD(b,r)`
  - what about m and n?

Euclid GCD: find linear coefficients

- given positive integers a and b
  - find d=GCD(a,b)
  - find integers m,n such that a*m + b*n = d. Do they always exist?
- recursion: if a>b and a=q*b+r then
  - `GCD(a,b) = GCD(b,r)`
  - $m_{(ab)} = n_{(br)}$
  - $n_{(ab)} = m_{(br)} - q*n_{(br)}$

Fibonacci numbers

- Problem defined with recursion
- F(n+2) = F(n) + F(n+1)
    - F(0) = 0; F(1) = 1

```
int Fibonacci (int n){
  if (n<=1) return n;
  else return Fibonacci(n-1) + Fibonacci(n-2);
}
```

Count characters

- preview the notion of ARRAY
- `char s[200];` //array of 200 characters
    - different type than class `string`
- can be accessed as `s[0], s[1], ..., s[199]`
    - `s[0]='H'; s[1]='e'; s[2]='l'; s[3]='l'; s[4]='o';`
    - `char a = s[3];`
- works for any type
    - `double d[10]; int i[100];`
- <u>C++ does not check for array bounds !!</u>

Count characters

- start counting at position 1
  - record 1 if character find,
  - keep looking at next position
- can be a loop
- can be a recursion

Binary search

- Find a specific value V in a sorted array A[ ]
- Start with array indices i=0, j=last, m=middle
- Compare A[m] to V and decide where in the array to look next
  - recursive call
  - or a loop
- Why binary search and not simply check all elements ?

Binary search

- How long is going to take? (worst case)
- In algorithms, how long means how many steps/instructions
  - as a function of input n = size of array
- we dont want an exact time/value
  - “linear” = like n = about CONSTANT * n
  - “quadratic” = like n^2 = about CONSTANT * n^2
  - CONST*log n, CONST*n*log n, etc
- Binary Search takes CONSTANT*log(n) steps, in worst case