Recitation Q/A 5/19

starts in 2 mins!

* Rec Eq (What to do when MT is not applicable)
* Induction

$T(n) = 3T(n/2) + n\log n$

using the Induction

T

① $T(n) = 3T(n/2) + n\log n$

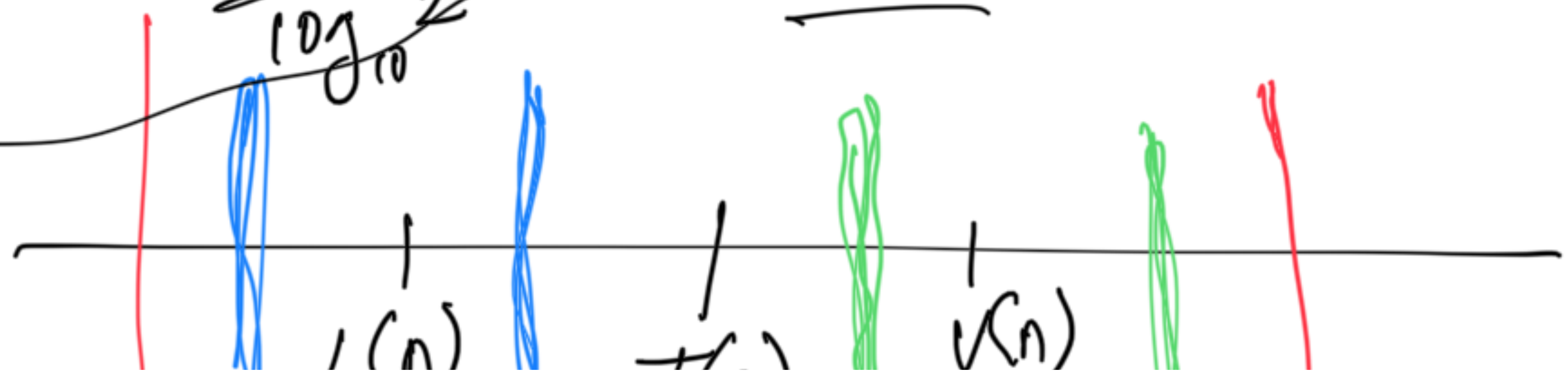
$\log_{10} 2 = 0.3$

$\log_{10} 3 = 0.48$

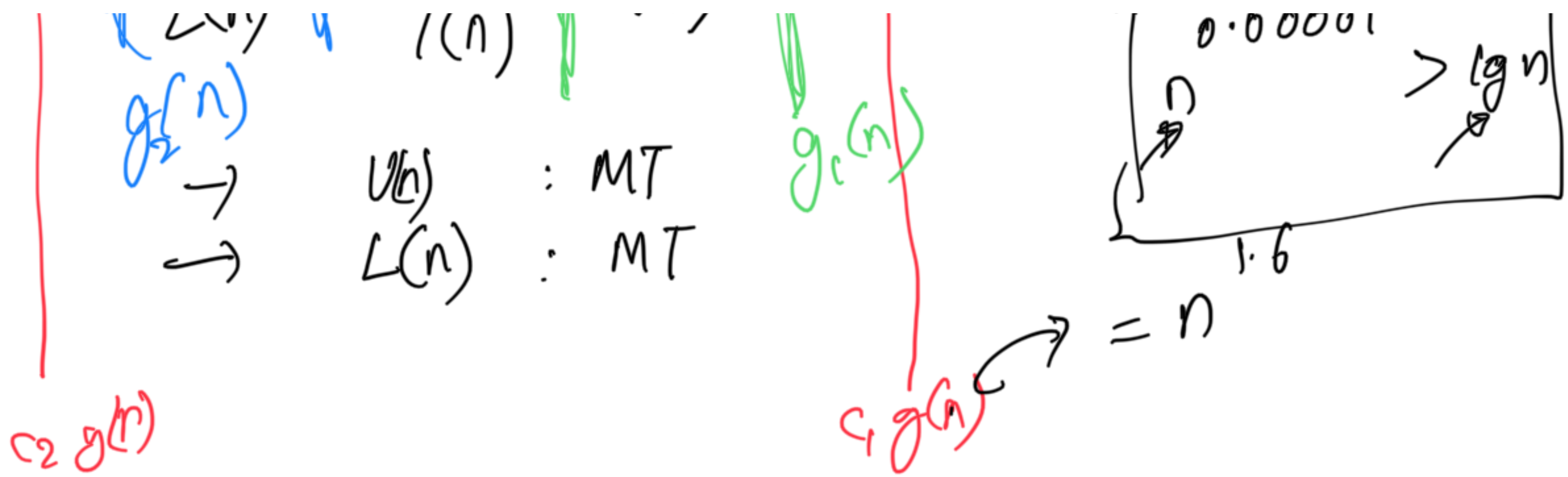
$\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2} = \frac{0.48}{0.3} \approx 1.6$

$n^{\log_2 3}$ vs $n\log n$

$n^{1.6}$ vs $\underline{n\log n}$



Number Line



$$L(n) = 3T(n/2) + n$$

$$U(n) = 3T(n/2) + n^{1.0001} \lg n \quad [\, n^{0.00001} \text{ is } > \log n \,]$$

MT : $L(n)$

$n^{1.6}$ vs n

$\Theta(n^{1.6})$

MT : $U(n)$

$n^{1.6}$ vs $\underline{n^{1.001} \lg n}$

$O(n^{1.6})$

$O(n^{[illegible]})$

$\therefore$ $T(n)$ is $\Theta(n^{1.6})$

Trick : Come up w/ $L(n)$ & $U(n)$ s.t MT on either of them gives the same answer

Quick Aside

$n^{1.6}$ Vs $n^{1.001} \log n$

$n \, n^{0.6}$ Vs $n \, n^{0.001} \log n$

$n^{0.6}$ Vs $n^{0.001} \log n$

be $n^{0.0001}$ is > $\log n$

$$\lim_{n \to \infty} \frac{\log n}{n^{0.001}}$$

$= 0$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 / \infty$$

$f(n)$ is $o(g(n))$

$g(n)$ is $o(f(n))$

Ex

$$T(n) = 4T(n/2) + \frac{n^2}{(\log n)^2}$$

$$\boxed{T(n) = 4\,T(n/2) + \frac{n^2}{\log n}}$$

(Repeated Substi)

Pf

$$\boxed{T(n) = 4T(n/2) + \frac{n^2}{(\log n)^2}}$$

$$n^{\log_2 2^2} \quad \text{Vs} \quad \frac{n^2}{(\log n)^2}$$

$$n^2 \quad \text{Vs} \quad \frac{n^2}{(\log n)^2}$$

Can't use MT

Aside : Can you still use the sandwitch trick
$L(n)$ / $U(n)$

Repeated Subs (change of Variables)

Pf.

$T(n) = T(\sqrt{n}) + 1$

Use $m = \log n$

How to figure out the change

$$T(n) = \boxed{4T(n/2)} + \boxed{\frac{n^2}{(\log n)^2}} \qquad T(1) = 1$$

(base case)

R.S : get rid of the T on RHS

$\boxed{4 \text{ subproblems of } \frac{\text{size}}{2}}$ Div

&

$\boxed{\frac{\text{size}^2}{(\log \text{size})^2}}$ Conquer

$$\boxed{T(2^k) = 4^k(1 + 1 + 2^2 + 3^2 + \cdots k^2)}$$

$$= 4^k\left(1 + \frac{k(k+1)(2k+1)}{6}\right)$$

$$= \Theta(4^k k^3)$$

get it back to n

$$n = 2^k$$

$$\log_2 n \qquad 3)$$

$$T(n) = \Theta\left(4^{\log_2 n}\,(\log n)^3\right)$$

$$= \Theta\left(n^{\log_2 4}\,(\log n)^3\right)$$

$$\boxed{T(n) = \Theta\left(n^2\,(\log n)^3\right)}$$ final

Use the same trick with

$$T(n) = T(\sqrt{n}) + 1$$

$$T(n) = 4T(n/2) + \frac{n^2}{\log n}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\boxed{1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2}$$

$$T(n) = 4\,T(n/2) + \frac{n^2}{(\log n)^2}$$

MESS

$$T(n) = 4^k\,T(n/2^k) + n^2\left[\left(\frac{1}{\log n}\right)^2 + \left(\frac{1}{\log \frac{n}{2}}\right)^2 + \cdots\right]$$

$$+ \frac{1}{\ }$$

$$\log \frac{n}{2^{k-1}}$$

$$n \rightarrow \frac{n}{2} \rightarrow \frac{n}{2^2} \cdots \rightarrow \frac{n}{2^{k-1}}$$

$$\frac{n}{2^{k-1}} = 1$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$G(n) = G(n-1) + G(n-2) + G(n-3)$$

$$F(n) = F(n-1) + F(n-2)$$