



# Recitation 5/26

## Heap

→ def : Binary Tree like structure which is as complete as possible

→ data structure

heap  
(conceptual)

← some other low level ds  
(implementation)

array

→ property / invariant : → Max heap  
→ the root is max (recursively true)

→ operations

→ find\_max

complexity  
 $O(1)$

→ insert

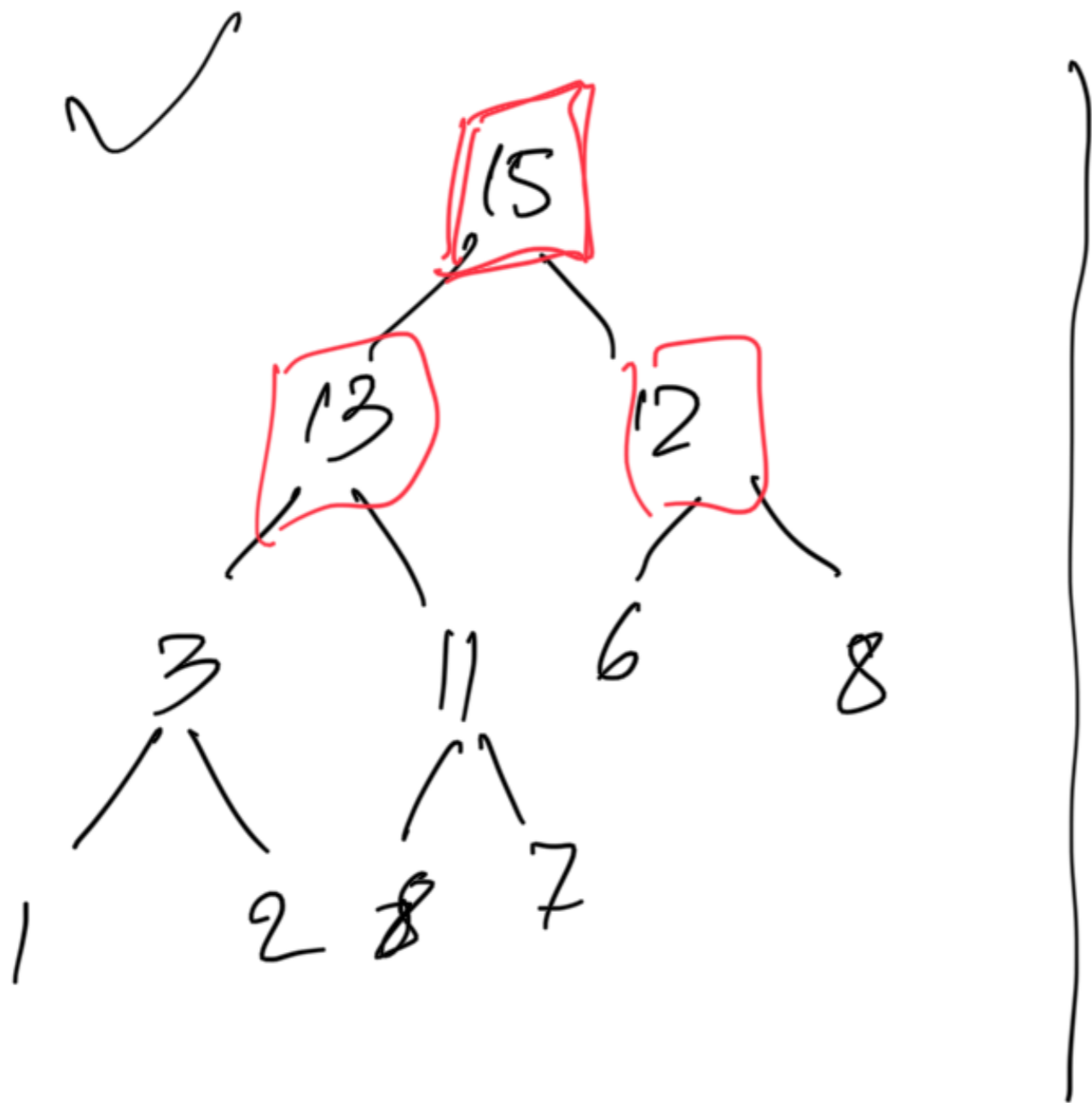
$O(\log n)$

→ delete

$O(\log n)$

→ build  
→ sort  
→ code implementation

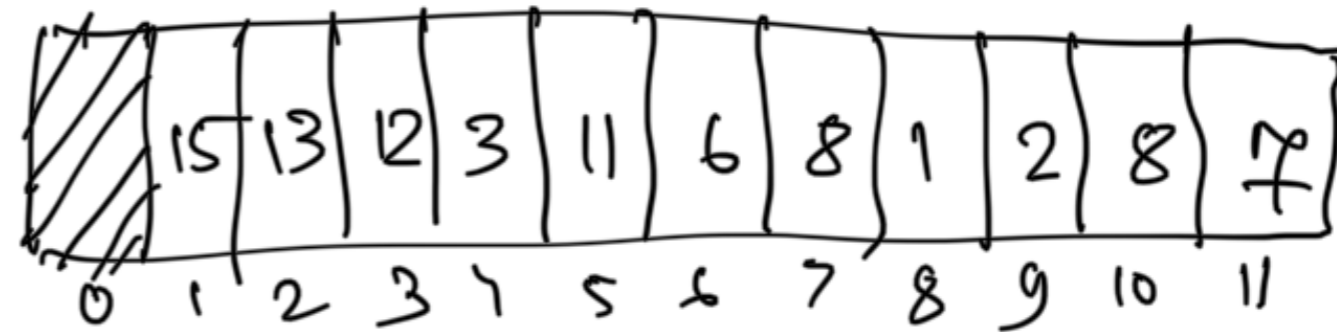
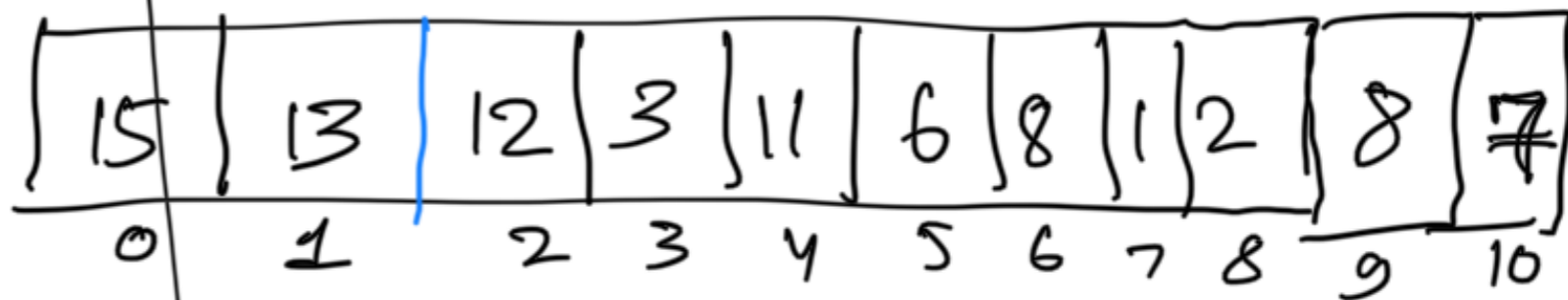
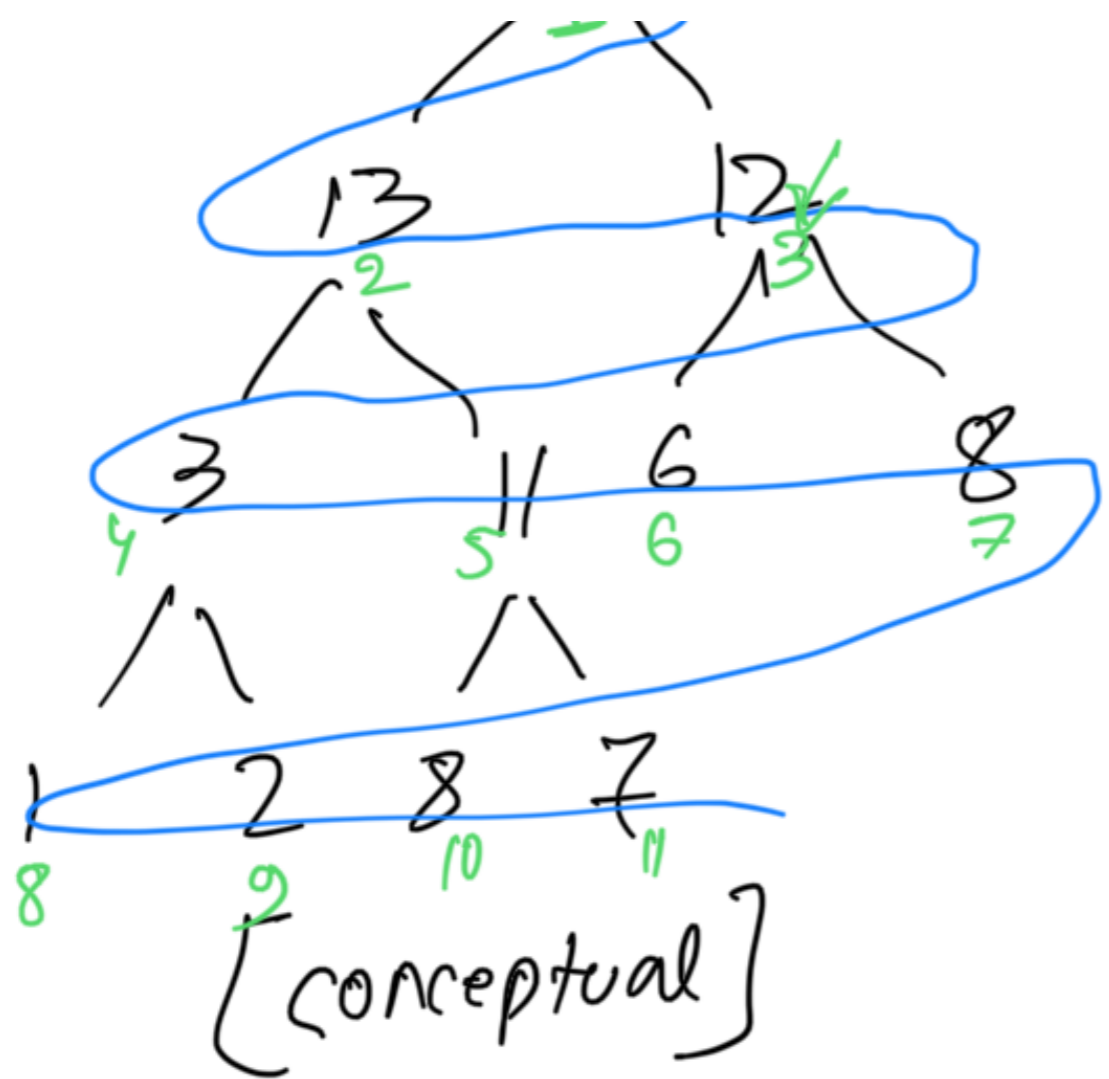
$O(n)$   
 $O(n \log n)$



datastructure

heap ← Array (low level ds)





[Reality]

Crucial operation in Implementation:-

Comparison

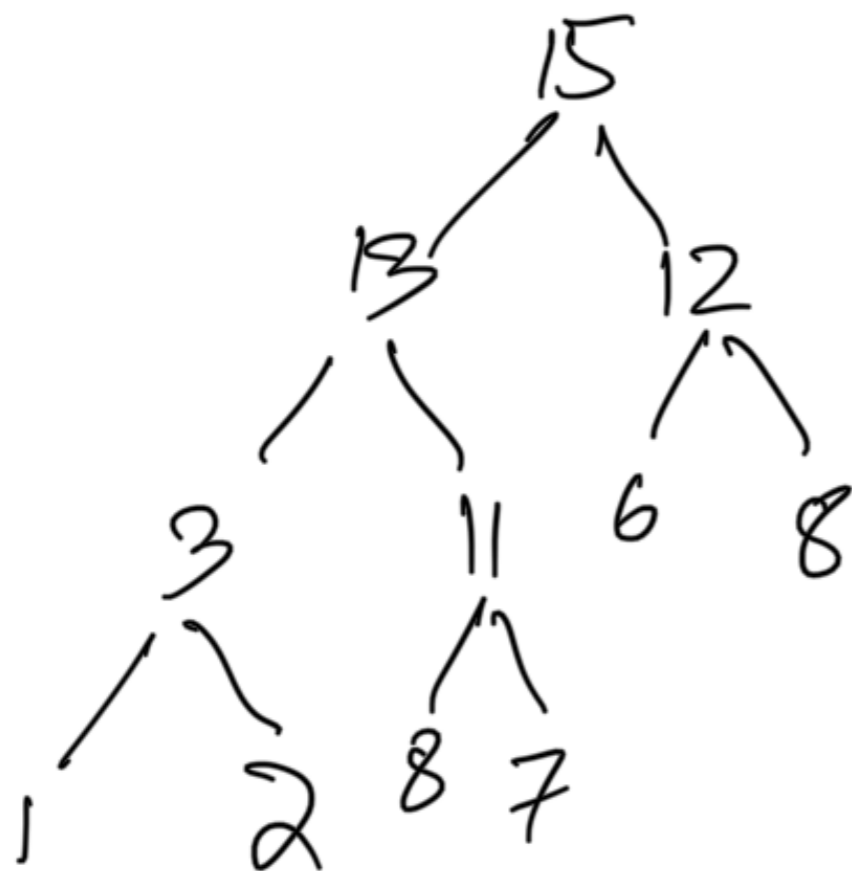




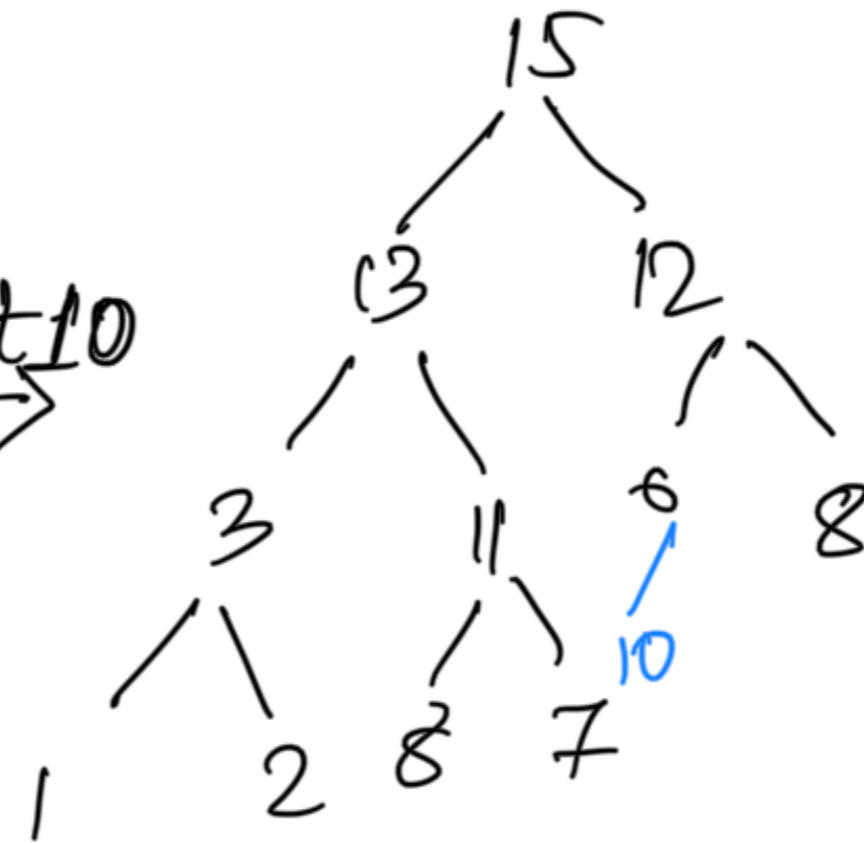
Operations

→ find\_max :  $O(1)$  lookup

→ insert :



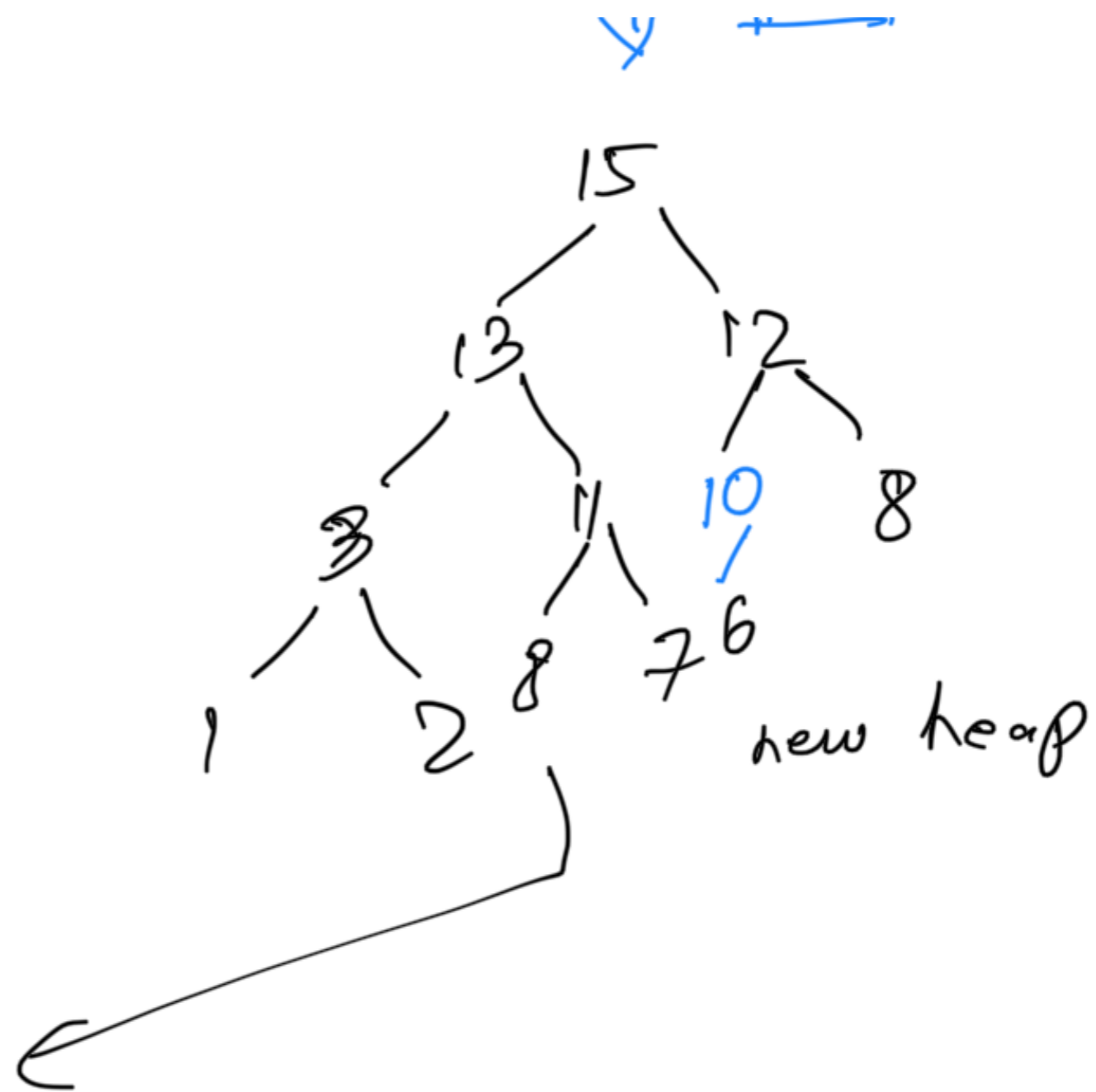
insert 10



(insert at the last spot)

→ invariant is false

→ **Fix** the heap



Algo :

→ insert at the last spot

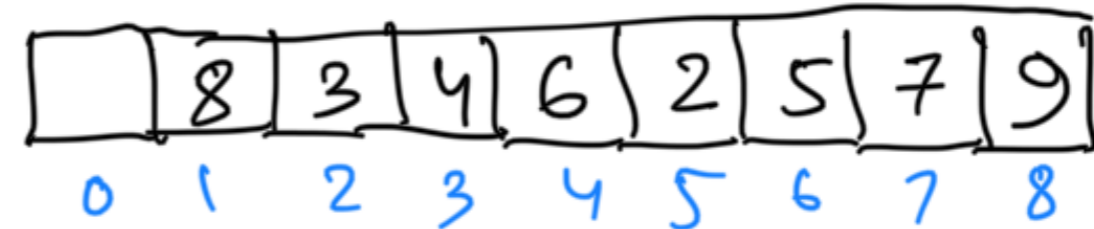
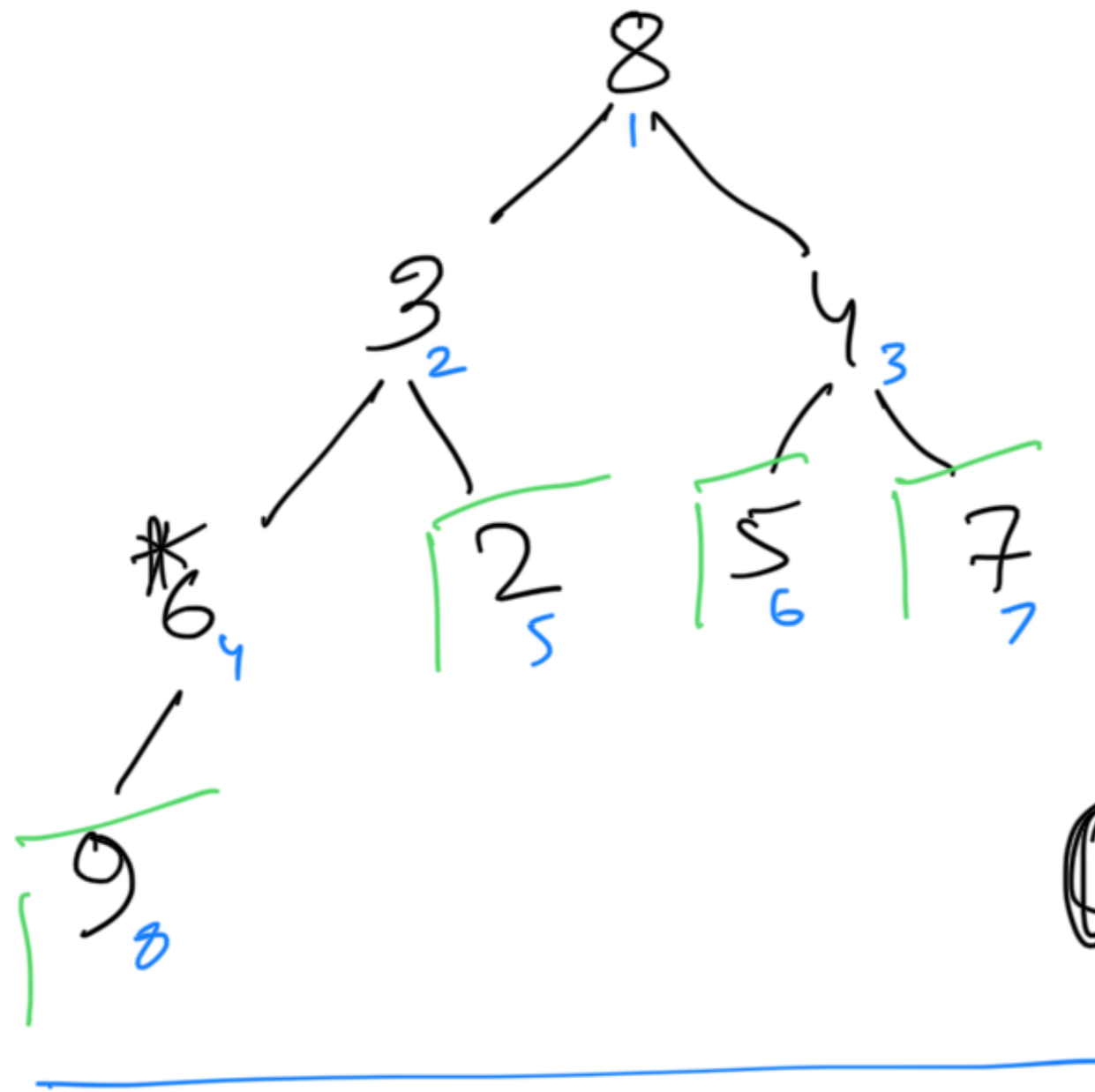
→ bubble it up until you hit the right spot, swapping elements as you go.

build

→ All leaves are already heaped

- bubble down the non-heap elements
- keep building the heap bottom up.

8, 3, 4, 6, 2, 5, 7, 9



$O(n)$

```
def build(.)
  heaped = size
  while heaped > 0
    bubble_down(heaped)
    heaped --
```

elements[heaped+1 : ]  
have been heaped

oth heaped = 8  
[ ] ← initially

↑ st  
bubble down(8)

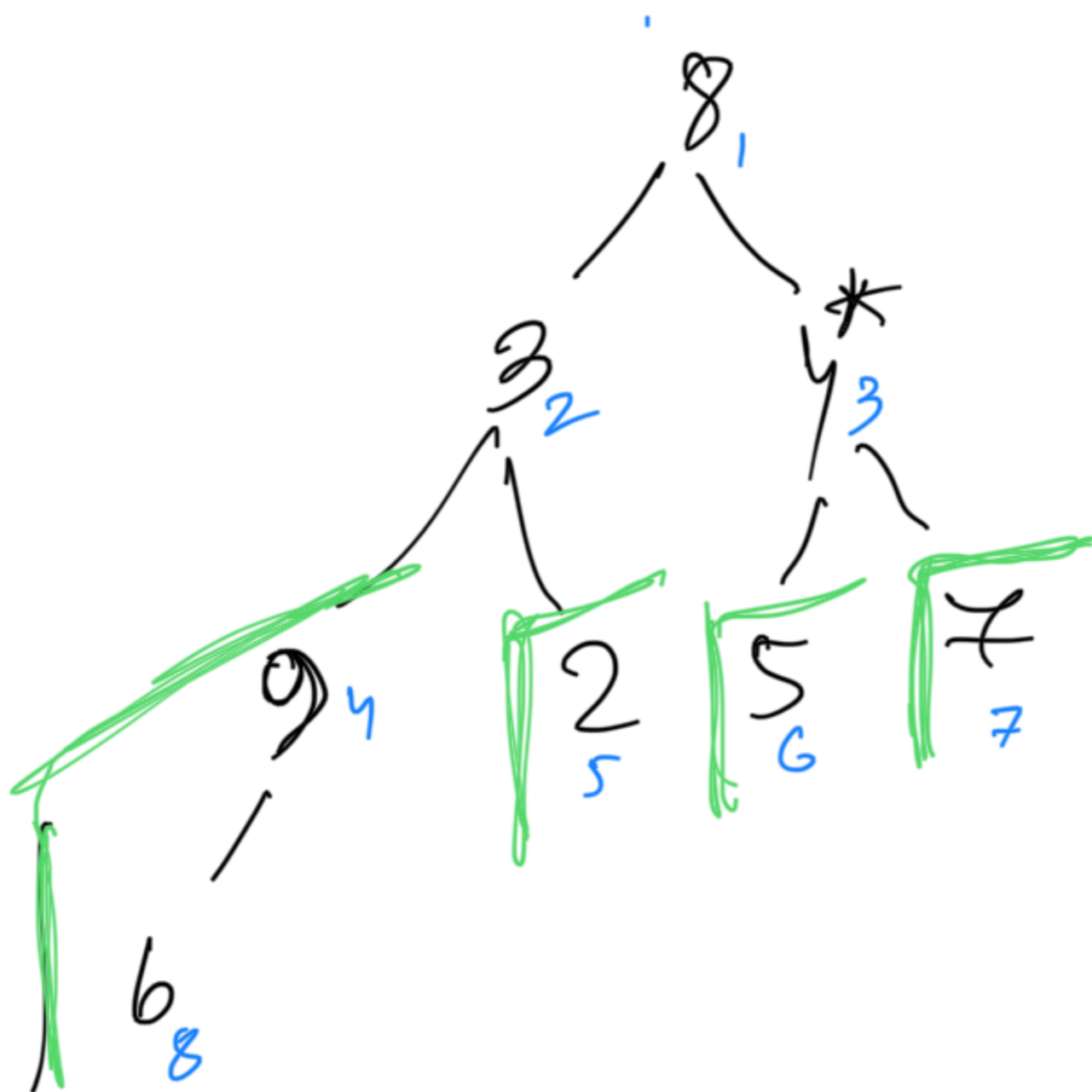
heaped = 7

heaped = 4

bubble-down(4)

elements [4+1 : ]

have been heaped





heap = 3

bubble\_down(3)

elements (3+1 % 2)  
heap

