15-10 *Planning an investment strategy*

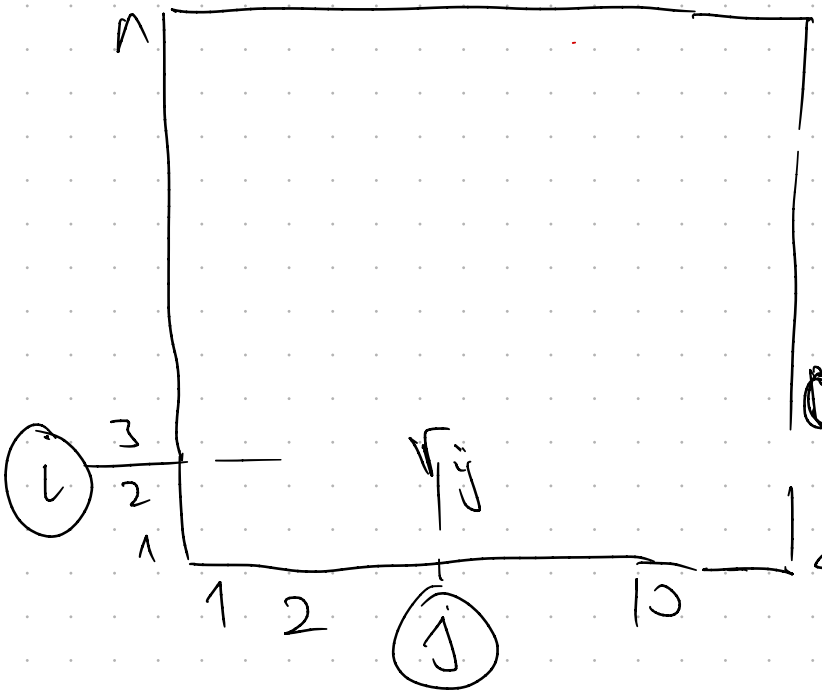
Your knowledge of algorithms helps you obtain an exciting job with the Acme Computer Company, along with a $10,000 signing bonus. You decide to invest this money with the goal of maximizing your return at the end of 10 years. You decide to use the Amalgamated Investment Company to manage your investments. Amalgamated Investments requires you to observe the following rules. It offers n different investments, numbered 1 through n. In each year j, investment i provides a return rate of r_{ij}. In other words, if you invest d dollars in investment i in year j, then at the end of year j, you have dr_{ij} dollars. The return rates are guaranteed, that is, you are given all the return rates for the next 10 years for each investment. You make investment decisions only once per year. At the end of each year, you can leave the money made in the previous year in the same investments, or you can shift money to other investments, by either shifting money between existing investments or moving money to a new investement. If you do not move your money between two consecutive years, you pay a fee of f_1 dollars, whereas if you switch your money, you pay a fee of f_2 dollars, where $f_2 > f_1$.

a. The problem, as stated, allows you to invest your money in multiple investments in each year. Prove that there exists an optimal investment strategy that, in each year, puts all the money into a single investment. (Recall that an optimal investment strategy maximizes the amount of money after 10 years and is not concerned with any other objectives, such as minimizing risk.)

b. Prove that the problem of planning your optimal investment strategy exhibits optimal substructure.

c. Design an algorithm that plans your optimal investment strategy. What is the running time of your algorithm?

d. Suppose that Amalgamated Investments imposed the additional restriction that, at any point, you can have no more than $15,000 in any one investment. Show that the problem of maximizing your income at the end of 10 years no longer exhibits optimal substructure.



r_{ij} = return (stock i, year j)

$d = 10{,}000$

$f_2 = f_2 - f_1$; $f_1 = 0$

(pay f_1 anyway)

$c[j+1]$ = best return after year j

that is years $j+1 : 10$

$s[j]$ = investment for year j

(all money → s[j] stock)

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at each year j = 10 : -1 : 1 // back from last year
    q = argmax_i {r_ij} // best stock at year j
    if( d·r_{s[j+1],j} > d·r_{q,j} - f2 )
        s[j] = s[j+1] // keep same stock as next year
        c[j] = c[j+1] + d·r_{s[j+1],j}
    else
        s[j] = q // change into best stock this year
        c[j] = c[j+1] + d·r_{q,j} - f2
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