$M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ we know that $M^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$

we want M^n, without doing n multiplications

trick: only need $\log(n)$ multiplications

$M \cdot M = M^2$; $M^2 \cdot M^2 = M^4$; $M^4 \cdot M^4 = M^8$ ...

what powers of M do we need?

Example: $n = 77 = 64 + 8 + 4 + 1$

$2^6 + 2^3 + 2^2 + 2^0$

$M^{77} = M^{2^6} \cdot M^{2^3} \cdot M^{2^2} \cdot M^{2^0}$

(M)

$M \cdot M = M^2$

$M^2 \cdot M^2 = M^4$ ✓

$M^4 \cdot M^4 = M^8$ ✓

$M^8 \cdot M^8 = M^{16}$

$M^{16} \cdot M^{16} = M^{32}$

$M^{32} \cdot M^{32} = M^{64}$ ✓

6 multiplications

$M^{77} = M^{64} \cdot M^8 \cdot M^4 \cdot M$ 3 multiplications

total: 9 multiplications instead of 76.