

Matrix Multiplication using Strassen's Eq.

$$\begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \times \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

$C_{n \times n} \qquad A_{n \times n} \qquad B_{n \times n}$

$C_1, C_2, C_3, C_4, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 - \frac{n}{2} \times \frac{n}{2}$

$$\begin{aligned} C_1 &= [A_1 \ A_2] \cdot \begin{bmatrix} B_1 \\ B_3 \end{bmatrix} = \underline{A_1 B_1} + \underline{A_2 B_3} \\ C_2 &= [A_1 \ A_2] \cdot \begin{bmatrix} B_2 \\ B_4 \end{bmatrix} = \underline{A_1 B_2} + \underline{A_2 B_4} \\ C_3 &= [A_3 \ A_4] \cdot \begin{bmatrix} B_1 \\ B_3 \end{bmatrix} = \underline{A_3 B_1} + \underline{A_4 B_3} \\ C_4 &= [A_3 \ A_4] \cdot \begin{bmatrix} B_2 \\ B_4 \end{bmatrix} = \underline{A_3 B_2} + \underline{A_4 B_4} \end{aligned}$$

} 8 multiple.

Trick: Only use 7 multiplication (Strassen's)

7 products (matrices)

→ Compute C_1, C_2, C_3, C_4 only using addition/subtraction

$$P_1 = A_1 (B_2 - B_4)$$

$$P_2 = (A_1 + A_2) B_4$$

$$P_3 = (A_3 + A_4) B_1$$

$$P_4 = A_4 (B_3 - B_1)$$

$$P_5 = (A_1 + A_4) (B_1 + B_4)$$

$$P_6 = (A_2 - A_4) (B_3 + B_4)$$

$$P_7 = (A_1 - A_3) (B_1 + B_2)$$

$$C_1 = P_5 + P_4 - P_2 + P_6$$

$$C_2 = P_1 + P_2$$

$$C_3 = P_3 + P_4$$

$$C_4 = P_5 + P_1 - P_3 - P_7$$

done! $C = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$

7 multiplications of size $\frac{n}{2}$, n^2 combination \Rightarrow
 $T(n) = 7 \cdot T(\frac{n}{2}) + \Theta(n^2)$