General linear programs

In the general linear-programming problem, we wish to optimize a linear function subject to a set of linear inequalities. Given a set of real numbers $a_1, a_2, ..., a_n$ and a set of variables $x_1, x_2, ..., x_n$, a ***linear function*** f on those variables is defined by

$$f(x_1, x_2, \ldots, x_n) = a_1x_1 + a_2x_2 + \cdots + a_nx_n = \sum_{j=1}^{n} a_jx_j .$$

If b is a real number and f is a linear function, then the equation

$f(x_1, x_2, ..., x_n) = b$

is a ***linear equality*** and the inequalities

$f(x_1, x_2, ..., x_n) \leq b$

and

$f(x_1, x_2, ..., x_n) \geq b$

are ***linear inequalities***. We use the term ***linear constraints*** to denote either linear equalities or linear equalities. In linear programming, we do not allow strict inequalities. Formally a ***linear-programming problem*** is the problem of either minimizing or maximizing a linear function subject to a finite set of linear constraints. If we are to minimize, then we call the linear program a ***minimization linear program***, and if we are to maximize, then we call the linear program a ***maximization linear program***.

This remainder of this chapter will cover the formulation and solution of linear programs. Although there are several polynomial-time algorithms for linear programming, we shall not study them in this chapter. Instead, we shall study the simplex algorithm, which is the oldest linear-programming algorithm. The simplex algorithm does not run in polynomial time in the worst case, but it is fairly efficient and widely used in practice.