

# Logical Types for Scheme

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# What do these languages have in common?

- COBOL
- Scheme
- Ruby
- Haskell

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- COBOL [Komondoor 05]
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- Ruby [Furr 09]
- Haskell [Vytiniotis 10]

New static checks

# What do these languages have in common?

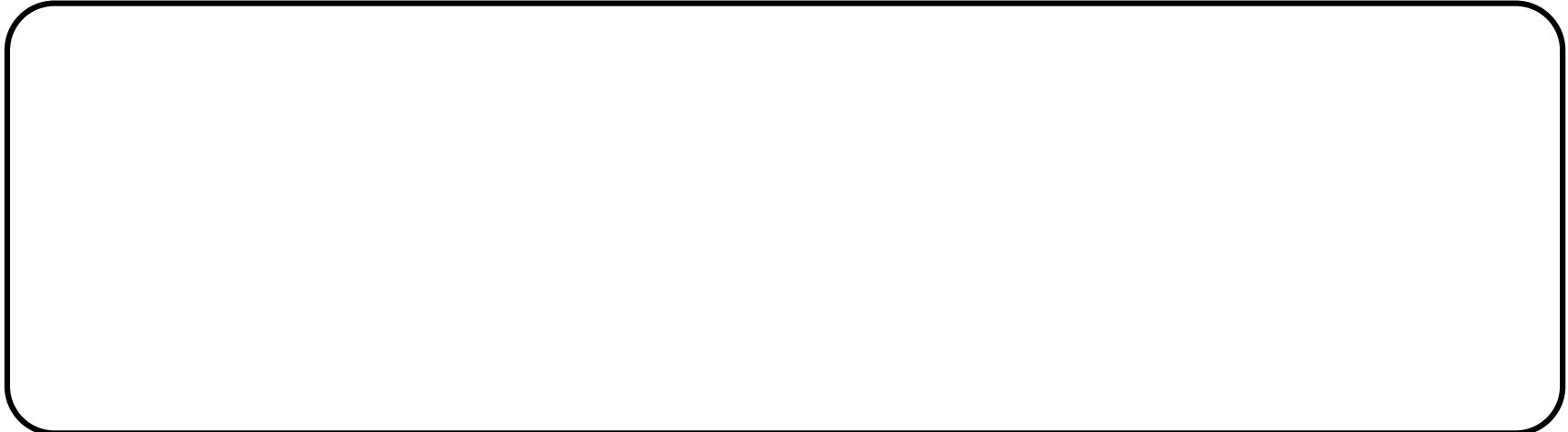
- COBOL [Komondoor 05]
- Scheme [Tobin-Hochstadt 06]
- Ruby [Furr 09]
- Haskell [Vytiniotis 10]

Millions of lines of code



# Types for Existing Languages

# What's Hard?



How programmers reason

# What's Hard?

Simple Types

How programmers reason

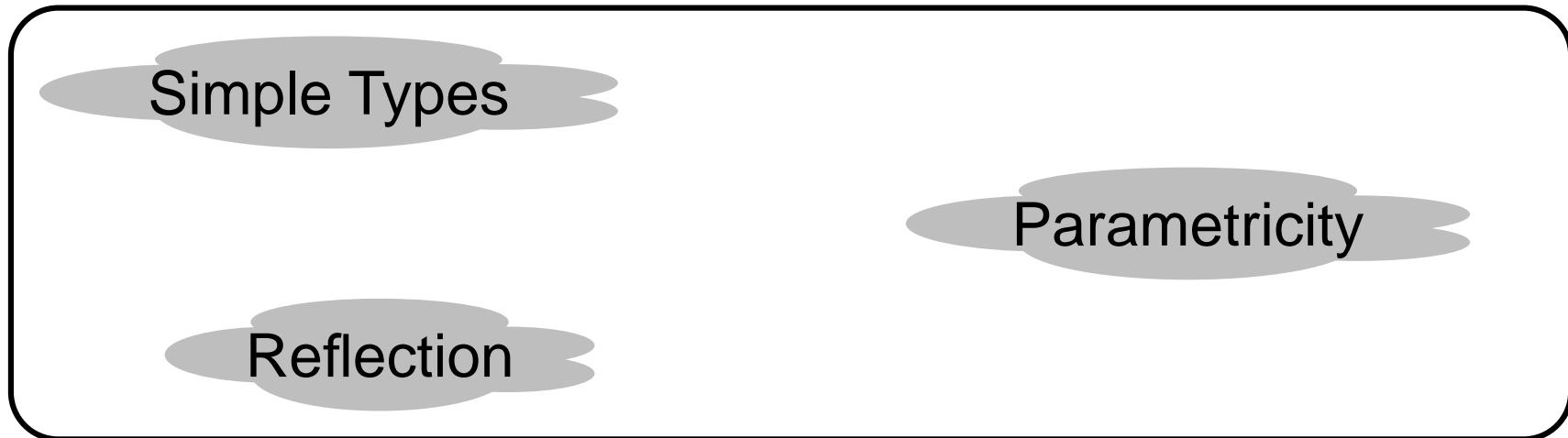
# What's Hard?

Simple Types

Reflection

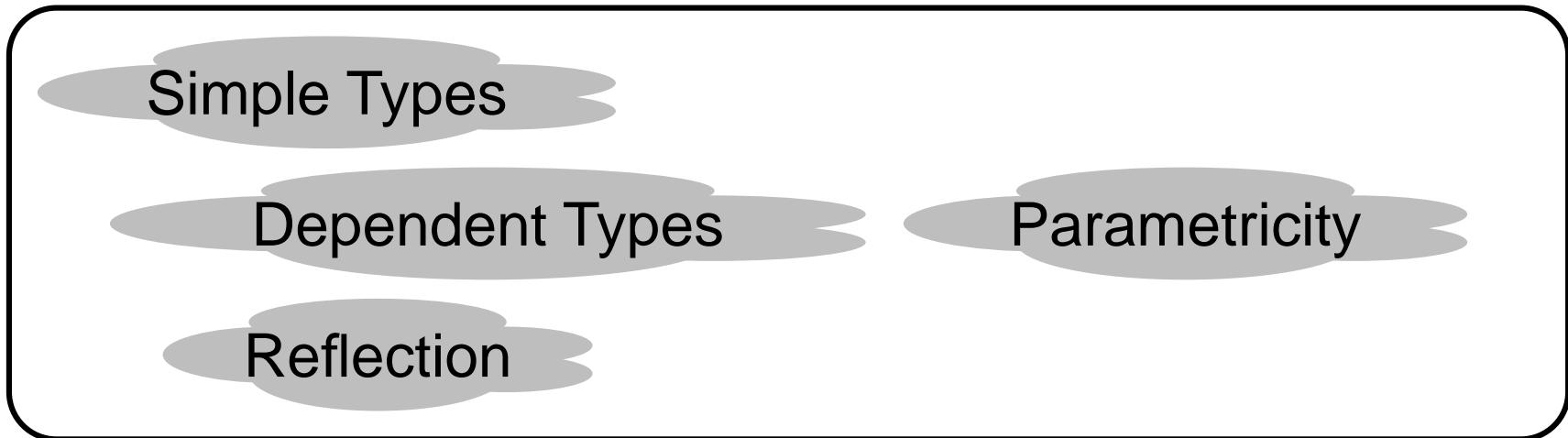
How programmers reason

# What's Hard?



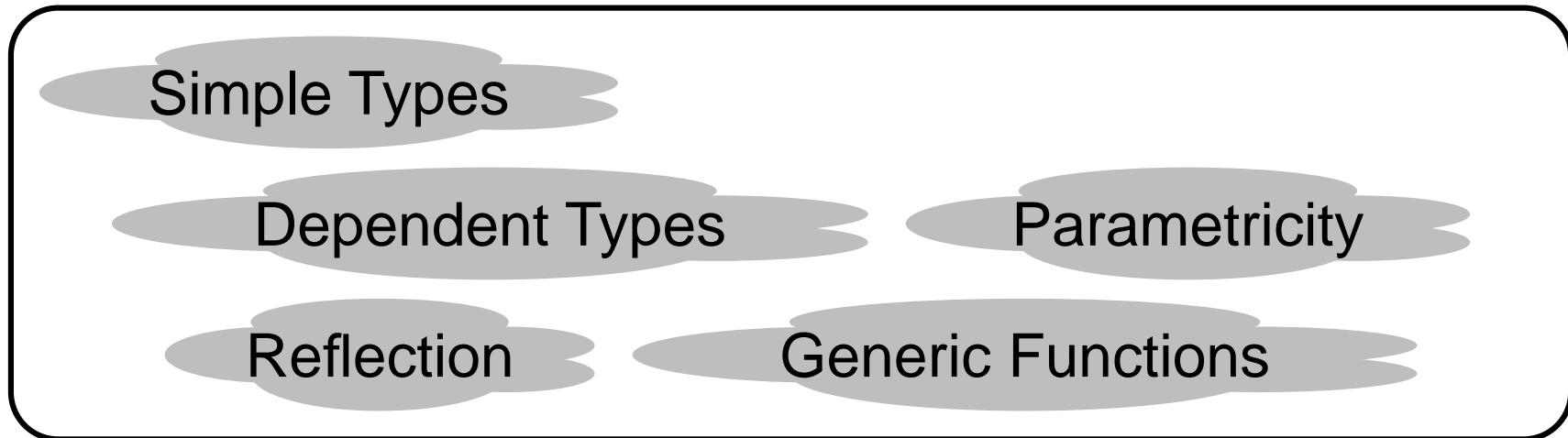
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# What's Hard?



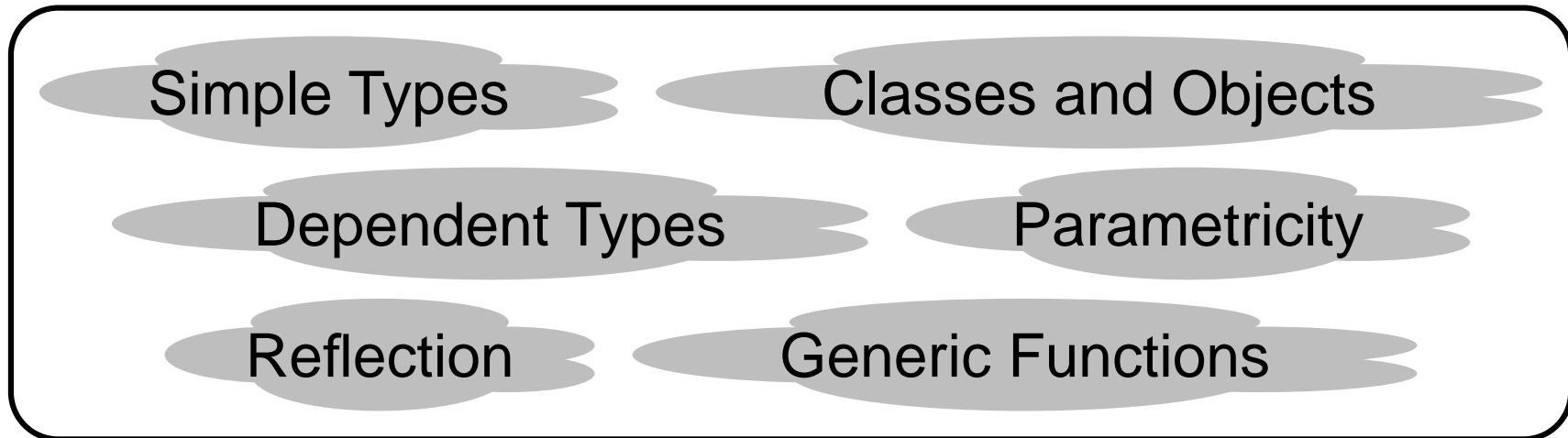
How programmers reason

# What's Hard?



How programmers reason

# What's Hard?



How programmers reason

# Checking Existing Code

- New static checking is valuable for existing code
  - Maintenance, Optimization, Trust
- Work with existing idioms
  - Survey, Analyze, Design, Validate

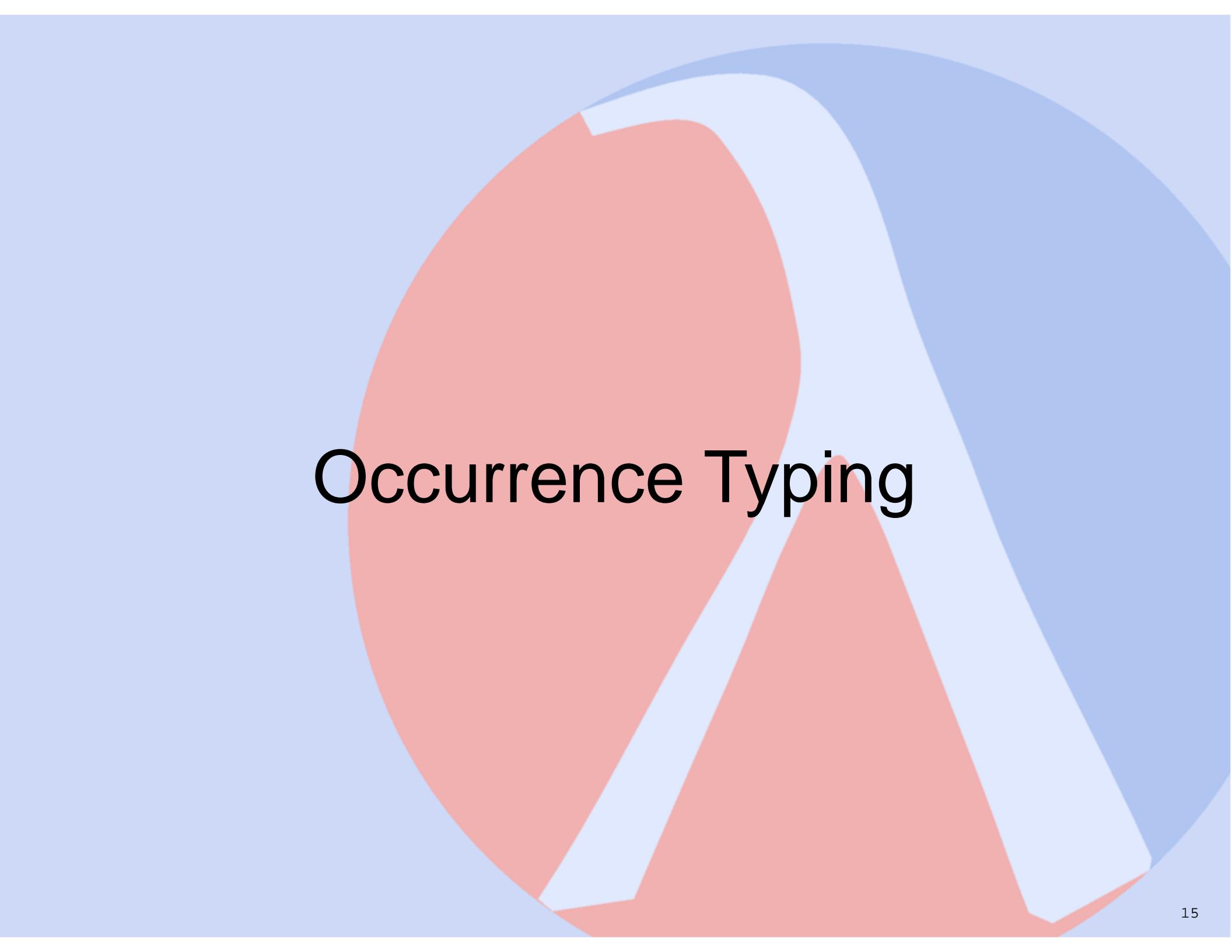
# What Can We Learn?

- New points in the design space

Ruby, Scheme, ...

- New type system ideas

Occurrence Typing



# Occurrence Typing

# Simple Examples

```
#lang typed/scheme

(: twice : Any -> Number)
(define (twice x)
  (if (number? x)
      (* 2 x)
      0))
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# Simple Examples

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#lang typed/scheme

(: twice : (U Number String) -> Number)
(define (twice x)
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---

Number<sub>x</sub>

# Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(or (number? x) (string? x))
          (twice x)]
        [else 0]))
```

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$\text{Number}_x \vee \text{String}_x$

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$\text{Number}_x \vee \text{String}_x \quad (\text{U Number String})_x$

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$\overline{\text{Number}_x} \vee \overline{\text{String}_y}, \text{Number}_x \quad \overline{\text{String}_y}$

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$(U \text{Number} \text{String})_y , \overline{\text{String}_y} \quad \text{Number}_y$

# Data Structures

```
#lang typed/scheme

(: twice-car : (Pair Any Any)  -> Number)
(define (twice-car x)
  (if (number? (car x))
      (* 2 (car x))
      0))
```

# Data Structures

```
#lang typed/scheme

(: twice-car : (Pair Any Any)  -> Number)
(define (twice-car x)
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```

Number\_car(x)

# Abstraction

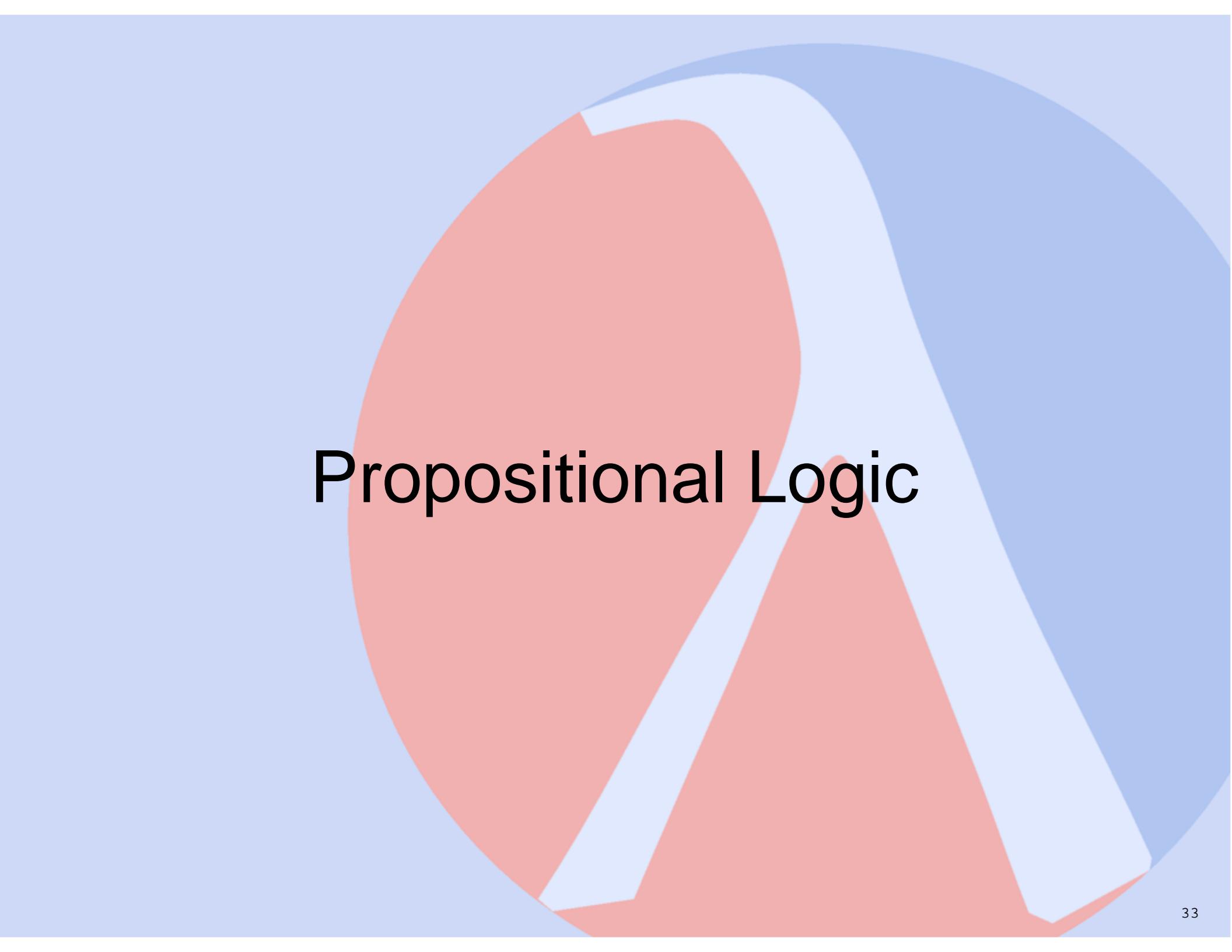
```
#lang typed/scheme

(: car-num? : (Pair Any Any) -> Boolean : Number @ car)
(define (car-num? x)
  (number? (car x)))
```

# Abstraction

```
#lang typed/scheme

(: car-num? : (Pair Any Any) -> Boolean : Number @ car)
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# Propositional Logic

# Judgments

$$\Gamma \quad e : T ; \phi_1 \mid \phi_2$$

# Judgments

$$\Gamma \quad \textcolor{red}{e} : T ; \phi_1 \mid \phi_2$$

$e ::= n \mid c \mid (\lambda x : T . e) \mid (e e) \mid (\text{if } e e e)$

# Judgments

$$\Gamma \quad e : \textcolor{red}{T} ; \phi_1 \mid \phi_2$$

**T** ::= Number | (U T ...) | #t | #f | (x:T -> T : φ|φ)

# Judgments

$$\Gamma \quad e : T ; \phi_1 \mid \phi_2$$
$$\phi ::= T_{\pi(x)} \mid \overline{T_{\pi(x)}} \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \phi_1 \supset \phi_2$$

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$$\Gamma ::= \phi \dots$$

# Judgments

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# Judgments

$$\Gamma \quad \phi$$
$$\frac{}{\text{Number}_x \vee \text{String}_y}, \text{ Number}_x \quad \frac{}{\text{String}_y}$$

# Typing

(if e<sub>1</sub> e<sub>2</sub> e<sub>3</sub>)

# Typing

(**if**  $e_1$   $e_2$   $e_3$ )

$\Gamma \quad e_1 : T_1 ; \phi_{-+} \mid \phi_{--}$

# Typing

(if  $e_1$   $e_2$   $e_3$ )

$\Gamma \quad e_1 : T_1 ; \phi_- + | \phi_- -$

$\Gamma, \phi_- + \quad e_2 : T ; \phi 1_- + | \phi 1_- -$

$\Gamma, \phi_- - \quad e_3 : T ; \phi 2_- + | \phi 2_- -$

# Typing

(**if**  $e_1$   $e_2$   $e_3$ )

$\Gamma \quad e_1 : T_1 ; \phi_-+ | \phi_- -$

$\Gamma, \phi_-+ \quad e_2 : T ; \phi_1_-+ | \phi_1_- -$

$\Gamma, \phi_- - \quad e_3 : T ; \phi_2_-+ | \phi_2_- -$

# Typing

$\Gamma \quad (\text{if } e_1 \ e_2 \ e_3) : T ; \phi_{1\_+} \vee \phi_{2\_+} \mid \phi_{1\_‐} \vee \phi_{2\_‐}$

$\Gamma \quad e_1 : T_1 ; \phi\_+ \mid \phi\_‐$

$\Gamma, \phi\_+ \quad e_2 : T ; \phi_{1\_+} \mid \phi_{1\_‐}$

$\Gamma, \phi\_‐ \quad e_3 : T ; \phi_{2\_+} \mid \phi_{2\_‐}$

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# Typing

$\Gamma \quad x : T$

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$\Gamma \quad x : T$

$\Gamma \quad T_x$

# Try Typed Scheme

Installer and Documentation  
**<http://www.plt-scheme.org>**

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