College of Computer & Information Science — Fall 2006
Northeastern University — Handout 10
CSU690: Algorithms and Data — 13 October 2006

Problem Set 4 (due Tuesday, October 24)

1. (5 + 5 = 10 points) Uniqueness of MSTs when all weights are distinct

- Suppose T_1 and T_2 are distinct minimum spanning trees for graph G. Let (u, v) be the lightest edge (smallest weight edge) among all edges that are in T_1 and but not in T_2. Let (x, y) be any edge that is in T_2 and not in T_1. Show that $w(x, y) \geq w(u, v)$.
- Prove that if the weights on the edges of a connected, undirected graph are distinct, then there is a unique minimum spanning tree.

2. (6 + 6 = 12 points) Bottleneck MSTs

Chapter 4, Exercise 9, page 192.

3. (2 + 10 = 12 points) Nesting boxes

We are given a set of n empty boxes $B_i = (h_i, w_i, \ell_i)$, $1 \leq i \leq n$, where h_i, w_i, and ℓ_i denote the height, width, and length of box B_i. Let us say that box B_i *nests* within box B_j if and only if box B_i can be oriented so that it is *strictly smaller* than box B_j in each corresponding dimension. For example, even though the box $B_i = (5, 1, 1)$ is taller than the box $B_j = (2, 6, 3)$, B_i can be placed inside B_j by orienting it as a $(1, 5, 1)$.

(a) Describe an $O(1)$ time algorithm to determine whether one box nests within another.

(b) We would like to compute the maximum length k of a sequence of boxes $B_{i_1}, \ldots, B_{i_k}$ such that B_{i_j} nests inside $B_{i_{j+1}}$, $1 \leq j < k$. Design and analyze a polynomial-time algorithm for the problem. (*Hint:* One can use either dynamic programming or express the problem as a shortest-path problem in a suitably defined graph.)

4. (13 points) Word segmentation

Chapter 6, Exercise 5, page 316.

5. (13 points) Defending against robot attacks

Chapter 6, Exercise 8, page 319.