College of Computer & Information Science
Northeastern University
CS5800: Algorithms

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Handout 28
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Sample Solution to Problem Set 7

1. Problem 8.22 of text.

Answer:

a) One can verify if $E' \subseteq E$ is a feedback arc set (FAS) in polynomial time by removing E' from E and using BFS to check if there is a cycle in $(V, E - E')$.

So we can prove FAS is in NP.

b) Suppose C is a vertex cover of G with size b. For any vertex $v_i \in C$, suppose its corresponding vertices in G' are w_i and w_i', so we add the edge (w_i, w_i') into E'.

After we process every vertex in C, the edge set E' we get is exactly the FAS of G with size b. It is because for vertices w_i and w_i', when we remove the edge (w_i, w_i'), all the edges that connect to w_i could not be in any cycle, since w_i has no out-edge.

Similarly all the edges that connect to w_i' could not be in any cycle either, since w_i' has no in-edge.

c) For any edge (v_i, v_j) in G, suppose its corresponding vertices in G' are w_i, w_i', w_j and w_j', and the corresponding edges are (w_i, w_i'), (w_j, w_j'), (w_i', w_j) and (w_j', w_i).

- If E' is a FAS of G with size b, obviously there is at least one edge of the above four edges that is in E', otherwise it will form a cycle.
- Moreover edge e has at least one vertex that belongs to (w_i, w_j). If w_i is the vertex of e, we will add v_i into C, otherwise we add v_i into C.

After the above processing, C is a vertex cover of G with size (at most) b.