Problem Set 4 (due Wednesday, March 9)

1. (10 points) Rate-Optimal Spanning Tree

You are working at a telecommunications company that has access to a communication network G connecting a set V of nodes with a set E of undirected edges (links). Each link e in E has an associated bandwidth b_e. For any path P in G that connects two nodes, say u and v, we define the *rate* $b(P)$ of P to be the minimum, over all edges e in P, of b_e. For any pair u and v of vertices in G, the *best rate achievable between u and v in G* is the maximum of $b(P)$ over all paths P from u to v in G.

The era of cost-cutting has hit your firm as well. One idea proposed for trimming costs is to not use the entire network G, but instead use a *spanning tree* T of G.

(a) Show that there exists a spanning tree T of G that has the following (seemingly incredible) property: for *every pair* u and v of vertices in G, the best rate achievable between u and v in T is equal to the best rate achievable between u and v in G.

(b) Give an efficient algorithm to compute a spanning tree that satisfies the property stated in (a).

2. (20 points) Problem 5.21.

3. (10 points) Problem 5.29.