Problem Set 3 (due Wednesday, February 23)

1. (10 points) Problem 3.16 of text.

2. (10 points) One-way connected graphs

Recall that a directed graph G is strongly connected if for any two vertices u and v, there is a path from u to v *and* there is a path from v to u. We say G is *one-way connected* if for any two vertices u and v, either there is a path from u to v *or* there is a path from v to u (or both).

For instance, the graph over 3 vertices u, v, w and two edges $u \to v$ and $v \to w$ is one-way connected. On the other hand, the graph over 4 vertices u, v, w, x with four edges $u \to v$, $u \to w$, $v \to x$, and $w \to x$ is not one-way connected since neither there is a path from v to w nor there is a path from w to v.

Design a linear-time algorithm for determining whether a given directed graph G is one-way connected.

3. (10 points) Network forensics

You are the chief network administrator of a large corporate network, and have just been informed of a major virus infection in the network. The entire network has been shut down to prevent further damage. You and your forensics team get down to business and immediately start poring over the network logs to figure out what happened and when. Your network consists of n computers which we label C_1, C_2, through C_n. You are able to collect all communication information in the form of m triples of the form (C_i, C_j, t), which means that C_i and C_j communicated with each other at time t.

You know that the virus first infected computer C_1 from an external source at time 0. From then on, there were no further infections from outside the network. But the virus is very infectious. So if C_i communicated with C_j at time t, and if one of them is infected prior to the communication, then the other will certainly be infected at time t.

Given the set of all communication triples, you would like to determine (a) which computers have been infected at the current time, (b) the precise times at which these computers were first infected, and (c) for each infected computer, the path by which the infection first reached the computer. Design an efficient (time polynomial in n and m) algorithm to solve this problem, and analyze its worst-case running time. You may assume for convenience that all times listed in the communication triples are integers.

4. (10 points) Problem 4.13 of text.