College of Computer & Information Science — Spring 2011
Northeastern University — Handout 12
CS5800: Algorithms — 5 March 2011

Some Practice Problems for Midterm

1. Derive an asymptotically tight bound for the following recurrence. Assume that $T(n)$ is $\Theta(1)$ for $n \leq 4$.

$$T(n) = T(n/2) + T(3n/4) + n^2.$$

2. Problem 1.42 of text.

3. A *hamiltonian path* of a directed graph G is a simple path in G that visits every vertex in G exactly once.

Design a linear time algorithm to determine whether a given directed acyclic graph has a hamiltonian path.

4. For each of the following claims, indicate whether it is true or false. Briefly justify your answers.

(a) If T is a minimum spanning tree of a weighted undirected graph G, then the minimum-weight edge of T is also a minimum-weight edge of G.

(b) Let G be a given weighted undirected graph. Let T_1 and T_2 be two minimum spanning trees of G that differ in only one edge. Let e_1 be the edge in T_1 that is not in T_2 and e_2 be the edge in T_2 that is not in T_1. Prove or disprove the following statement: The cycle obtained when e_1 is added to T_2 contains e_2.

(c) Let G be a weighted directed graph and s be a vertex in G. Let p denote a shortest path from s to t. If we increase the weight of every edge in the graph by 1, then p remains a shortest path from s to t.

5. Problem 4.21 (a) of text.