College of Computer & Information Science | Summer 2011
Northeastern University | Handout 1
Directed Study: Algorithms | 29 June 2011

Final Exam

Problem 1. (15 points) Recurrence

Solve the following recurrence. You may assume that $T(1) = 1$.

$$T(n) = 5T(n/5) + n.$$

Problem 2. (15 points) Heavy-hitter: the most frequent element

Given a list L of n elements, the *frequency* of an element x in L is the number of times x occurs in L. For instance, if L is the list $[A, B, B, A, C, C, C, A, B, A, D]$, then the frequencies of A, B, C, and D in L are 4, 3, 3, and 1, respectively.

Give an algorithm that takes as input a list L of n elements and returns the element in L that has the highest frequency. Analyze the worst-case running time of your algorithm. The more efficient your algorithm is in terms of its worst-case running time, the more credit you will get.

Problem 3. (20 points) Identifying kingpins

You work for a public relations firm and have just acquired complete information on the Washington lobbying network. In particular, you have a list V of n persons (congressmen, bureaucrats, lobbyists, businessmen, etc.) and for each person i in V, a list of persons in V that i can *influence*. (Note that it is possible that i can influence j but j cannot influence i.)

Having just studied graph algorithms, you immediately capture the above information by a directed graph G with V as the set of vertices and the set E of edges defined as follows.

$$E = \{(i, j) : i \text{ can influence } j\}.$$

We call a person i a *kingpin* if for every other person $j \in V$, there is a path from i to j in G.

Give a polynomial-time algorithm to determine *all* kingpins in the given lobbying network. If there are no kingpins, then your algorithm must indicate so. State the running time of your algorithm. The more efficient your algorithm is in terms of its worst case running time, the more credit you will get.

Problem 4 (15 points) Dijkstra's Algorithm

Show a complete run of Dijkstra's algorithm on the graph given on page 127 of text, with source vertex being A. Assume the edges are directed in alphabetical order; i.e., from A to B, A to C, B to C, etc.

Problem 5 (10 + 10 = 20 points) Short-answer questions on minimum spanning trees

For each of these questions, if your claim is "True", then give a brief proof; if your claim is "False", then provide a counterexample.

(a) True or False: For any connected weighted undirected graph G, any minimum spanning tree T of G, and vertices u and v of T, the unique path from u to v in T is the shortest path from u to v in G.

(b) True or False: For any connected weighted undirected graph G, if edge (u, v) has minimum weight among all edges of G then (u, v) belongs to some minimum spanning tree of G.

Problem 6 (15 points) Huffman encoding

Exercise 5.14 of text.