College of Computer & Information Science — Spring 2008
Northeastern University — Handout 9
CSG714: Theory of Computation — 13 February 2008

Problem Set 3 (due Friday, February 22)

1. (5 + 5 = 10 points) Window size in the proof of Cook-Levin Theorem

In the proof of Cook-Levin Theorem, we captured the legal moves of a Turing machine by considering a 2-row 3-column window of the grid containing the computation history of a Turing machine running on a particular input. Show why the proof would have failed if we had used

(a) 2-row 2-column windows;

(b) two entire rows as our window (i.e., a 2-row, $n^k + 3$-column window).

2. (15 points) Generic complete problems

Show that all languages in TIME($T(n)$) reduce to the language

$$\{\langle M, x\rangle : M \text{ is a deterministic TM that accepts } xinT(|x|) \text{ time}\}.$$

Is the above language in TIME($T(n)$)?

Repeat the above exercise for nondeterministic time classes.

3. (15 points) PUZZLE is NP-complete

Problem 7.26 from Sipser's text.

4. (10 + 10 = 20 points) 0-1 Linear equations

Let A be an $m \times n$ matrix over the integers and $\mathbf{b}$ be a vector of length m over the integers. Define *LINSOL* to be the language $\langle A, \mathbf{b}\rangle$ such that there exists a length-n vector $\mathbf{x}$ with components 0s or 1s that satisfies $A\mathbf{x} = \mathbf{b}$.

(a) Prove that *LINSOL* is NP-complete.

(b) Consider the sublanguage of *LINSOL* with A restricted to all zeros except for at most two 1s in each row and $\mathbf{b}$ restricted to all 1s. Show that there is a polynomial algorithm to recognise this language.

Hint: Note that if x and y are restricted to be in $\{0, 1\}$, then $x + y = 1$ if and only if exactly one of x and y is 1.

5. (10 points) Space complexity

Prove that if NSPACE(n) = DSPACE(n), then NSPACE($S(n)$) = DSPACE($S(n)$) for all $S(n) \geq n$.