1 Solution of PS1-4

1.1 Self-reducibility of Maximum Matching

For maximum matching, the decision version is: Given a graph $G = (V, E)$ and an integer k, decides whether the maximum matching of G, denoted by $M(G)$, has size greater than or equal to k. The self-reducibility of maximum matching can be proved as follows.

Given a graph $G = (V, E)$, we can call oracle $O(\log(|E|))$ to decide $|M(G)|$ by binary searching. Then, we can use the following procedure to find the maximum matching:

1. $S \leftarrow \emptyset, G \leftarrow G(V, E), m \leftarrow |M(G)|$
2. If $m = |E|$, then $S \leftarrow S \cup E$, return S; Otherwise
3. Pick arbitrary edge $e \in E$, let $G' \leftarrow G - \{e\}$
4. Ask oracle whether $|M(G')| \geq m$
5. If yes, let $G \leftarrow G'$, repeat from step 3; Otherwise
6. Let $S \leftarrow S \cup \{e\}, G \leftarrow G' - \{\textit{all edges adjacent to e in G}\}$, $m \leftarrow m-1$, repeat from step 2

This procedure will find a maximum matching for G. It will call oracle $O(|E| + \log(|E|))$ times. For each edge picked, finding adjacent edge costs $O(|E|)$. Each edge will be picked at most once, so the total time cost is polynomial to the size of G.

1.2 Self-reducibility of Shortest Superstring

For shortest superstring, the decision version is: Given a set of strings $S = \{s_1, s_2, \ldots, s_n\}$ and an integer l, decides whether the shortest superstring of S, denoted by $\Gamma(S)$, has length less than or equal to l. The self-reducibility of shortest superstring can be proved as follows.

Given a set $S = \{s_1, s_2, \ldots, s_n\}$, we can call oracle $O(\log(\sum_{i=1}^{n} |s_i|))$ to decide $|\Gamma(S)|$ by binary searching. Then, we can use the following procedure to find the shortest superstring:

1. $S \leftarrow \{s_1, s_2, \ldots, s_n\}, l \leftarrow |\Gamma(S)|$

2. If $l = \sum_{s_i \in S} |s_i|$, then return $\bigwedge_{s_i \in S} s_i$, where $\bigwedge$ means the concatenation; Otherwise

3. Orderly go through pair (s_i, s_j), $i \neq j$. For all possible string s_{ijk}, which is constructed from overlapping the last k chars of s_i with the first k chars of s_j, let $S' \leftarrow S - \{s_i, s_j\} \cup \{s_{ijk}\}$. Ask oracle whether $|\Gamma(S')| \leq l$.

4. If yes, $S \leftarrow S'$, repeat from step 2; Otherwise continue to next pair (s_i, s_j) or next s_{ijk}.

This procedure will find a shortest superstring for S. For each pair (s_i, s_j), there are at most $|s_i|$ number of s_{ijk}. For each s_{ijk}, we call oracle once. So total call of oracle is $O((\sum_{i=1}^{n} |s_i|)^2)$. Other costs are linear, so the total time cost is polynomial.