Problems of the Week – 3 and 4

POW-3: Consistency of constraints in program analysis

Here's a problem that occurs in automatic program analysis. Consider a program that has n integer variables $x_1, \ldots, x_n$. Analysis of the program and its specification reveal several constraints, which are of four forms: *equality*, of the form $x_i = x_j$; *inequality*, of the form $x_i > x_j$; *strict inequality*, of the form $x_i \geq x_j$; and *disequality*, of the form $x_i \neq x_j$.

Give an algorithm that takes as input m constraints over n variables and determines whether the constraints can be satisfied. Analyze the running time of your algorithm. Make your algorithm as efficient as you can, in terms of its worst case running time.

POW-4: Minimal directed graphs

Call a directed graph *minimal* if which there is at most one simple path from any vertex to any other vertex. Give an $O(mn)$ time algorithm to determine whether a given directed graph with n vertices and m edges is minimal. If you are unable to achieve the desired running time, give the fastest algorithm you can.