College of Computer & Information Science | Fall 2014
Northeastern University | Handout 24
CS7800: Advanced Algorithms | 29 November 2014

Problem of the Week – 22

Hardness of independent set

Recall that an *independent set* of an undirected graph G is a subset V' of vertices such that no two vertices in V' have an edge in G. The IndependentSet problem is to find a maximum-size independent set in G. We know that IndependentSet is NP-complete. In this problem, we investigate the approximability of IndependentSet.

Define the *product* of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ as $G = (V, E)$, where

$$V = \{\langle v_1, v_2 \rangle : v_1 \in V_1, v_2 \in V_2\},$$

and there is an edge between vertices $\langle u_1, u_2 \rangle$ and $\langle v_1, v_2 \rangle$ in G if either $u_1 = v_1$ and $(u_2, v_2) \in E_2$ or $(u_1, v_1) \in E_1$. For a positive integer n, let G^n be defined by the recurrence relation $G^{i+1} = G^i \times G$ and $G^1 = G$.

(a) Prove that G has an independent set of size k if and only if G^t has an independent set of size k^t.

(b) Give a polynomial-time algorithm to construct an independent set of G of size $\lceil k^{1/t} \rceil$ from any independent set of G^t of size k.

(c) Using parts (a) and (b), argue that if there exists a constant c such that there is a polynomial-time c-approximation algorithm for IndependentSet, then one can obtain an arbitrarily good approximation – $(1 + \varepsilon)$-factor for $\varepsilon > 0$ arbitrarily small – for IndependentSet in polynomial time.