

Problems of the Week – 15 and 16

15. Sequencing jobs with deadlines

Earlier in the semester, we used matroid theory to solve a job scheduling problem optimally. There are hundreds of useful job scheduling formulations in the literature, and many of them happen to be NP-complete. Here is one.

Given a collection of n jobs, labeled 1 through n , with job i released at time r_i and requiring processing time p_i units, we would like to determine if all of the jobs can be executed sequentially in such a way that they all complete within their deadlines. Using a reduction from the subset sum problem, show that this scheduling problem is NP-complete.

16. Luckiest sheep

A wolf is located at node 0 of a ring consisting of $n + 1$ nodes $0, 1, \dots, n$, with an (undirected) edge between node i and node $(i + 1) \bmod (n + 1)$ for $0 \leq i \leq n$. Each node i , $i > 0$, has a sheep. The wolf starts a random walk on the ring from 0. In each step, it selects one of its two neighbors uniformly at random, moves there and gobbles up the sheep present there, if not already eaten. Which sheep is likely to be eaten the last? That is, for which i is the probability that the sheep at i is eaten last the maximum?