

## Problem of the Week – 11

### Feasibility and optimality

Suppose you are given a *black box* algorithm that takes as input integers  $n$ ,  $m$ ,  $m \times n$  matrix  $A$  with integer entries, and  $m \times 1$  vector  $b$  with integer entries, and returns whether there exists a real  $n \times 1$  vector  $x$  such that  $Ax \geq b$  (i.e, the black box returns a yes or no answer).

You are faced with the following problem.

Find  $x$  that minimizes  $c^T x$  subject to the constraint  $A'x \geq b'$ ,

where  $c$ ,  $A'$ , and  $b'$  are  $n' \times 1$  vector,  $m' \times n'$  matrix, and  $m' \times 1$  vector, all with integer entries, respectively. Show how to solve this problem by using the black box algorithm, where the number of calls you make is at most polynomial in  $n'$ ,  $m'$ , and the sizes of  $A'$ ,  $b'$ , and  $c$ .