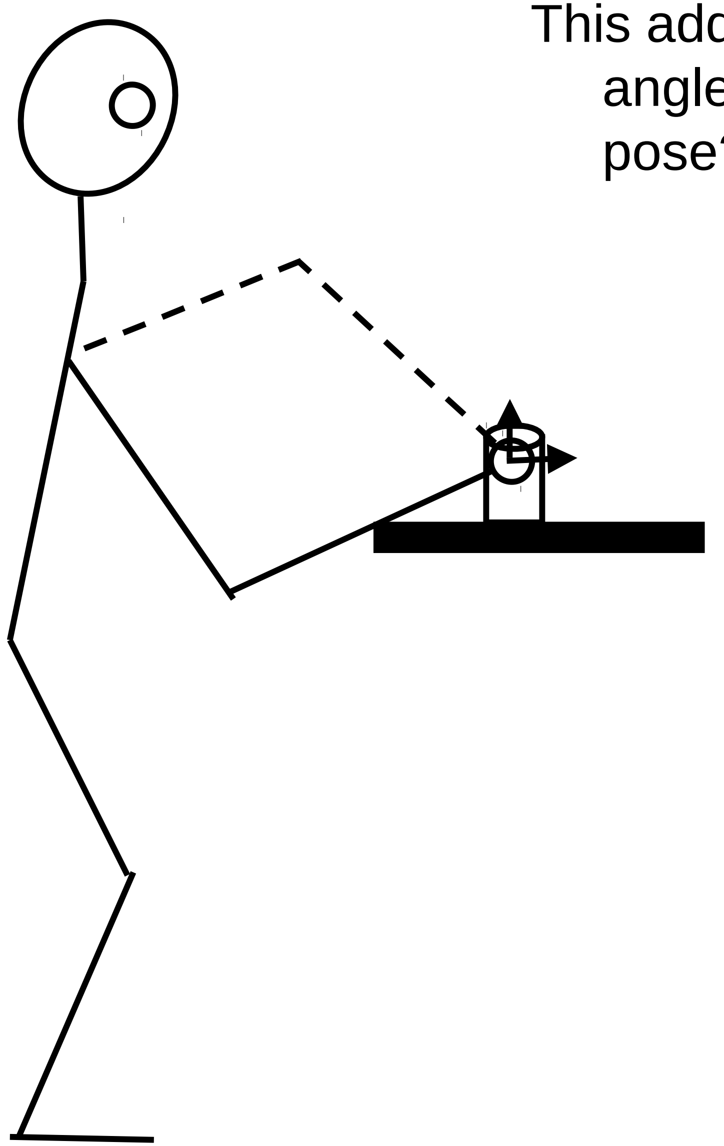
Inverse Kinematics

Robert Platt
Northeastern University

Inverse Kinematics



This addresses the obvious question: what joint angles will place my end effector in a desired pose?

Inverse kinematics

Closed form (analytical) solution: a sequence or set of equations that can be solved for the desired joint angles

- Potentially faster than an iterative solution
- A unique solution to all manipulator positions can be determined *a priori*.
    - Can guarantee “safe” joint configurations where the manipulator does not collide with the body.

Iterative (numerical) solution: numerical iteration toward a desired goal position (variation on Newton’s method)

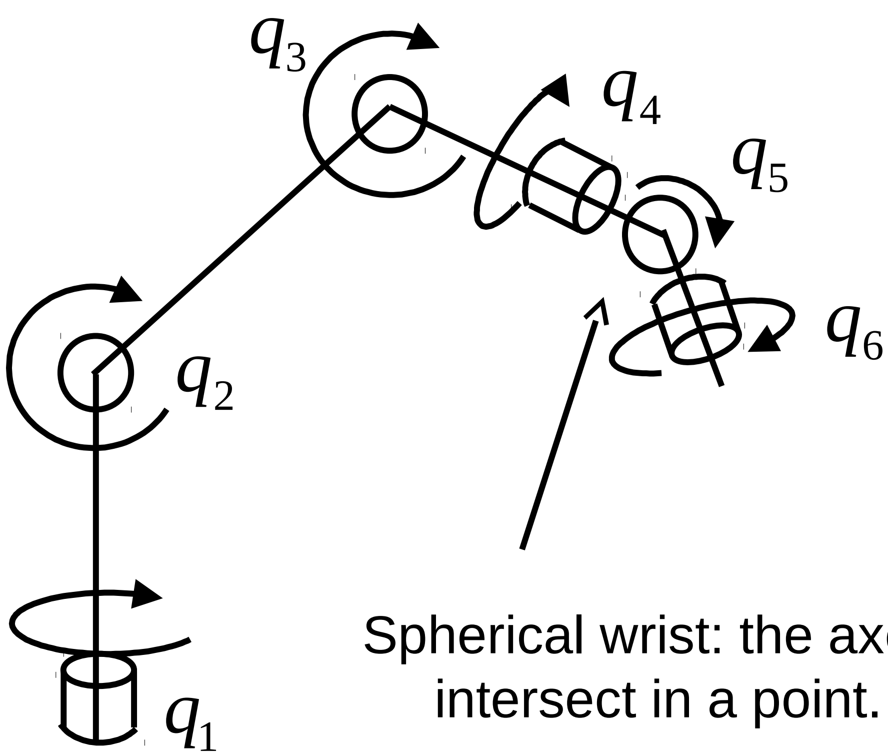
- Easier to think about
- Better suited to incremental displacements and control.

Inverse kinematics

There is no general analytical inverse kinematics solution

- All analytical inverse kinematics solutions are specific to a robot or class of robots.
    - based on geometric intuition about the robot
- I'll give one example – there are many variations.

Inverse kinematics

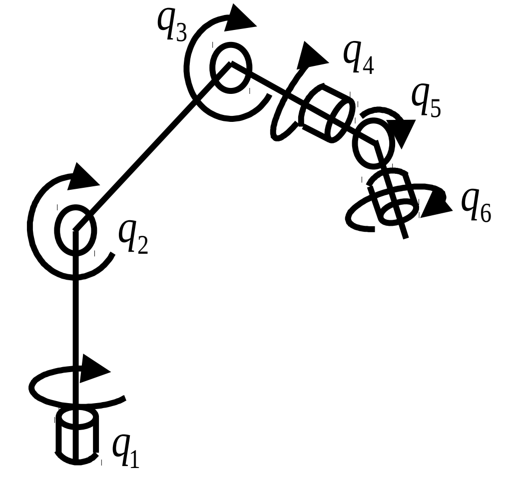


Spherical wrist: the axes of the last three joints intersect in a point.

Consider this 6-joint robot:

- this example is out of the book…

Inverse kinematics



Problem:

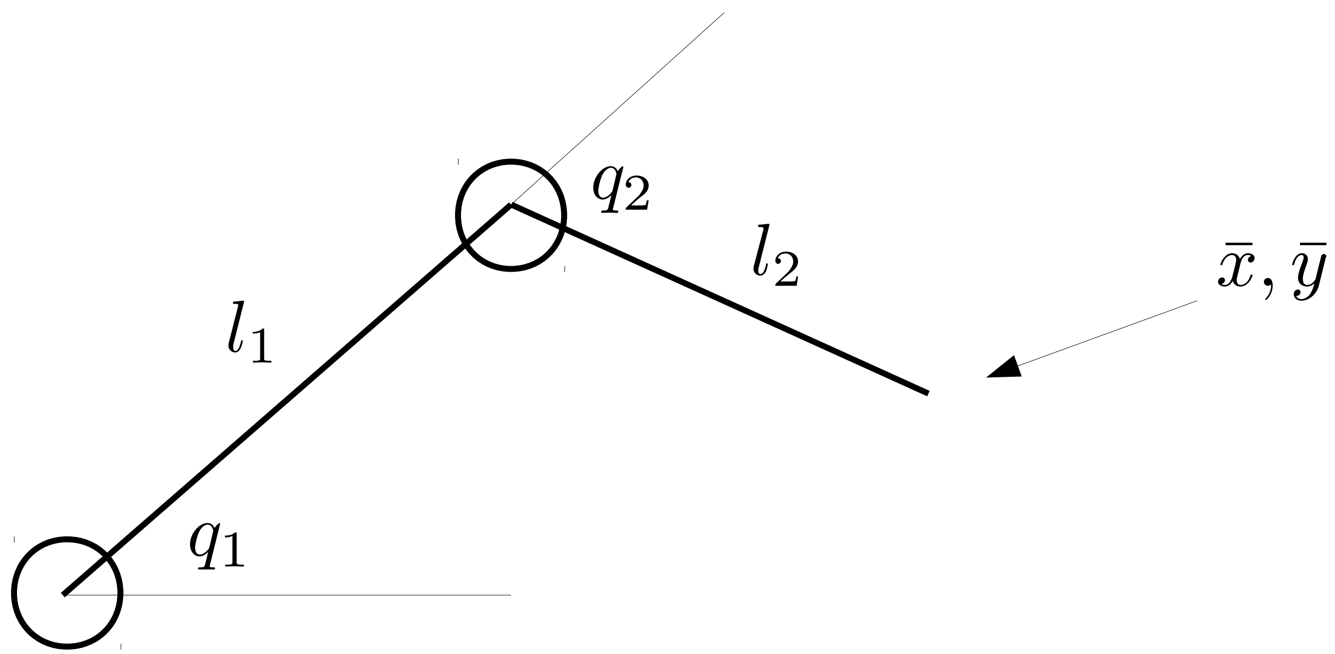
- Given: desired transform, $T_{eff} = \begin{pmatrix} R_{eff} & d_{eff} \\ 0 & 1 \end{pmatrix}$

- Find: $q = \begin{pmatrix} q_1 & q_2 & q_3 & q_4 & \cdots & q_n \end{pmatrix}$

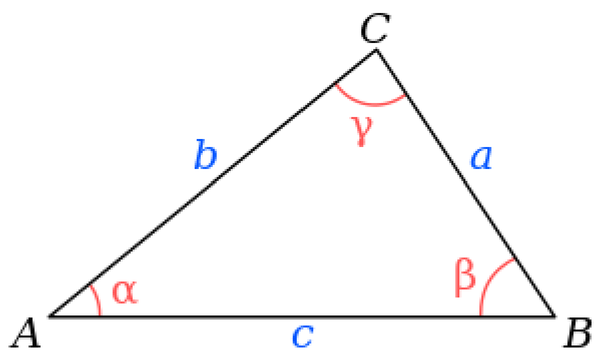
Note:

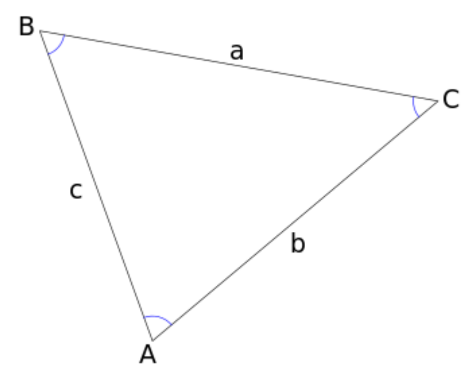
- The desired transform (pose) encodes six *degrees of freedom* (this info can be represented by six numbers)
  - Since we only have six joints at our disposal, there is no manifold of *redundant* solutions.
- For this manipulator, the problem can be decomposed into a position component (the first three joints) and an orientation component (the last three joints)
- The first three joints tell you what the position of the spherical wrist

In class exercise



Given l_1 and l_2, calculate joint angles that cause eff to reach $\bar{x}, \bar{y}$

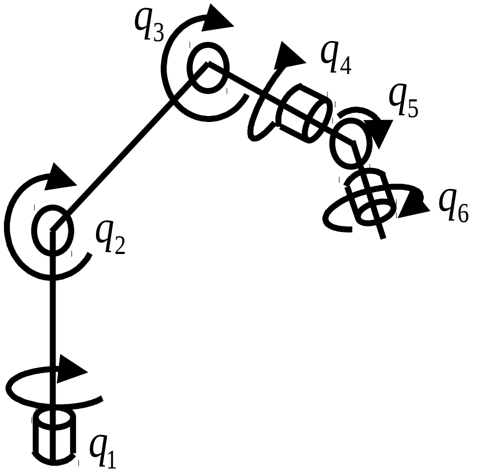




$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Example: Inverse kinematics



Solution:

- First, back out the position of the spherical wrist:

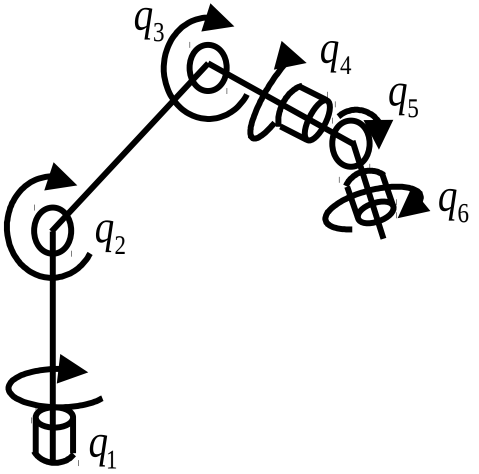
Since it's a spherical wrist, the last three joints can be thought of as rotating about a point.

- A constant transform exists that goes from the last wrist joint out to the end effector (sometimes this is called the "tool" transform): ${}^{sw}T_{eff}$

- Back out the position of the wrist:

$$ {}^{b}T_{sw} = {}^{b}T_{eff} \, {}^{sw}T_{eff}^{-1} $$

Example: Inverse kinematics



- Next, solve for the first three joints

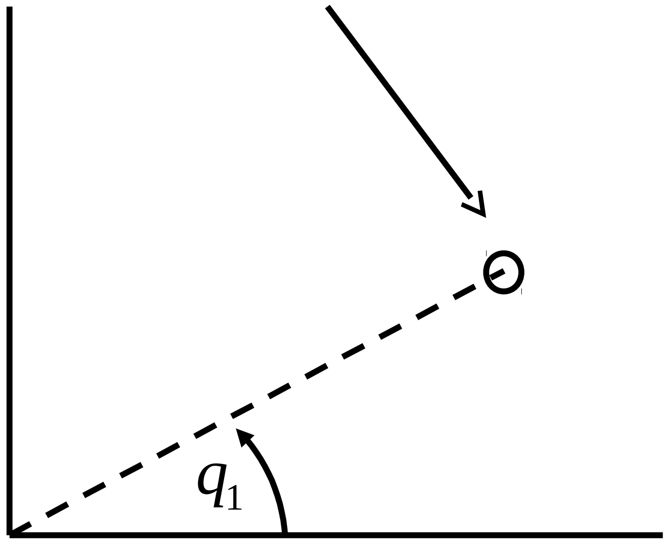
First, solve for q_1. (look down from above)

$$q_1 = a\tan 2\left(x_g, y_g\right)$$

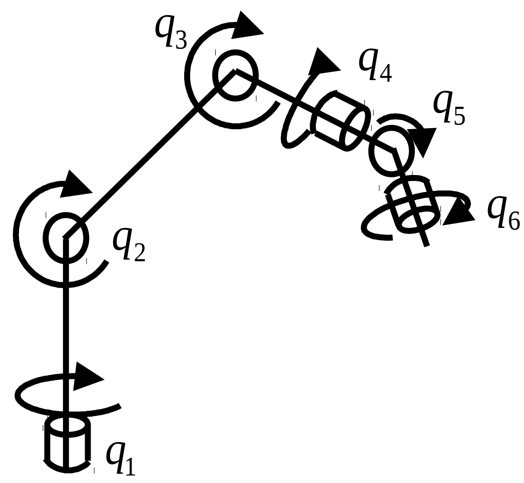
or

$$q_1 = a\tan 2\left(x_g, y_g\right) + \pi$$

Goal position in horizontal plane



Example: Inverse kinematics



Next, solve for q_3. (look at the manipulator orthogonal to the plane of the first two links)

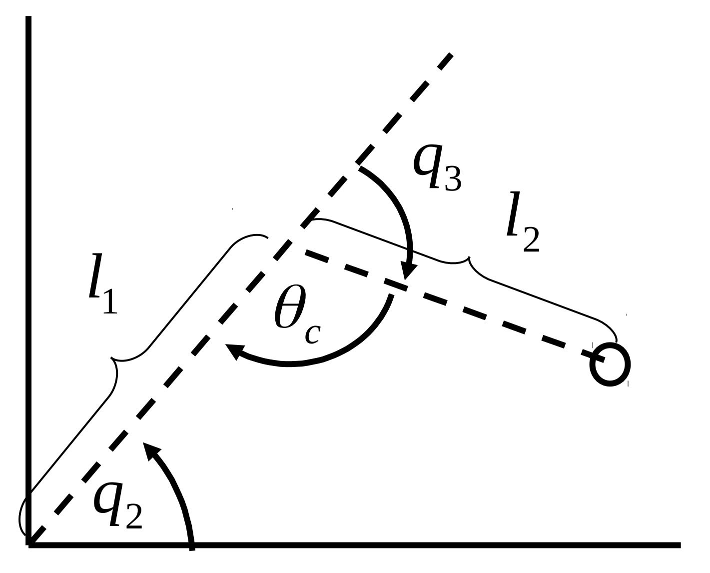
$$c^2 = a^2 + b^2 - 2ab\cos(\theta_c)$$

$$\cos(\theta_c) = -\frac{r_g^{\,2} + (z_g - h)^2 - l_1^{\,2} - l_2^{\,2}}{2l_1l_2} = -D$$

where $r_g^{\,2} = x_g^{\,2} + y_g^{\,2}$

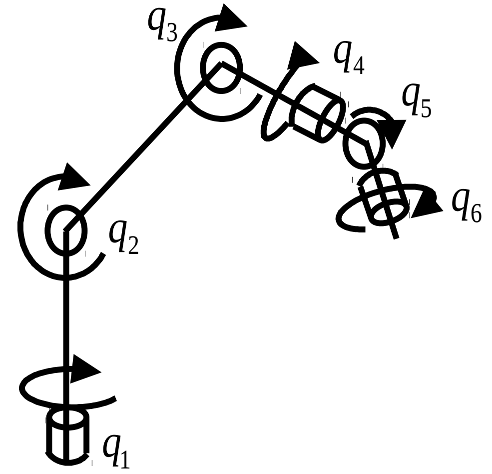
and h is the height of the first link

$$\tan(q_3) = \frac{\pm\sqrt{1 - D^2}}{D}$$



Example: Inverse kinematics

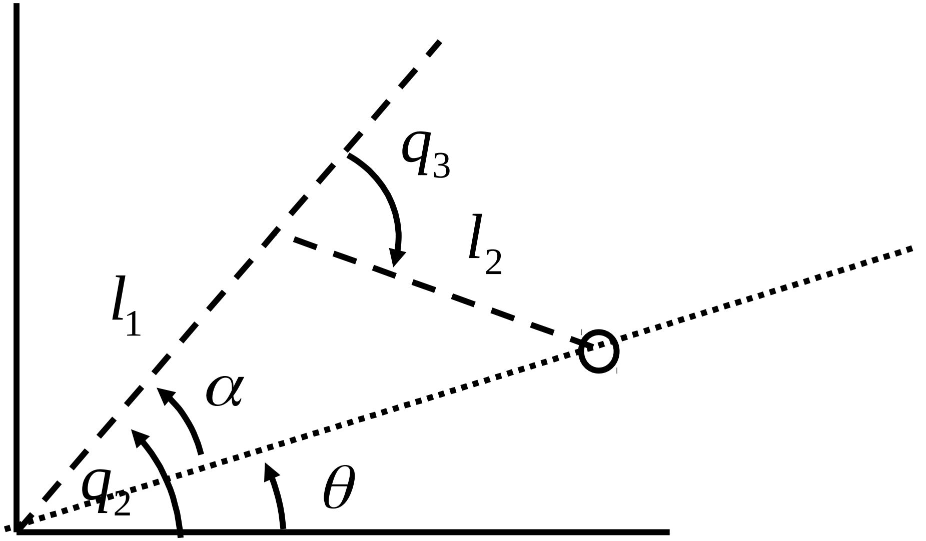
Next, solve for q_2. (continue to look at the manipulator orthogonal to the plane of the first two links)



$$\tan(\theta) = \frac{z_g - h}{\sqrt{x_g^2 + y_g^2}}$$

$$\tan(\alpha) = \frac{l_2 s_3}{l_1 + l_2 c_3}$$

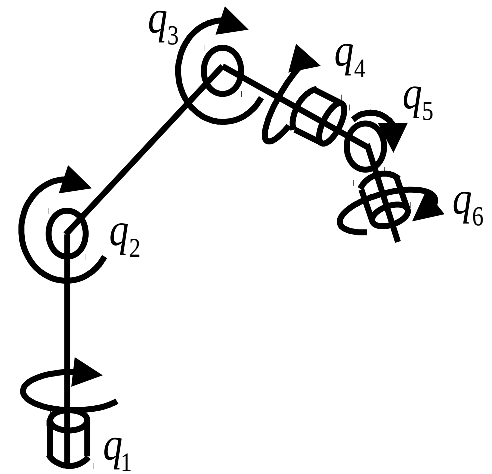
$$q_2 = \theta \pm \alpha$$



Example: Inverse kinematics

Finally, the last three joints completely specify the orientation of the end effector.

- Note that the last three joints look just like ZYZ Euler angles
  - Determination of the joint angles is easy – just calculate the ZYZ Euler angles corresponding to the desired orientation.



Remember: ZYZ Euler Angles

$$R_{zyz}(\varphi,\theta,\psi)=\begin{pmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$R_{zyz}(\varphi,\theta,\psi)=\begin{pmatrix}c_\varphi c_\theta c_\psi - s_\varphi s_\psi & -c_\varphi c_\theta s_\psi - s_\varphi c_\psi & c_\varphi s_\theta\\ s_\varphi c_\theta c_\psi + c_\varphi s_\psi & -s_\varphi c_\theta s_\psi + c_\varphi c_\psi & s_\varphi s_\theta\\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta\end{pmatrix}$$

$$\theta=\pm a\tan 2\left(\sqrt{1-r_{33}^{2}},r_{33}\right)$$

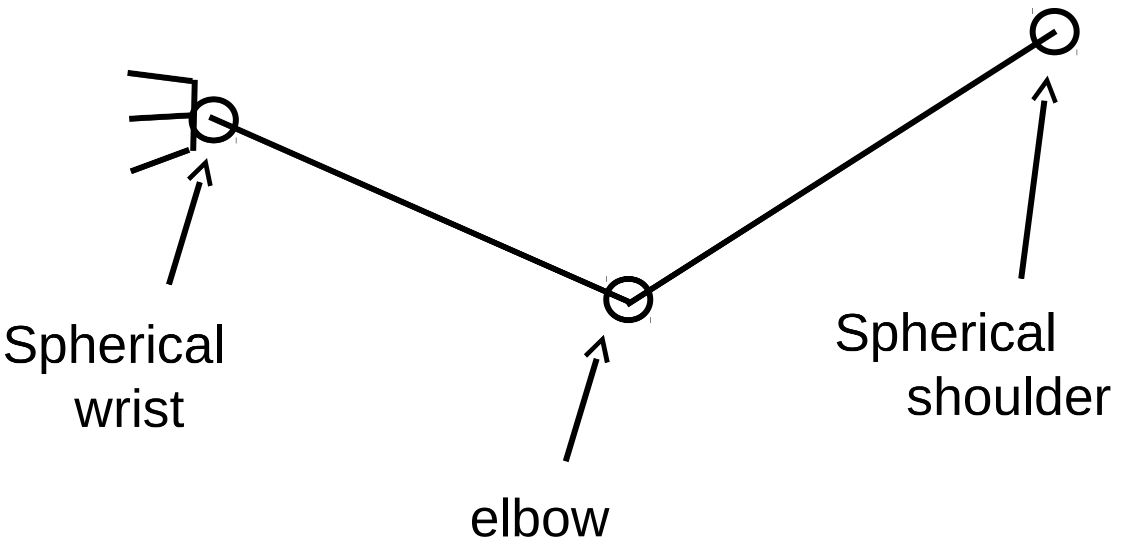
$$\phi=a\tan 2(r_{23},r_{13})+k\pi$$

$$\psi=a\tan 2(r_{32},r_{31})$$

Inverse kinematics for a humanoid arm

You can do similar types of things for a humanoid (7-DOF) arm.

- Since this is a redundant arm, there are a manifold of solutions…



General strategy:

1. Solve for elbow angle
2. Solve for a set of shoulder angles that places the wrist in the right position (note that you have to choose an elbow orbit angle)
3. Solve for the wrist angles