Homogeneous Transforms

Robert Platt
Northeastern University

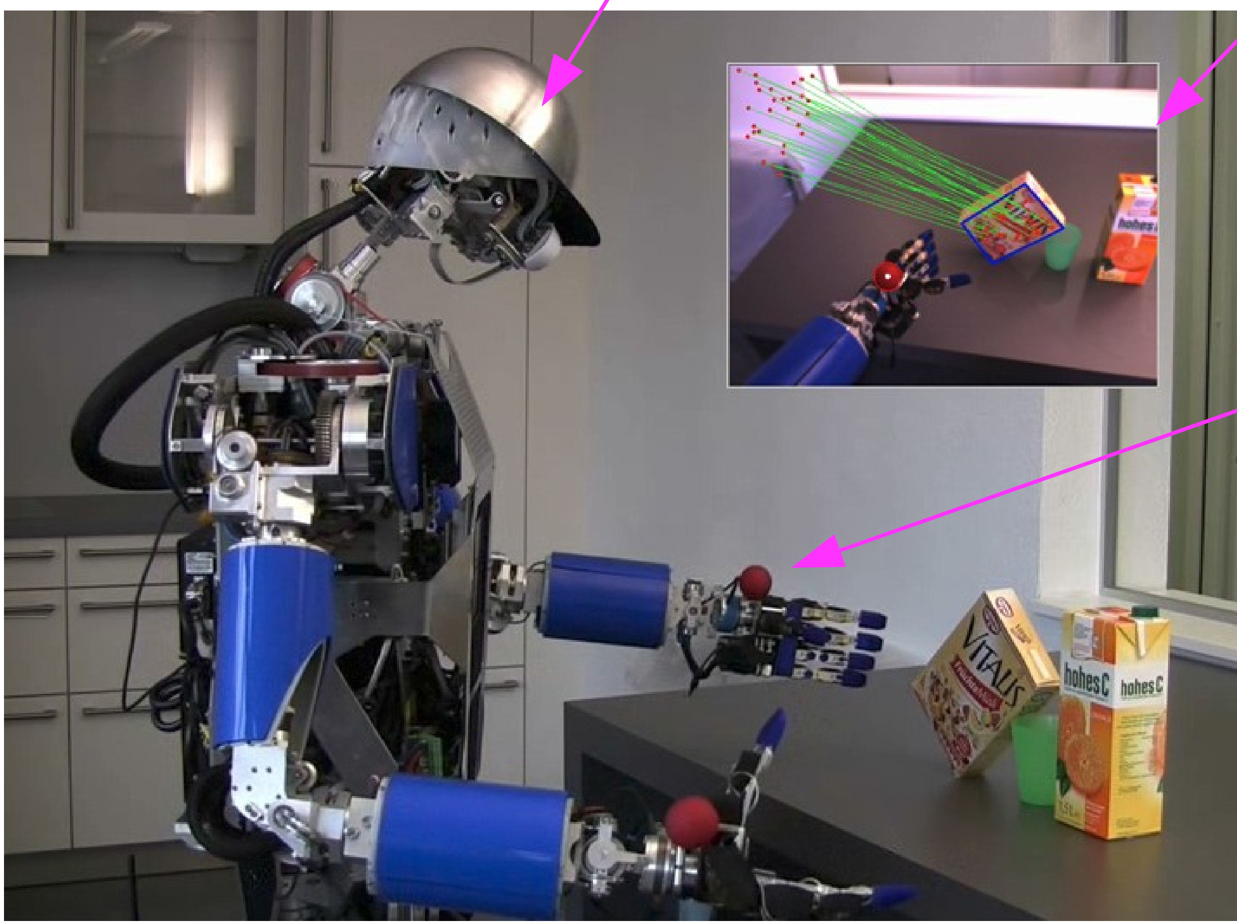
Why do we care about kinematics?

Joint encoders tell us head angle

Visual perception tells us object position and orientation (pose)

Need to know where hand is...

Need to tell the hand where to move!

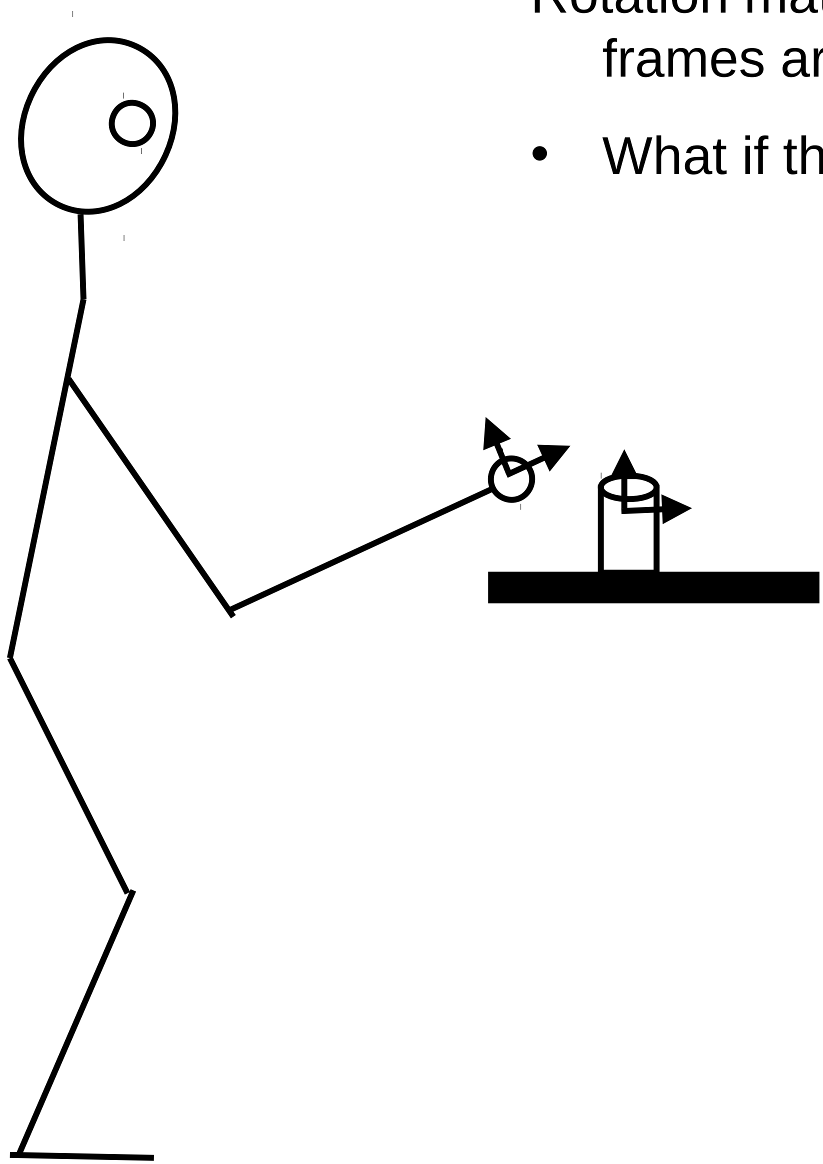


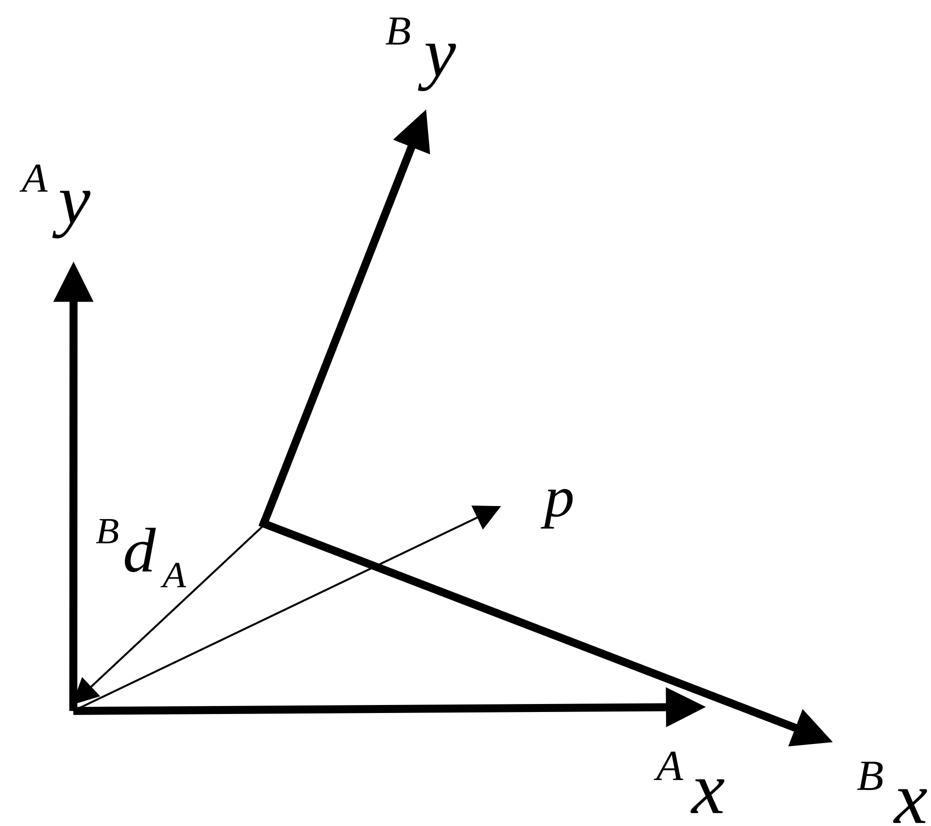
KIT Humanoid

Homogeneous transforms

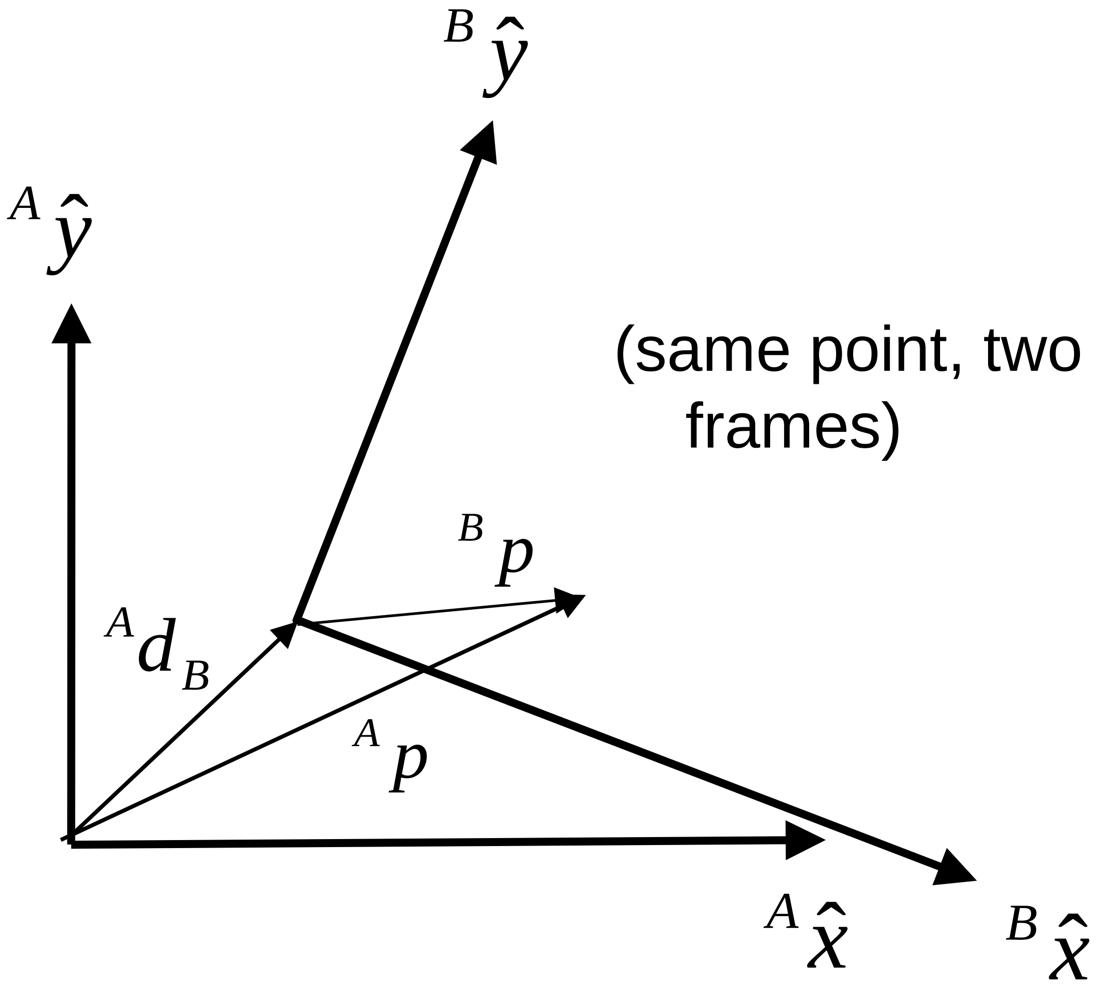
Rotation matrices assume that the origins of the two frames are co-located.

- What if they're separated by a translation?





Homogeneous transform



(same point, two reference frames)

$$^{A}p = {}^{A}R_{B}\,{}^{B}p + {}^{A}d_{B}$$

Homogeneous transform

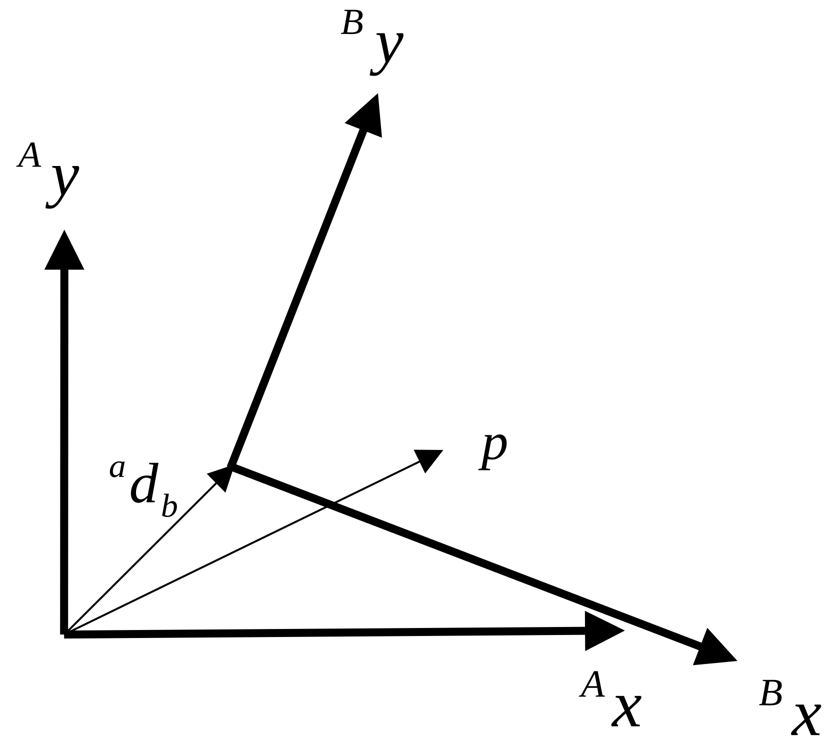
$$^{A}p = {}^{A}R_{B}\,{}^{B}p + {}^{A}d_{B}$$

$$= \begin{pmatrix} {}^{A}R_{B} & {}^{A}d_{B} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix}$$

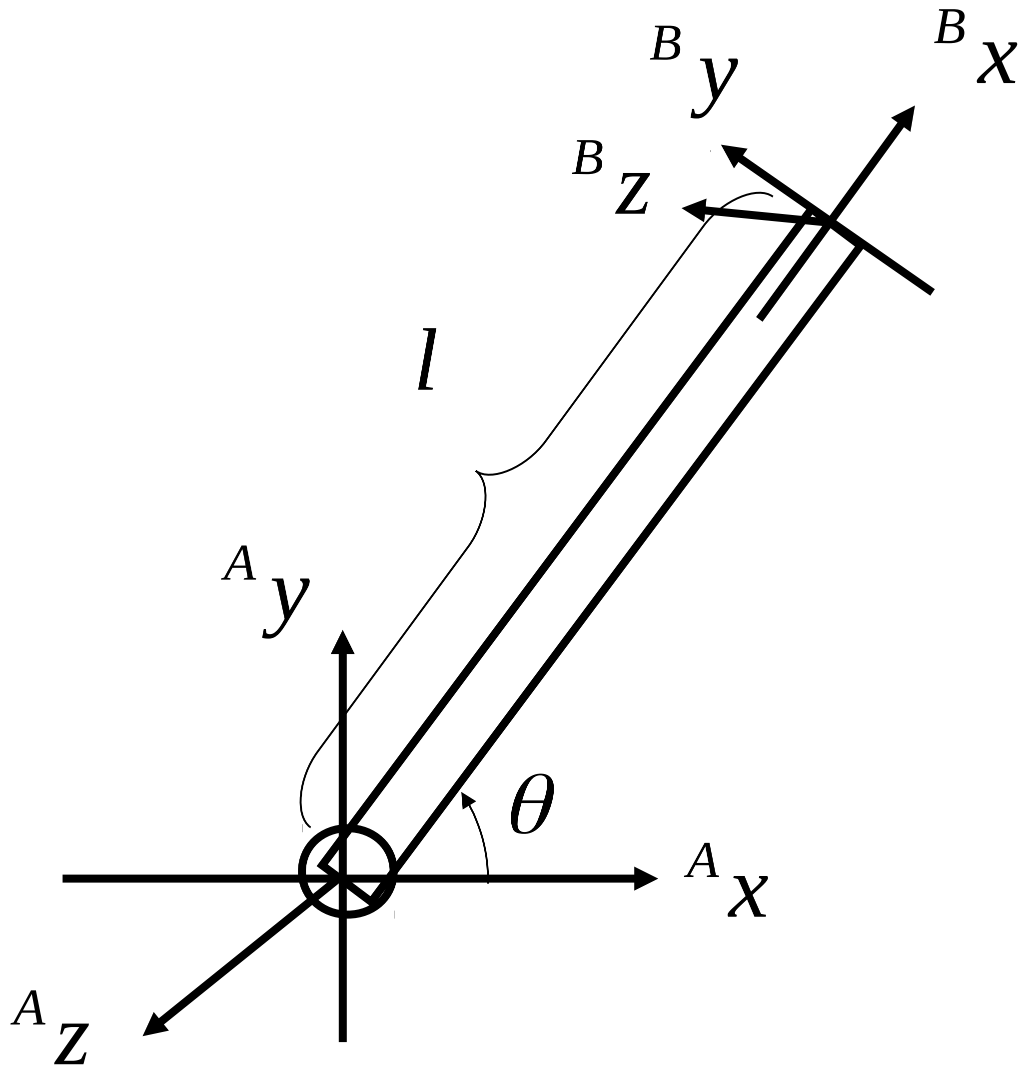
$$= \begin{pmatrix} r_{11} & r_{12} & r_{13} & {}^{A}d_{x} \\ r_{21} & r_{22} & r_{23} & {}^{A}d_{y} \\ r_{31} & r_{32} & r_{33} & {}^{A}d_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix} = {}^{A}T_{B} \begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix}$$

always zeros

always one



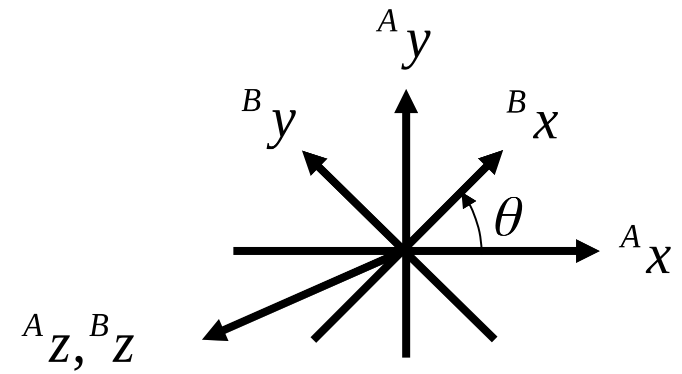
Example 1: homogeneous transforms



What's $^B T_A$?

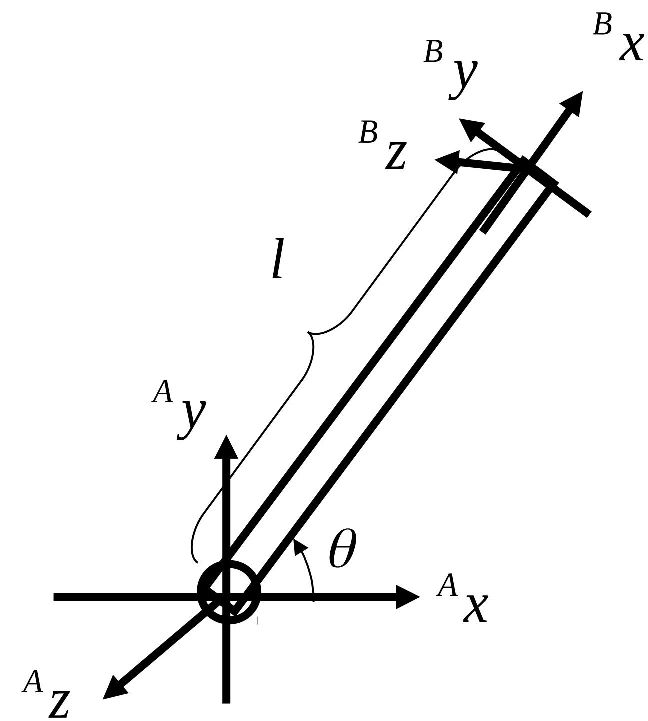
Example 1: homogeneous transforms

What's $^{B}T_{A}$?



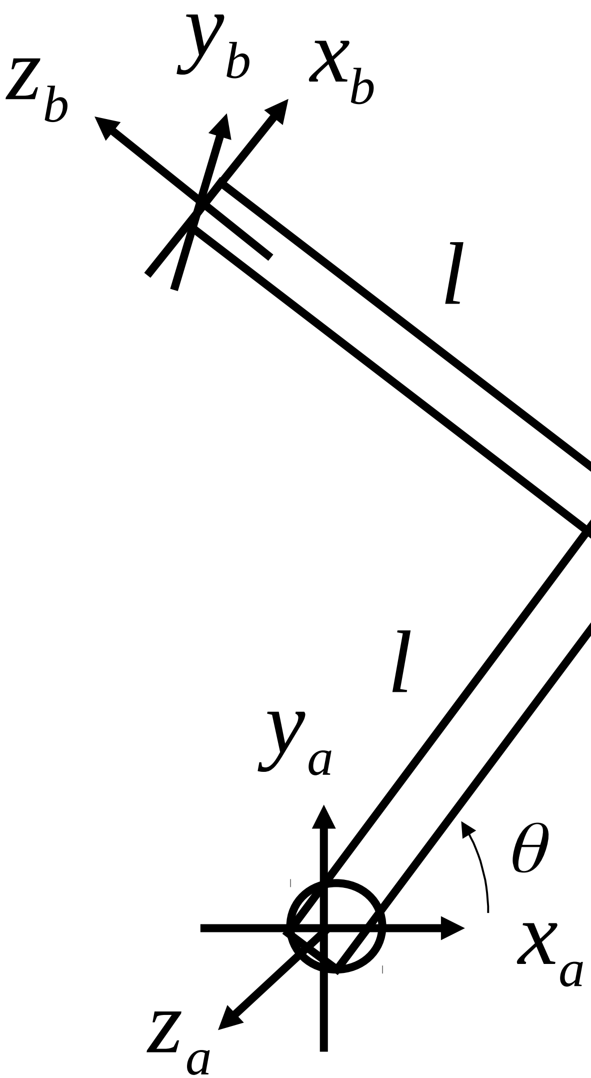
$$^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} \qquad ^{B}T_{A} = \begin{pmatrix} ^{B}R_{A} & ^{B}d_{A} \\ 0 & 1 \end{pmatrix}$$



$$^{B}T_{A} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & -l \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

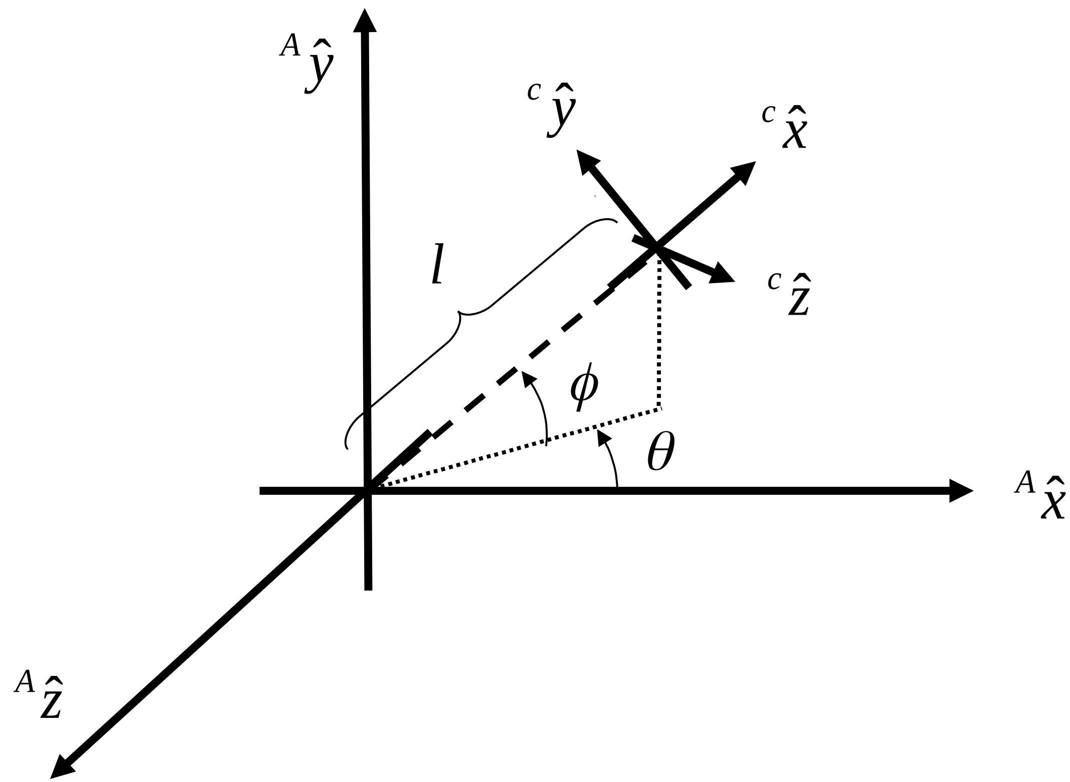
Think-pair-share



This arm rotates about the z_a axis.

Calculate: ${}^{a}T_{b}$

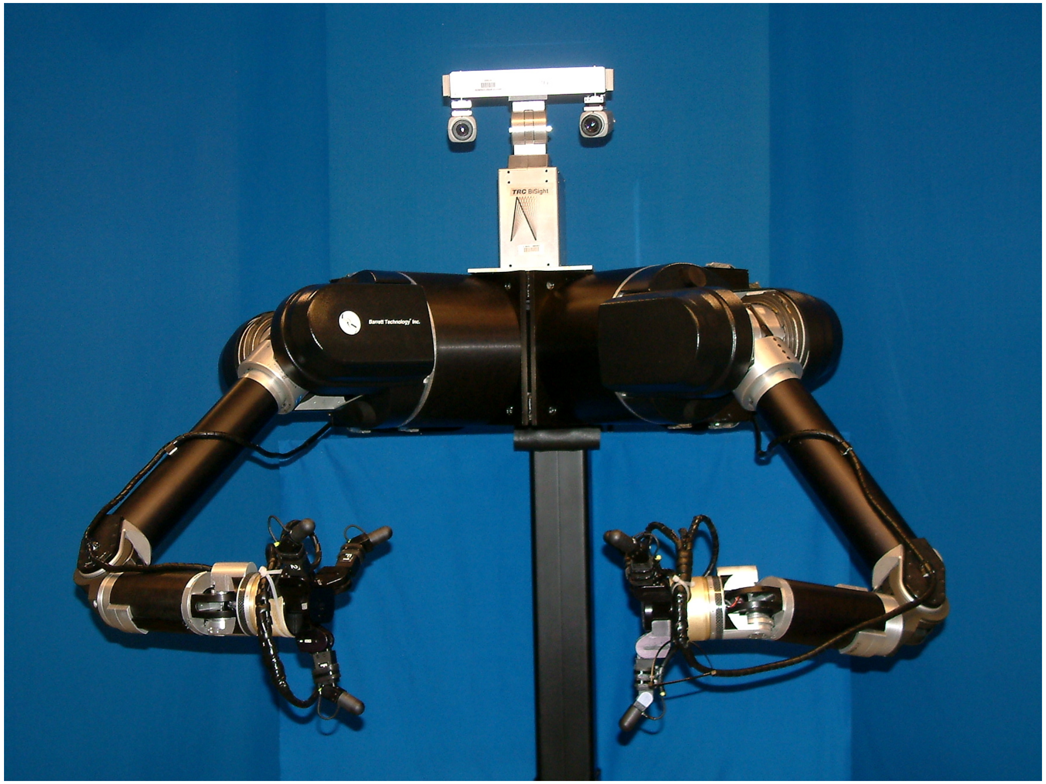
Example 3: homogeneous transforms



$$ {}^{a}R_{c} = {}^{a}R_{b}\,{}^{b}R_{c} = \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \begin{pmatrix} c_\varphi & -s_\varphi & 0 \\ s_\varphi & c_\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_\theta c_\varphi & -s_\varphi c_\theta & s_\theta \\ s_\varphi & c_\varphi & 0 \\ -s_\theta c_\varphi & s_\theta s_\varphi & c_\theta \end{pmatrix} $$

Outline the procedure for calculating ${}^{A}T_{C}$ and ${}^{C}T_{A}$

Forward Kinematics



- Where is the end effector w.r.t. the “base” frame?

Composition of homogeneous transforms

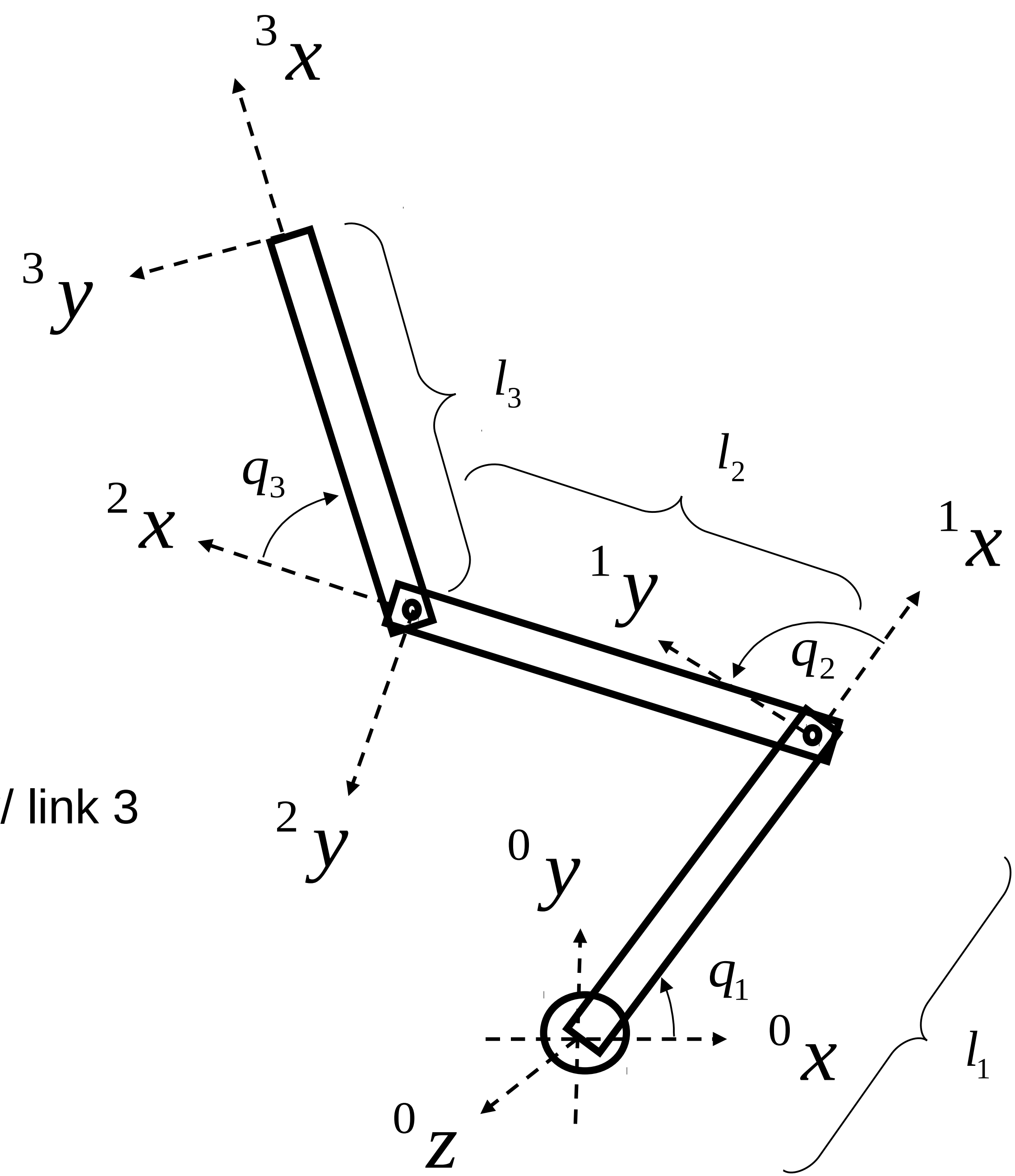
Base to *eff* transform

$$ {}^{0}T_{3} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3} $$

Transform associated w/ link 3

Transform associated w/ link 2

Transform associated w/ link 1

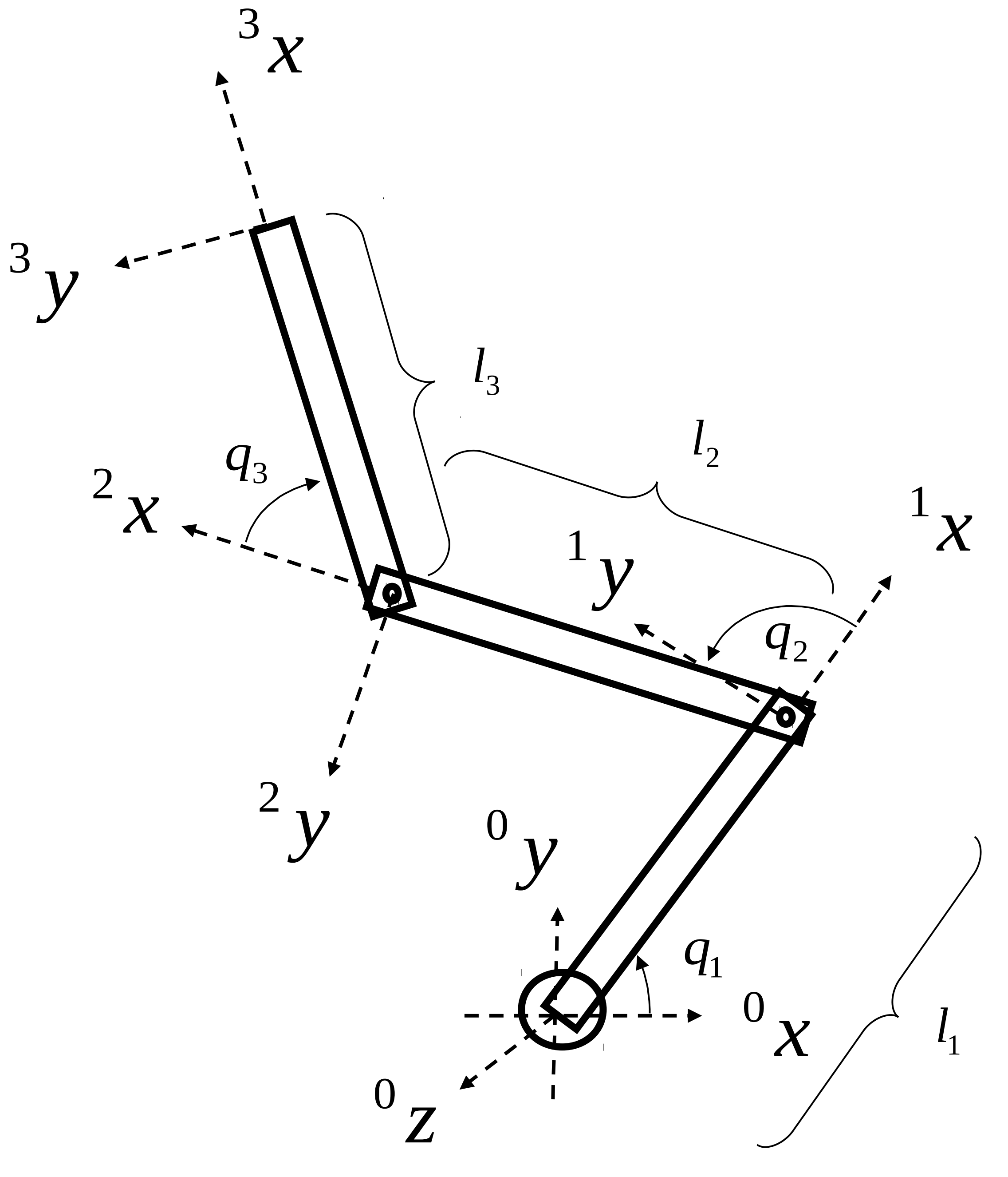


Forward kinematics: composition of homogeneous transforms

$$^0T_3 = {}^0T_1 \, {}^1T_2 \, {}^2T_3$$

$$^0T_1 = \begin{pmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

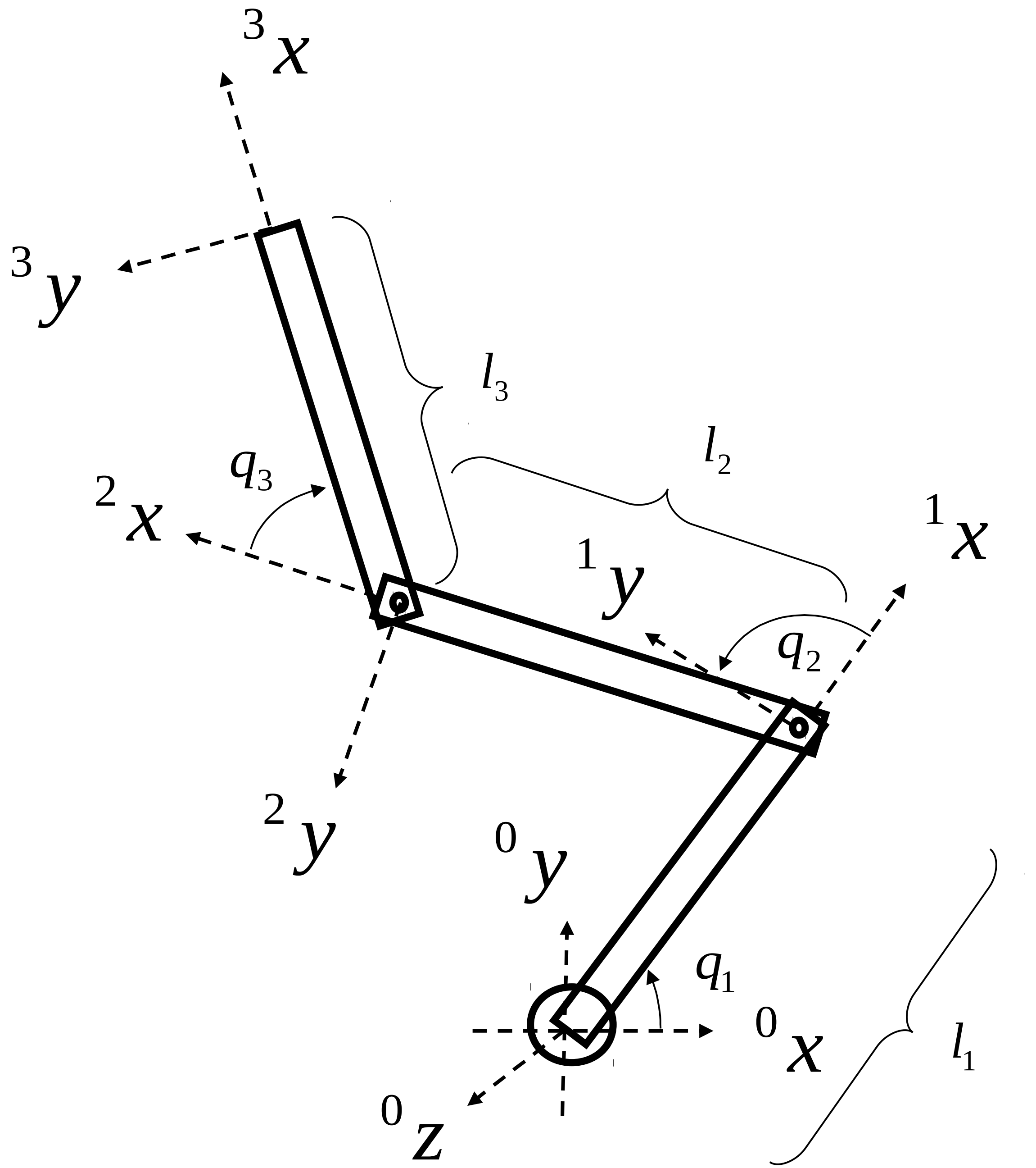
$$^1T_2 = \begin{pmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Forward kinematics: composition of homogeneous transforms

$$^{0}T_{3} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3}$$

$$^{2}T_{3} = \begin{pmatrix} c_3 & -s_3 & 0 & l_3 c_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Remember those double-angle formulas…

$$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \cos(\theta)\sin(\phi)$$

$$\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$$

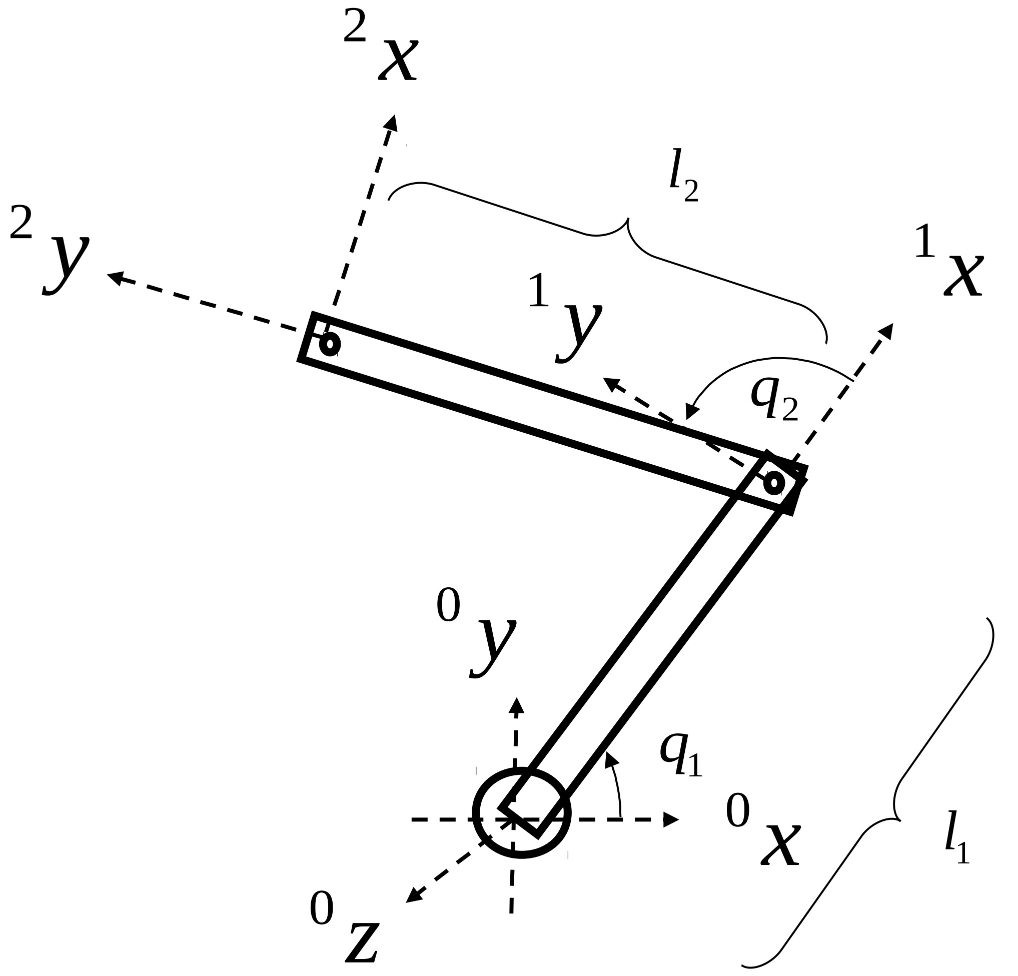
Forward kinematics: composition of homogeneous transforms

$$^{0}T_3 = {}^{0}T_1\, {}^{1}T_2\, {}^{2}T_3$$

$$^{0}T_3 = \begin{pmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_3 & -s_3 & 0 & l_3 c_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{0}T_3 = \begin{pmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Think-pair-share



Calculate $^2T_0$