Configuration Space

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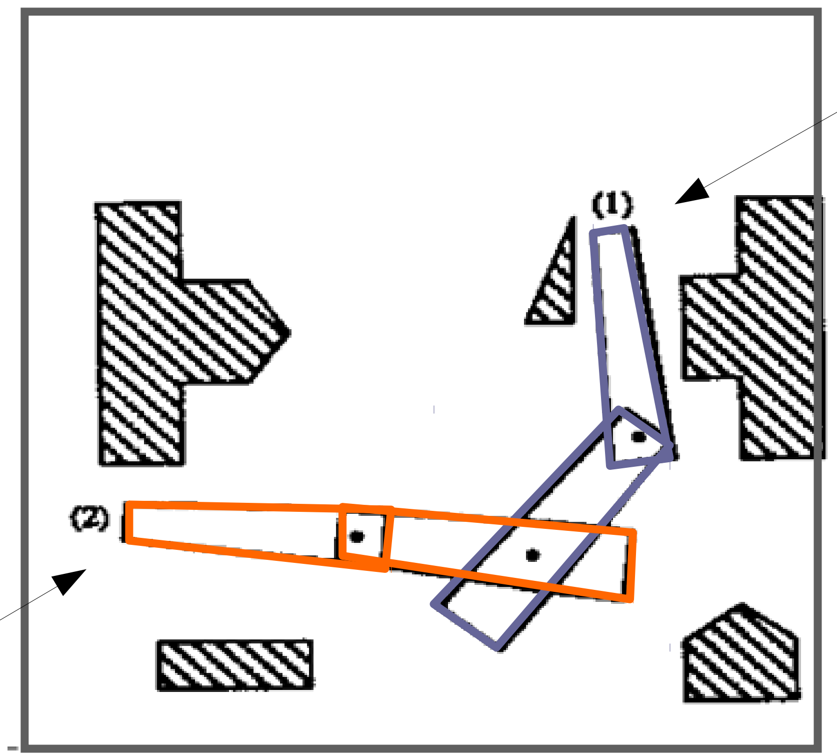
Problem we want to solve

<u>Given</u>:
– description of the robot arm (the manipulator)
– description of the obstacle environment
<u>Find</u>:
– path from start to goal that does not result in a collision

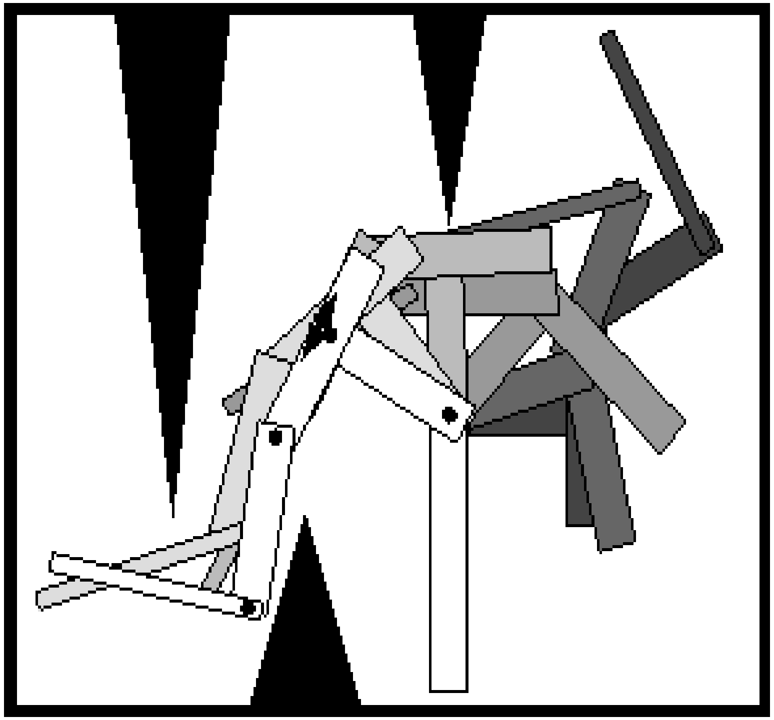
Starting configuration



Goal configuration

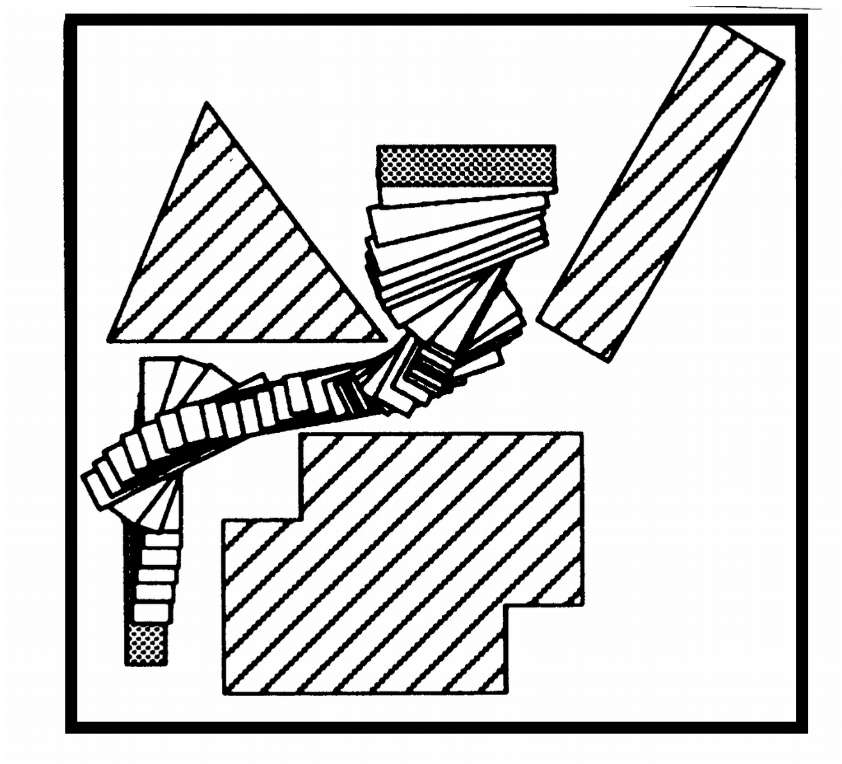
Problem we want to solve

This problem statement is actually very general
– manipulators



Problem we want to solve

This problem statement is actually very general
– manipulators
– mobile robots



Approach: plan in “configuration space”

Convert the original planning problem into a planning problem for a single point.

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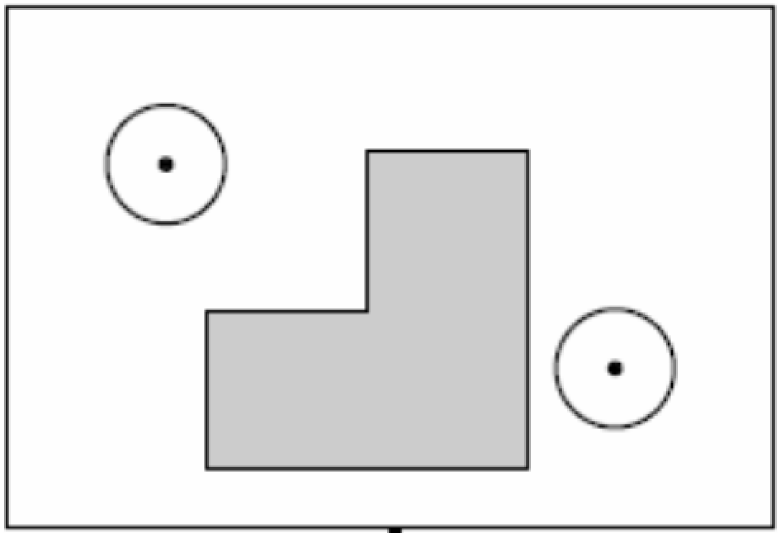
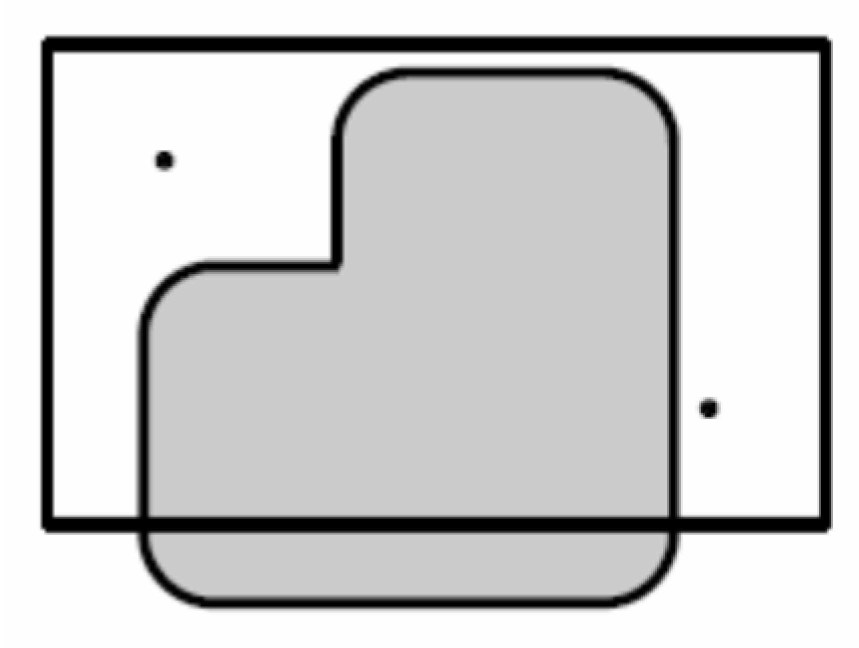
workspace

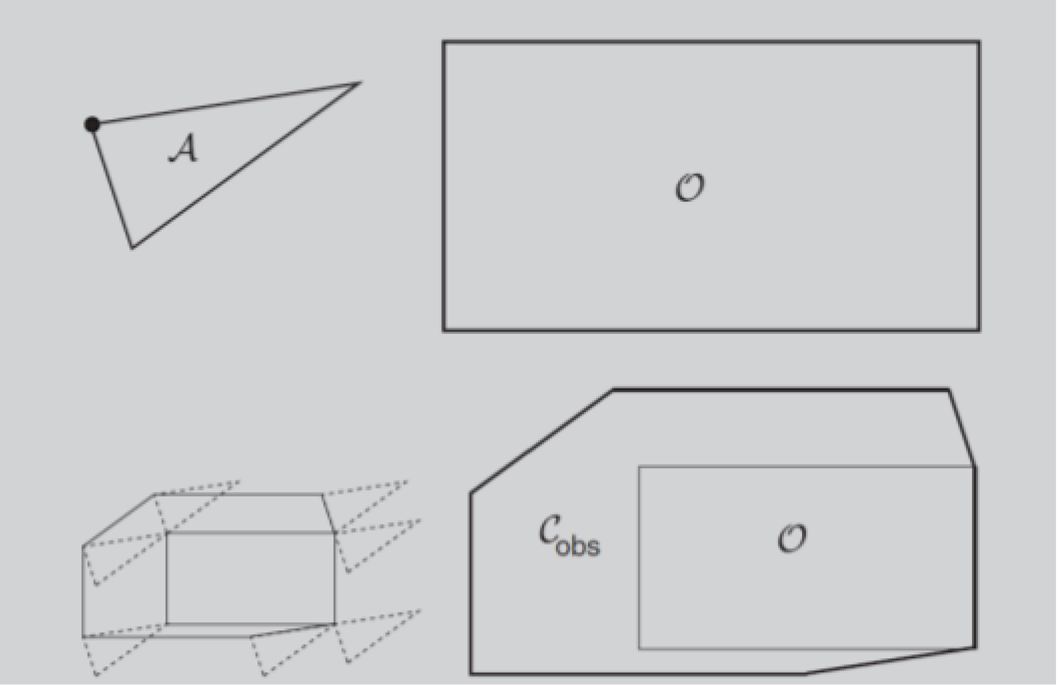


configuration space



Approach: plan in “configuration space”

Conversion to c-space: Minkowski Sum



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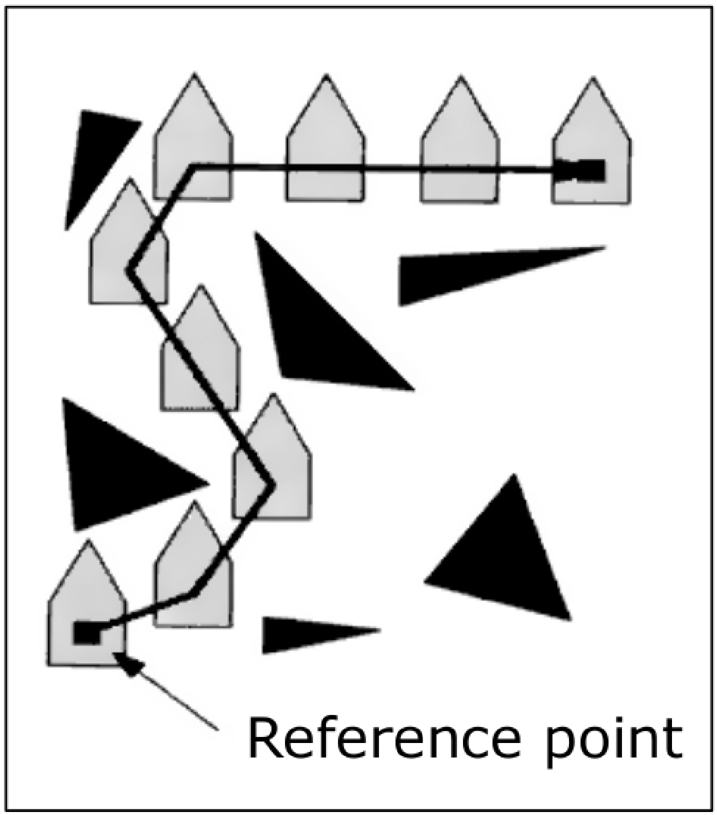
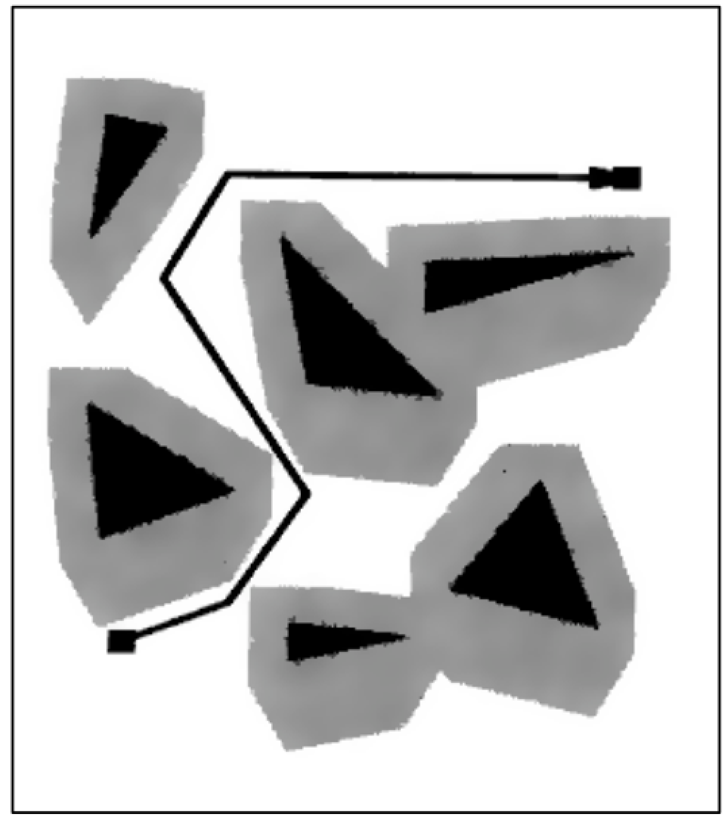
Convert the original planning problem into a planning problem for a single point.

workspace



configuration space



Approach: plan in “configuration space”

Convert the original planning problem into
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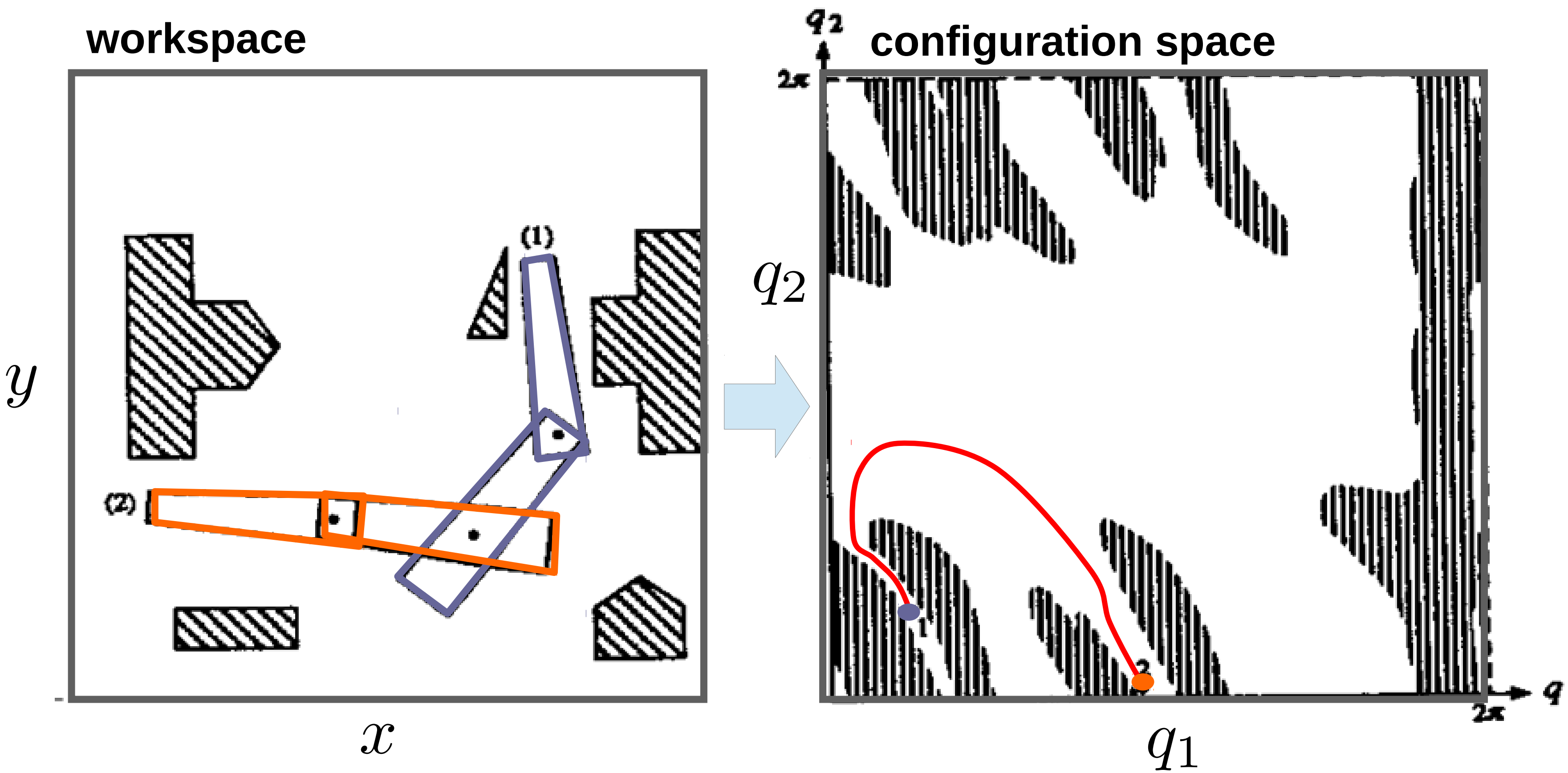
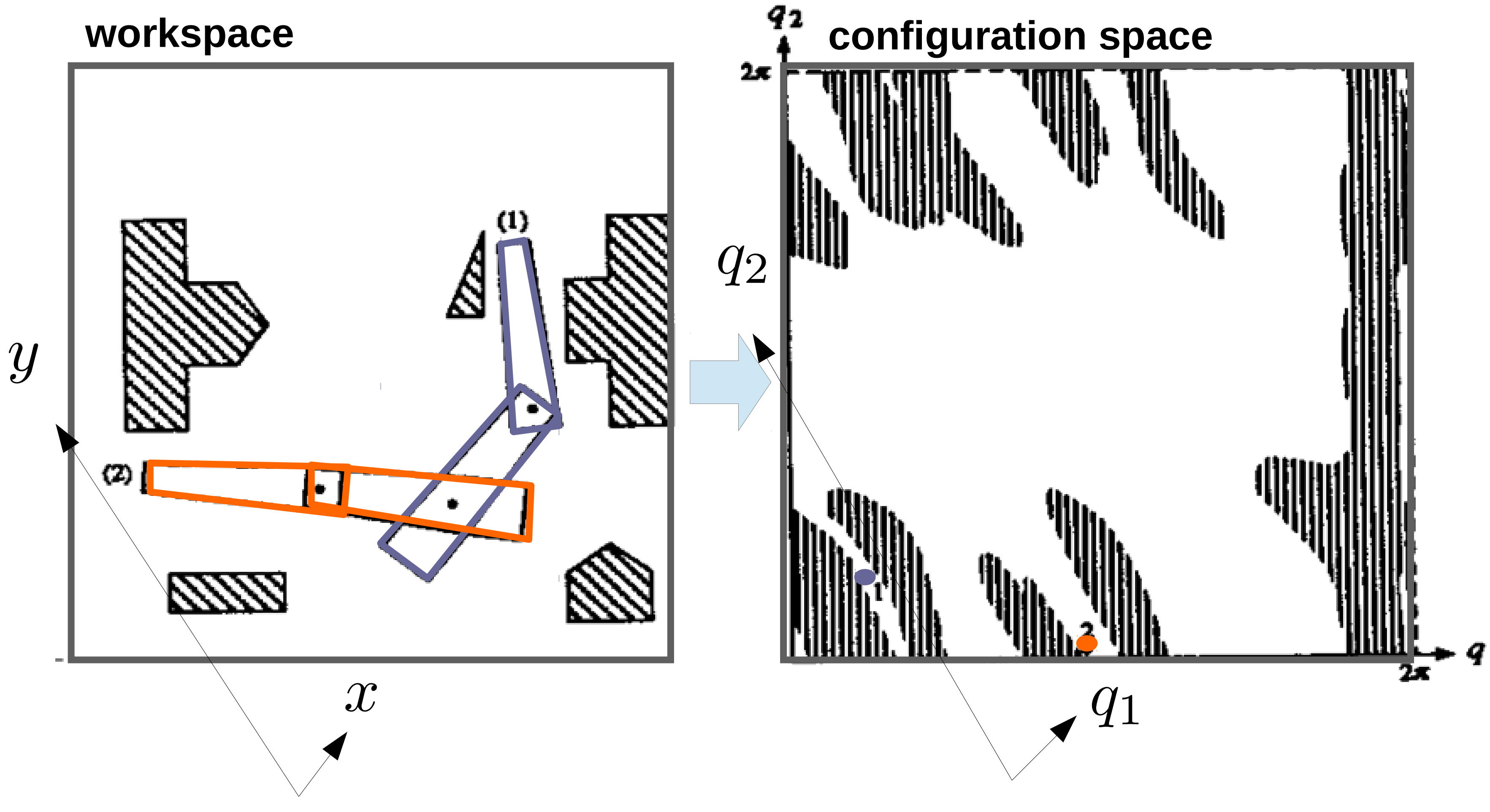


Original problem
– plan path for robot arm

Equivalent problem:
– plan path for a point

Approach: plan in “configuration space”

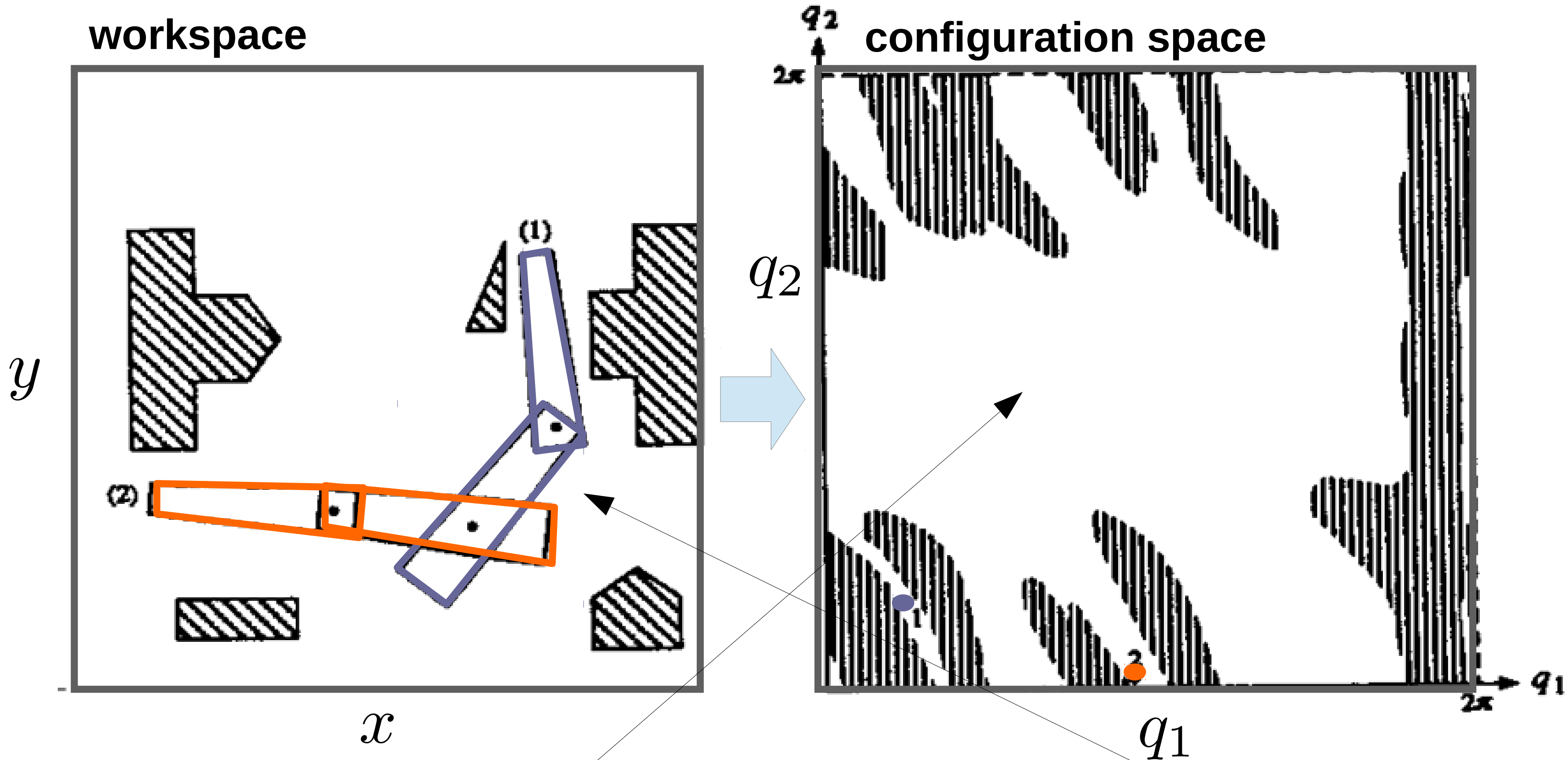
Notice the axes!



Cartesian space!

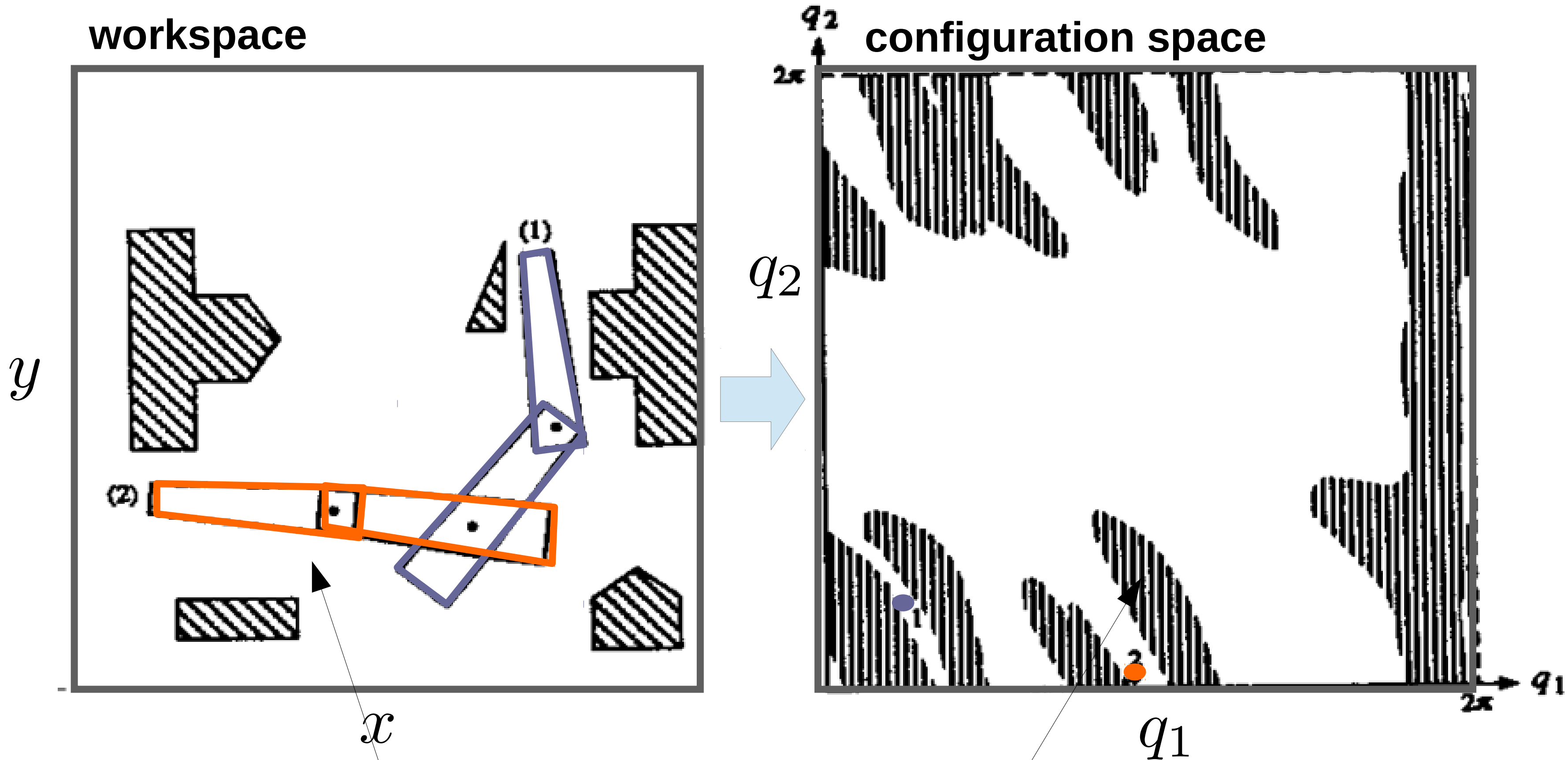
Joint angles!

Approach: plan in "configuration space"



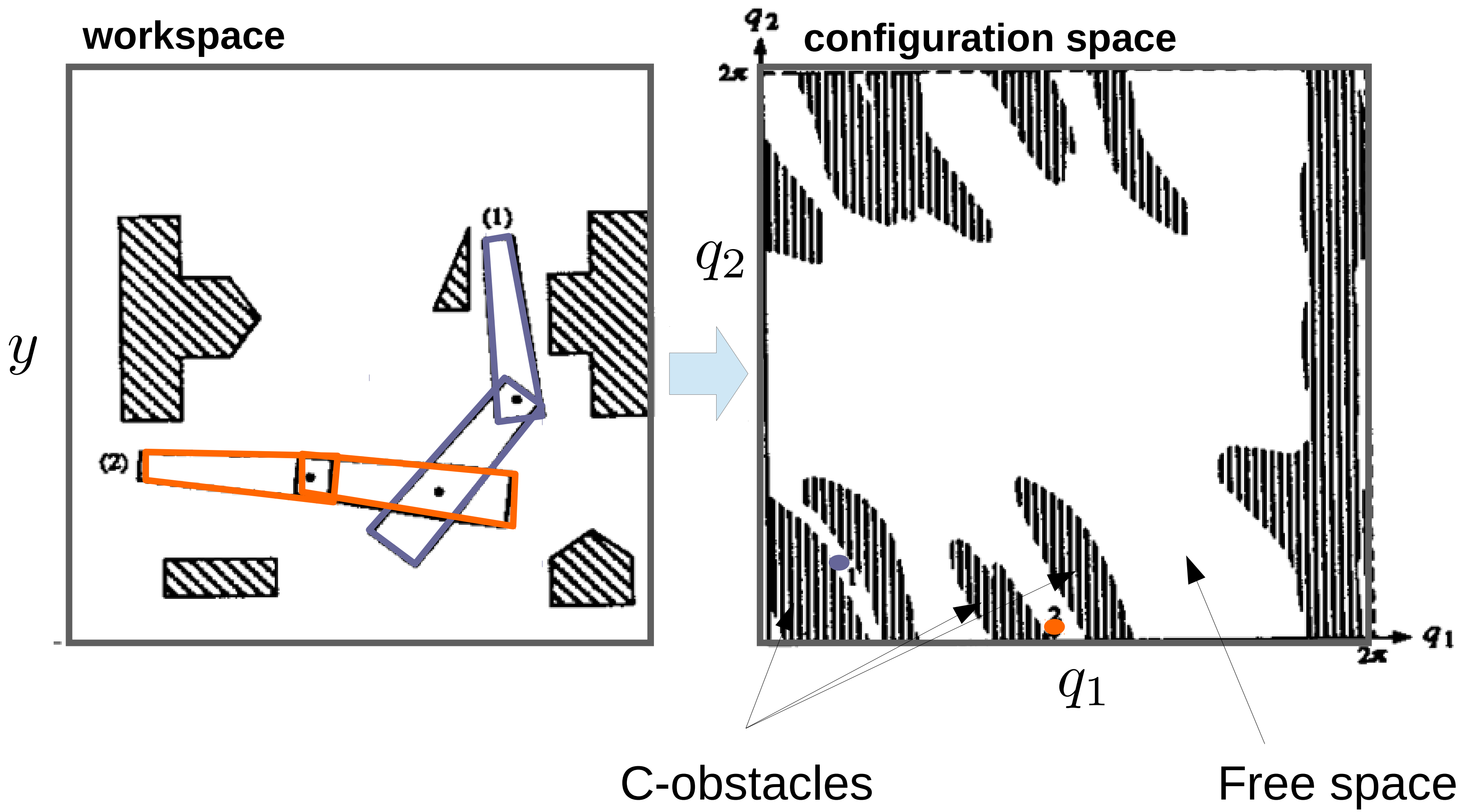
Every point here corresponds to a single robot configuration here

Approach: plan in "configuration space"

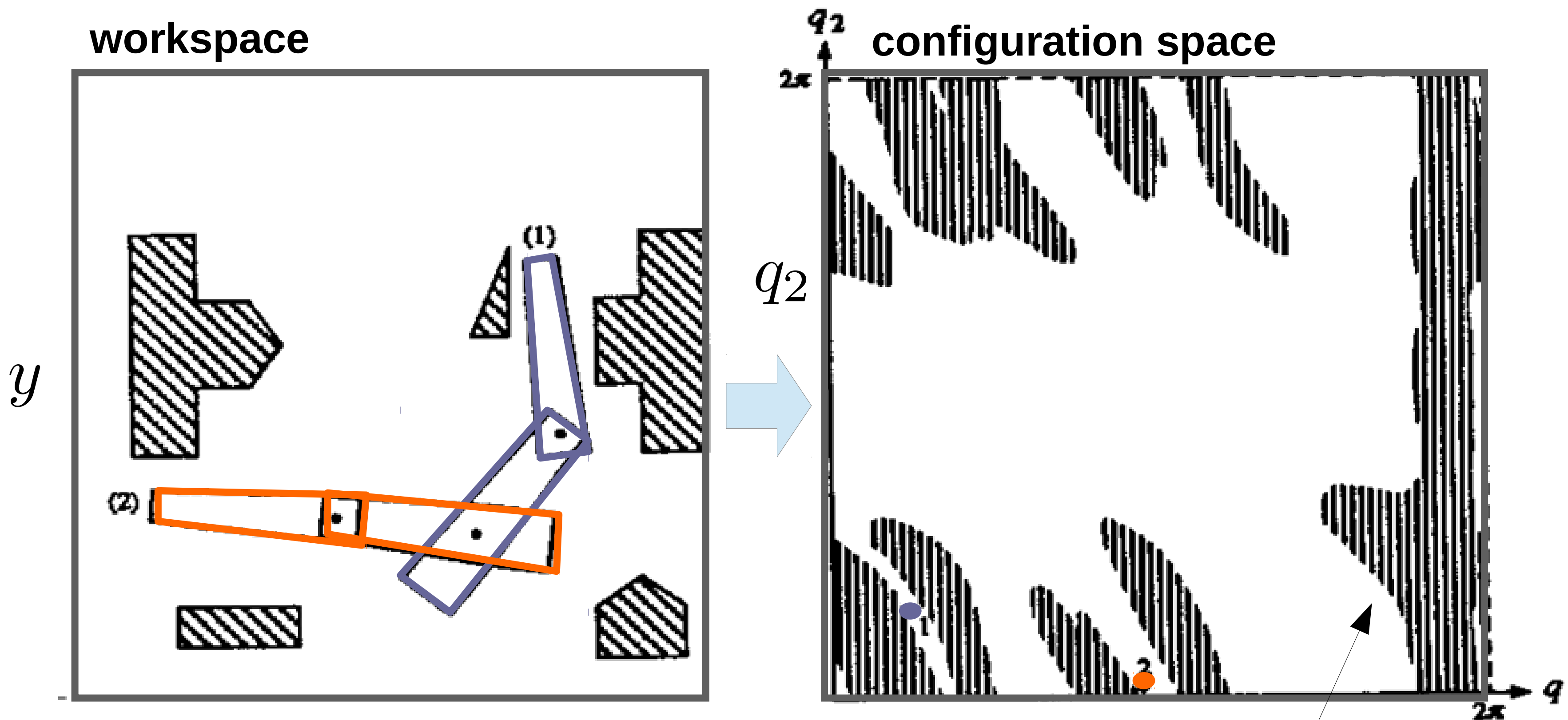


Every point that intersects an obstacle here corresponds to an arm configuration that intersects an obstacle

Approach: plan in "configuration space"



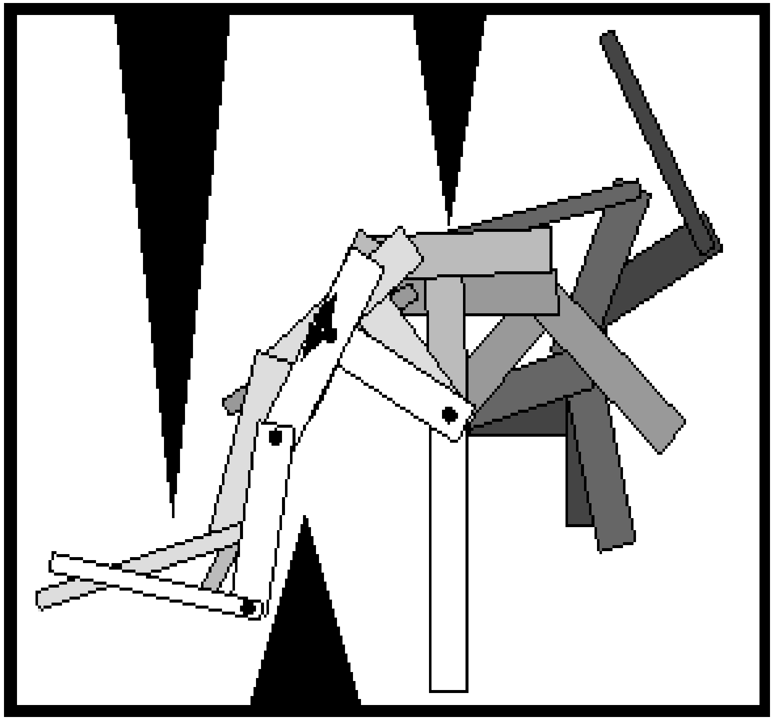
Question



Which object is responsible for this c-obstacle?

Configuration space

The dimension of a configuration space is the minimum number of parameters needed to specify the configuration of the robot completely.
– also called the number of “degrees of freedom” (DOFs)



Dimension = 3

Question

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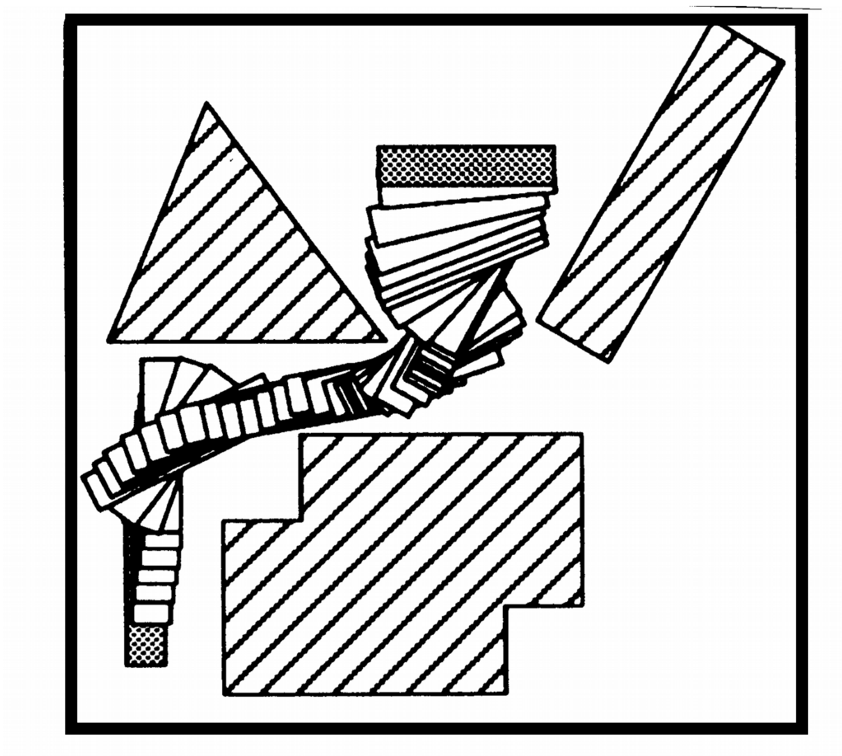
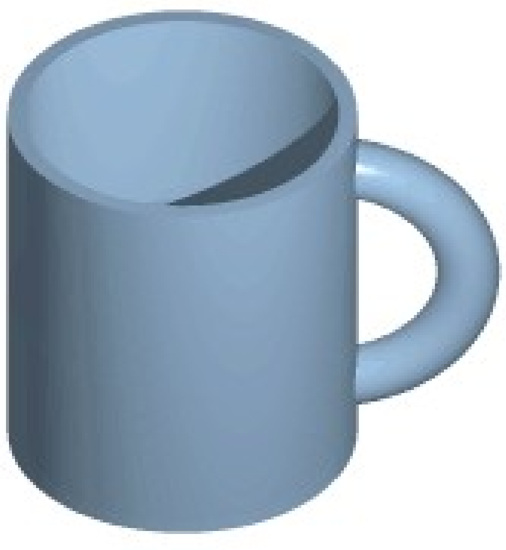


Dimension = ?

Topology of configuration space

<u>What is topology?</u>

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing



The topology of this mug is a torus

Topology of configuration space

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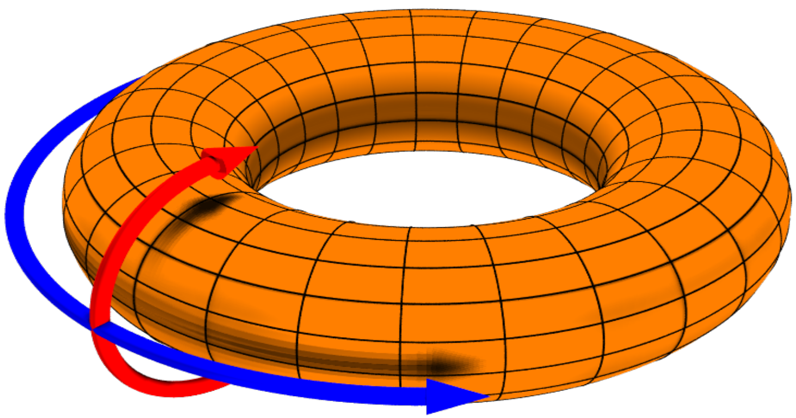

Torus: $C = S^1 \times S^1$

Topology of configuration space

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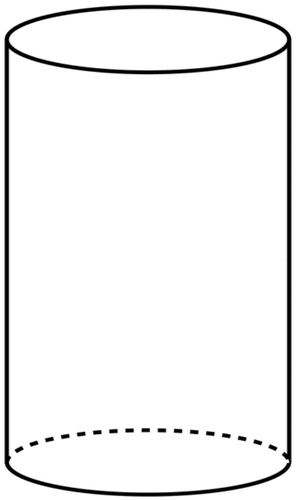

Cylinder: $C = R^2 \times S^1$

Question

<u>What is topology?</u>

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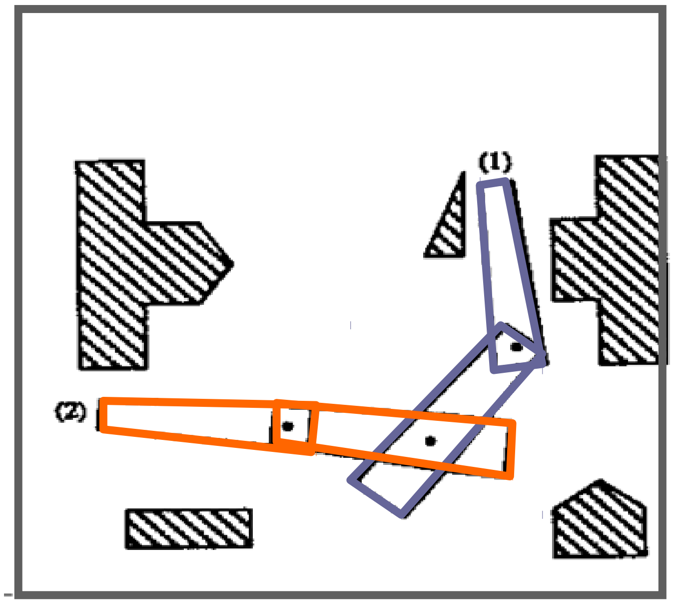

Configuration space: $C = ?$

Question

What is topology?

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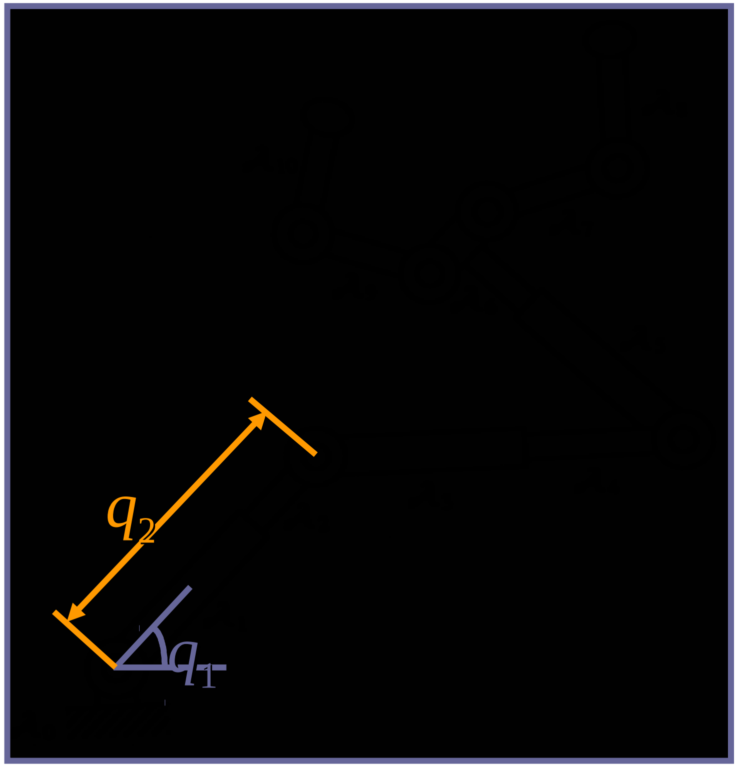


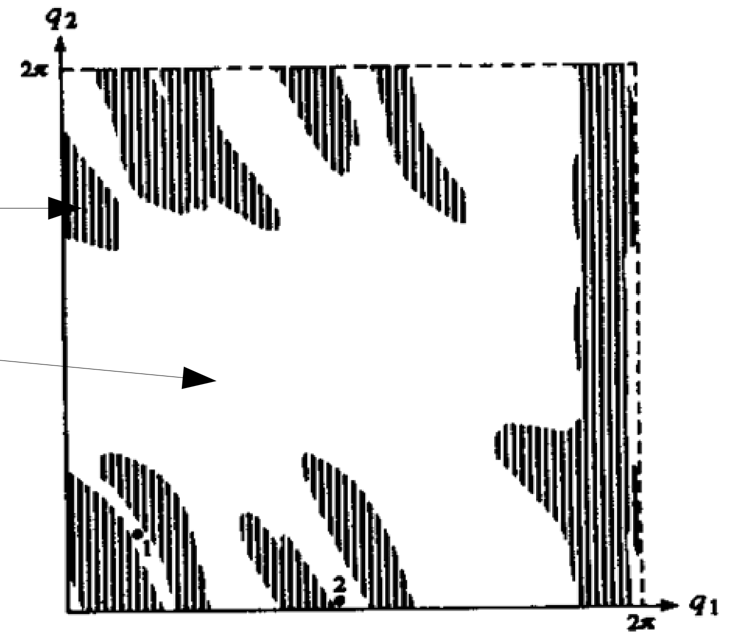
Configuration space: $C = ?$

Formalization of the path planning problem

Configuration space: $\mathcal{C}$

Obstacle space: $\mathcal{C}_{obs}$

Free space: $\mathcal{C}_{free} = \mathcal{C} \setminus \mathcal{C}_{obs}$



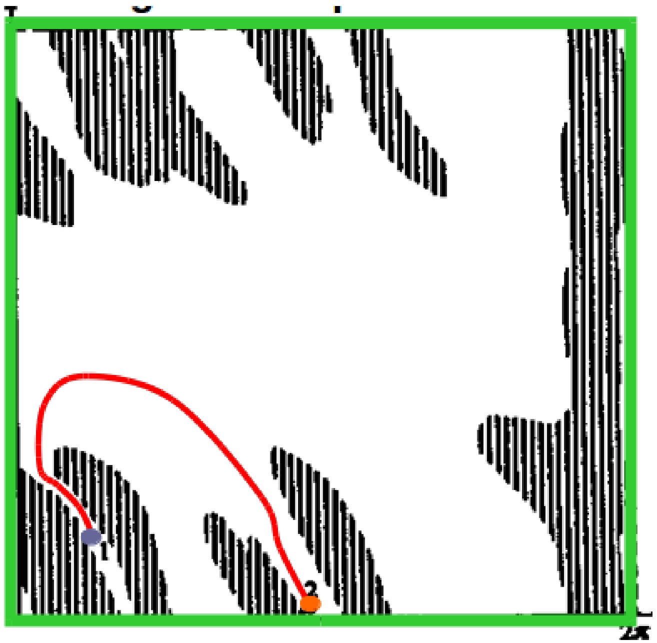
Path: $\sigma : [0, 1] \rightarrow \mathcal{C}$ where σ must be continuous

Collision-free path: $\sigma(\tau) \in \mathcal{C}_{free}, \tau \in [0, 1]$

Question

Define a path, $\sigma : [0, 1] \rightarrow \mathcal{C}$, that describes the unit circle in two dimensions

Formalization of the path planning problem



Given:

– configuration space $\mathcal{C}$

– free space $\mathcal{C}_{free}$

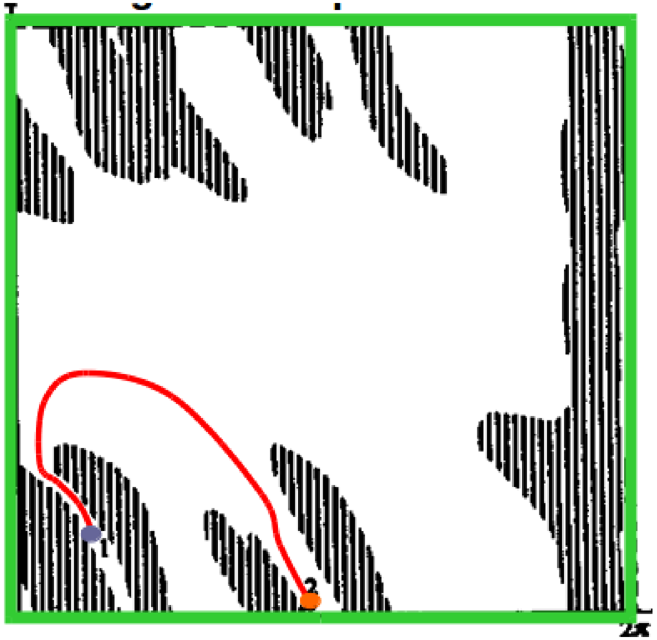
– start state $x_{init} \in \mathcal{C}_{free}$

– goal region $X_{goal} \subset \mathcal{C}_{free}$

Find:

– a collision-free path σ, such that $\sigma(0) = x_{init}$ and $\sigma(1) \in X_{goal}$

Question



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Find:

– a collision-free path σ, such that $\sigma(0) = x_{init}$ and $\sigma(1) \in X_{goal}$

How might we parameterize σ ?

Homotopic paths

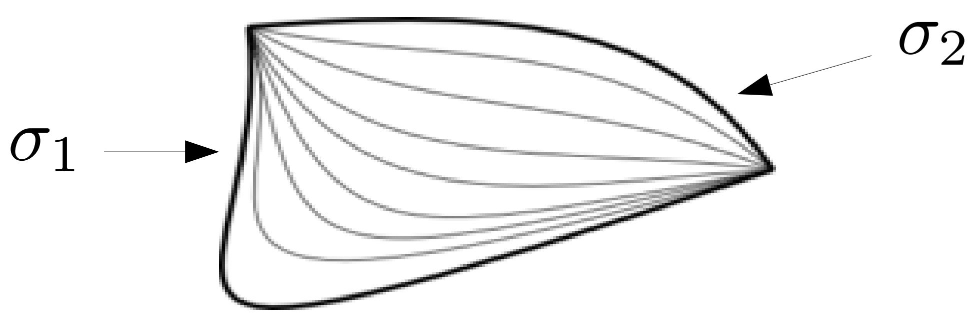
Two paths are homotopic if it is possible to continuously deform one into the other

<u>Formal definition:</u>

Let Σ_{free} denote the space of collision free paths.

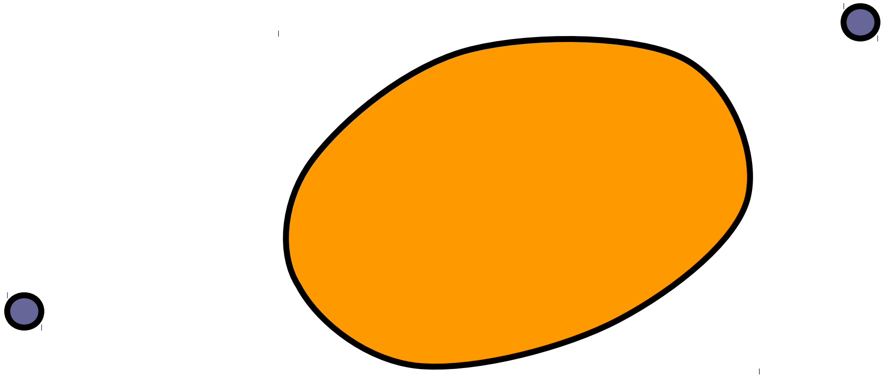
Two paths, σ_1 and σ_2 are homotopic if there exists a continuous function, $\Psi : [0,1] \rightarrow \Sigma_{free}$, such that

$\Psi(0) = \sigma_1$ and $\Psi(1) = \sigma_2$



Homotopic paths

Find distinct homotopic paths connecting these two points



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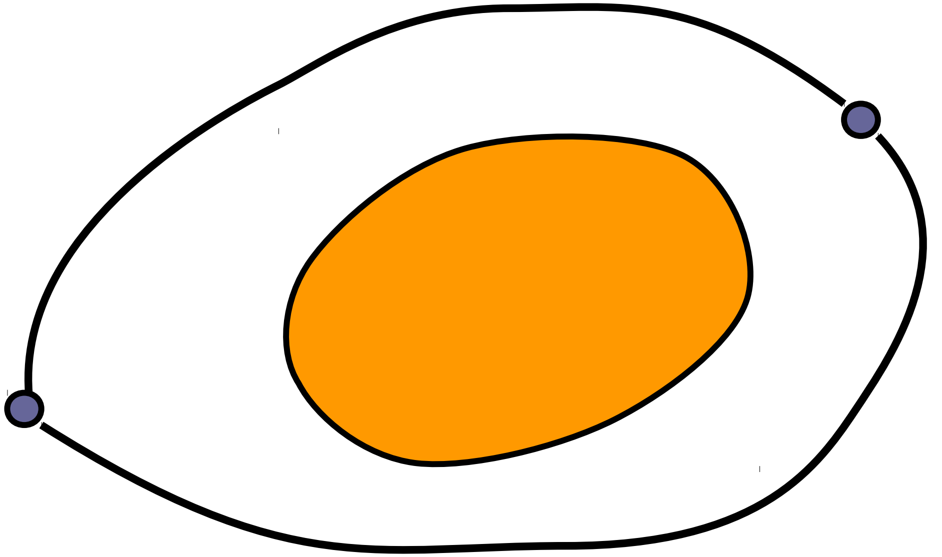

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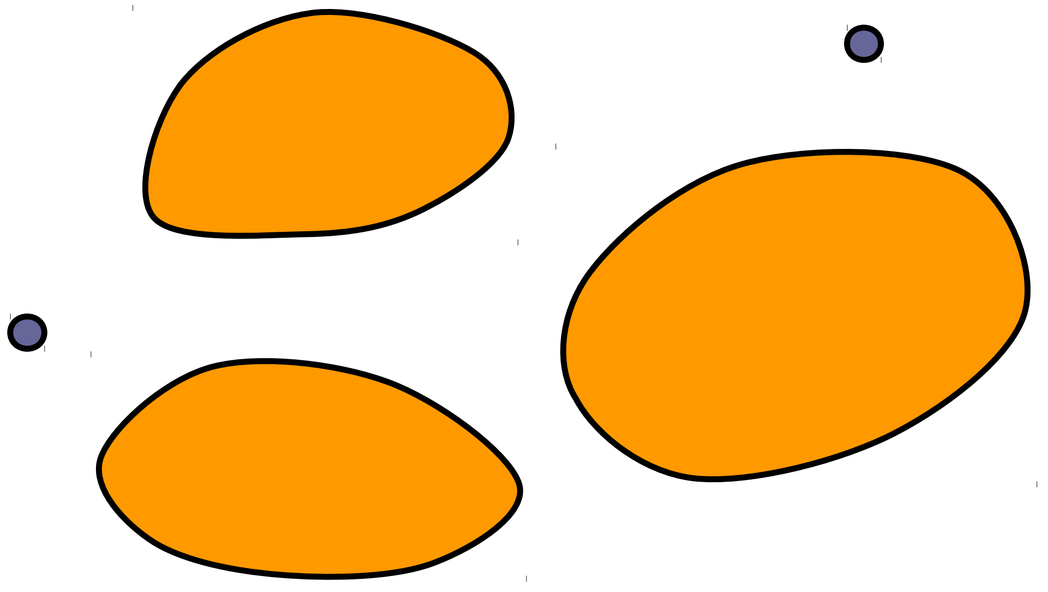

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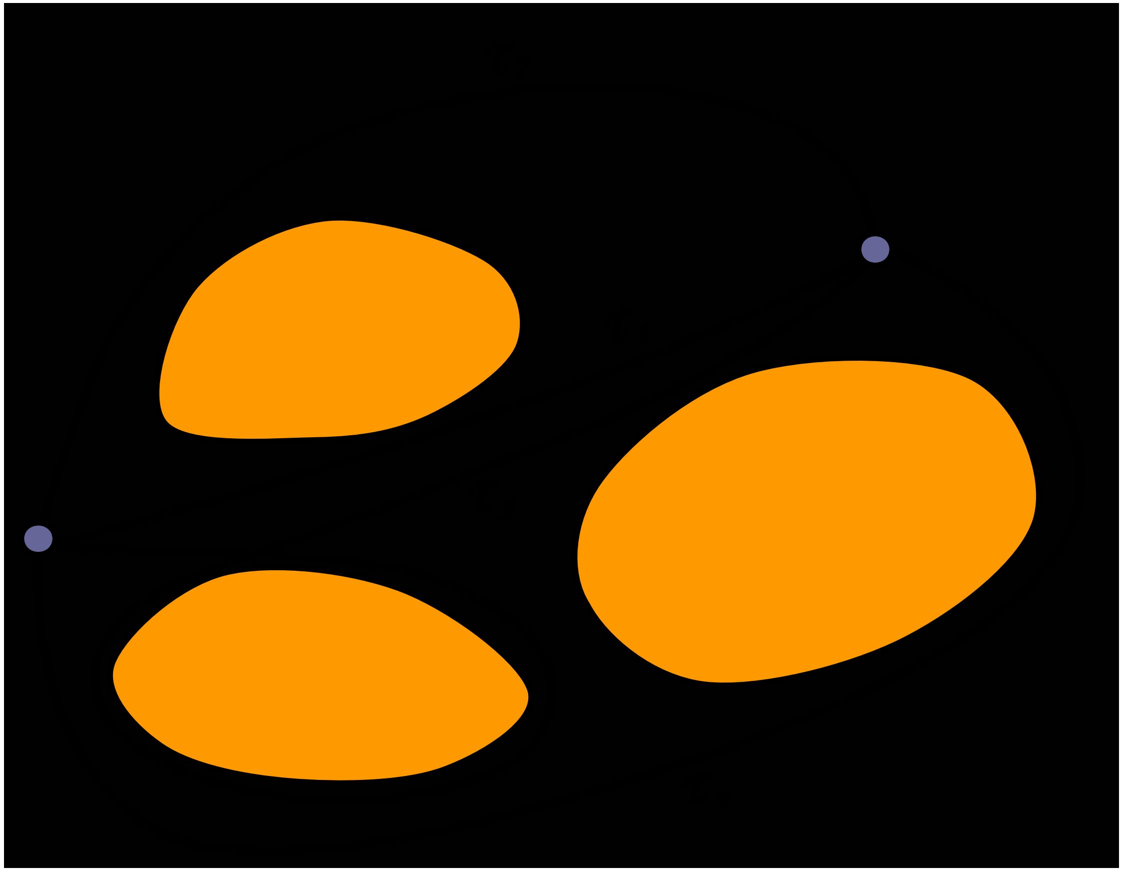

Connectedness of c-space

C is connected if every two configurations can be connected by a path.

C is simply-connected if any two paths connecting the same endpoints are homotopic.

Otherwise C is multiply-connected.