

Cartesian Control (Wrenches)

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Using the Jacobian for Statics

Up until now, we've used the Jacobian in the twist equation, $\xi = J\dot{q}$

Interestingly, you can also use the Jacobian in a statics equation:

$$\tau = J^T w$$

Joint torques

Cartesian wrench:

$$w = \begin{pmatrix} f \\ m \end{pmatrix}$$

force

moment (torque)

The diagram illustrates the statics equation $\tau = J^T w$. It shows the equation at the top center. Below it, two arrows point towards the equation: one from the left pointing to τ , and one from the right pointing to w . To the left of the first arrow is the text 'Joint torques'. To the right of the second arrow is the text 'Cartesian wrench:'. Below the Cartesian wrench definition, the equation $w = \begin{pmatrix} f \\ m \end{pmatrix}$ is shown. To the right of this equation, two horizontal arrows point left towards the f and m components respectively. The top arrow is labeled 'force' and the bottom arrow is labeled 'moment (torque)'.

Using the Jacobian for Statics

It turns out that both wrenches and twists can be understood in terms of a representation of displacement known as a *screw*.

- Therefore, you can calculate work by integrating the dot product:

$$W = \int (v \cdot f + \omega \cdot m) = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^T \begin{bmatrix} f \\ m \end{bmatrix} \quad \leftarrow \quad \begin{array}{l} \text{Work in Cartesian} \\ \text{space} \end{array}$$

$$W = \int \tau^T \dot{q} \quad \leftarrow \quad \text{Work in joint space}$$

Conservation of energy:
$$\int \tau^T \dot{q} = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^T \begin{bmatrix} f \\ m \end{bmatrix}$$

Using the Jacobian for Statics

$$\tau^T \dot{q} = \begin{bmatrix} f \\ m \end{bmatrix}^T \begin{bmatrix} v \\ \omega \end{bmatrix} \quad \longleftarrow \text{Incremental work (virtual work)}$$

$$\tau^T \dot{q} = \begin{bmatrix} f \\ m \end{bmatrix}^T J \dot{q}$$

$$\tau^T = \begin{bmatrix} f \\ m \end{bmatrix}^T J$$

$$\tau = J^T \begin{bmatrix} f \\ m \end{bmatrix}$$

Wrench-twist duality:

$$\tau = J^T w \quad \text{vs} \quad \xi = J \dot{q}$$

$$\tau = J^T w$$

New perspective on J^T control

Input: x^*

Output: q^*

1. repeat until dx is small:
2. init q to random joint configuration
3. repeat K times:
4. $x = FK(q)$
5. $dx = x^* - x$
6. $dq = \text{stepsize} * J^T dx$
7. $q = q + dq$
8. return $q^* = q$

Original statement of J^T transpose control

New perspective on J^T control

Input: x^*

Output: q^*

1. repeat until dx is small:
2. init q to random joint configuration
3. repeat K times:
4. $x = FK(q)$
5. $f = x^* - x$
6. $\tau = \text{stepsize} * J^T f$
7. $q = q + \tau$
8. return $q^* = q$

Statement of J^T transpose control as forces and torques