Cartesian Control (Wrenches)

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Using the Jacobian for Statics

Up until now, we've used the Jacobian in the twist equation, $\xi = J\dot{q}$

Interestingly, you can also use the Jacobian in a statics equation:

$$\tau = J^T w$$

Joint torques

Cartesian wrench: $w = \begin{pmatrix} f \\ m \end{pmatrix}$

force

moment (torque)

Using the Jacobian for Statics

It turns out that both wrenches and twists can be understood in terms of a representation of displacement known as a *screw.*

- Therefore, you can calculate work by integrating the dot product:

$$W = \int (v \cdot f + \omega \cdot m) = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^T \begin{bmatrix} f \\ m \end{bmatrix}$$ ← Work in Cartesian space

$$W = \int \tau^T \dot{q}$$ ← Work in joint space

Conservation of energy: $$\int \tau^T \dot{q} = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^T \begin{bmatrix} f \\ m \end{bmatrix}$$

Using the Jacobian for Statics

$$\tau^T \dot{q} = \begin{bmatrix} f \\ m \end{bmatrix}^T \begin{bmatrix} v \\ \omega \end{bmatrix}$$ ← Incremental work (virtual work)

$$\tau^T \dot{q} = \begin{bmatrix} f \\ m \end{bmatrix}^T J \dot{q}$$

$$\tau^T = \begin{bmatrix} f \\ m \end{bmatrix}^T J$$

$$\tau = J^T \begin{bmatrix} f \\ m \end{bmatrix}$$

Wrench-twist duality:

$\tau = J^T w$ vs $\xi = J\dot{q}$

$$\tau = J^T w$$

New perspective on J^T control

```
Input: x*
Output: q*
1. repeat until dx is small:
2.         init q to random joint configuration
3.         repeat K times:
4.                 x = FK(q)
5.                 dx = x*-x
6.                 dq = stepsize * J^T dx
7.                 q = q + dq
8. return q* = q
```

Original statement of J^T transpose control

New perspective on J^T control

```
Input: x*
Output: q*
1. repeat until dx is small:
2.         init q to random joint configuration
3.         repeat K times:
4.                 x = FK(q)
5.                 f = x*-x
6.                 tau = stepsize * J^T f
7.                 q = q + tau
8. return q* = q
```

Statement of J^T transpose control as forces and torques