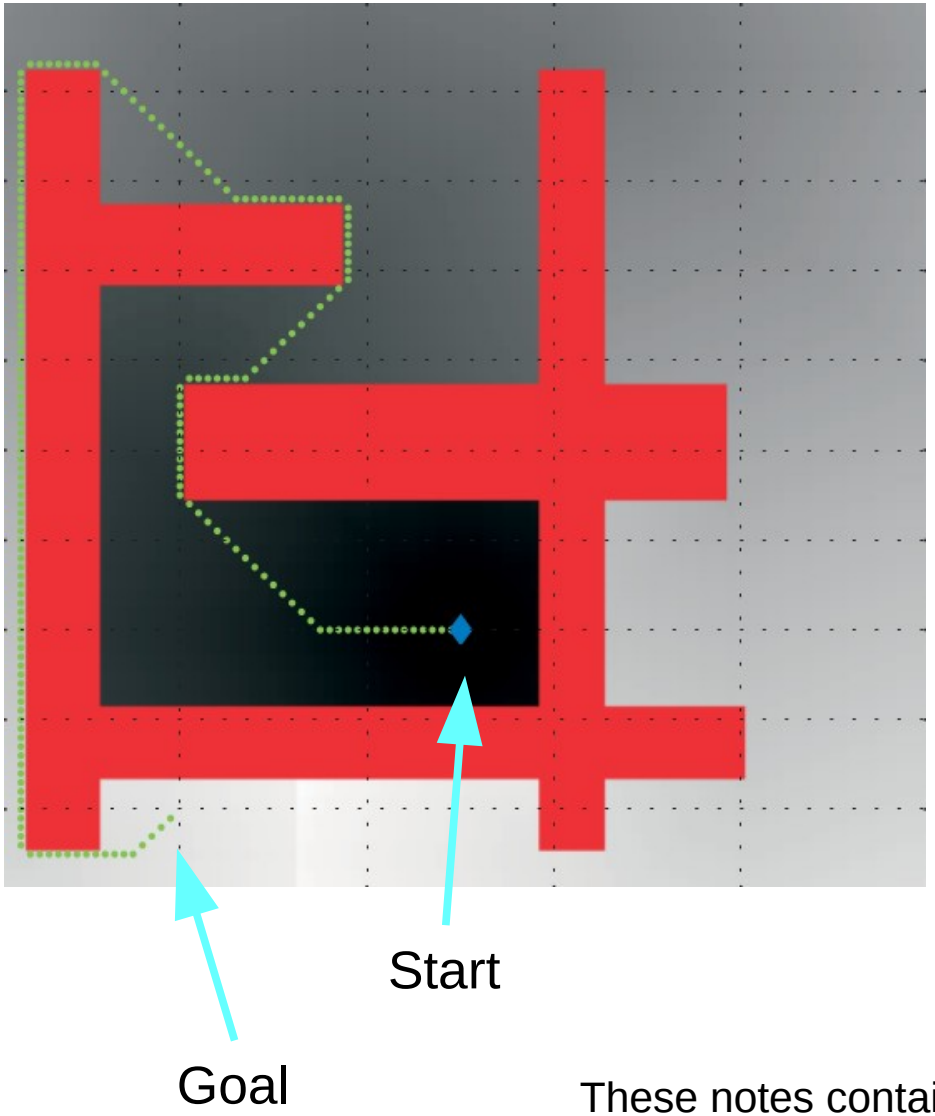
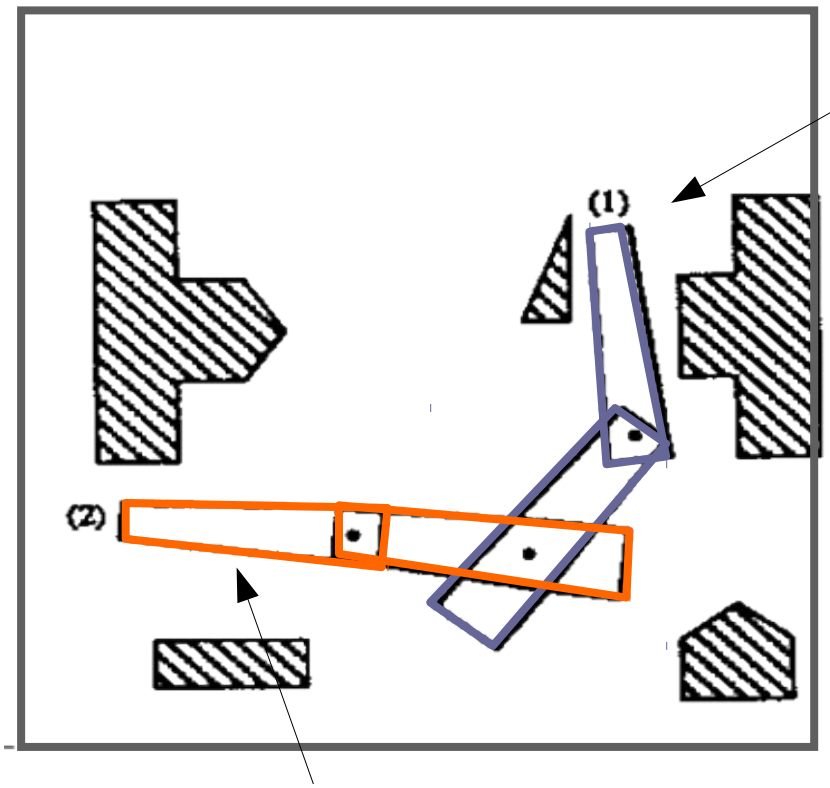
# Path planning: BUGs and wavefront




How do you plan a path for a robot from start to goal?

These notes contain materials from Peter Corke's book and from Howie Choset's lecture materials.

# Path planning: BUGs and wavefront




Starting configuration

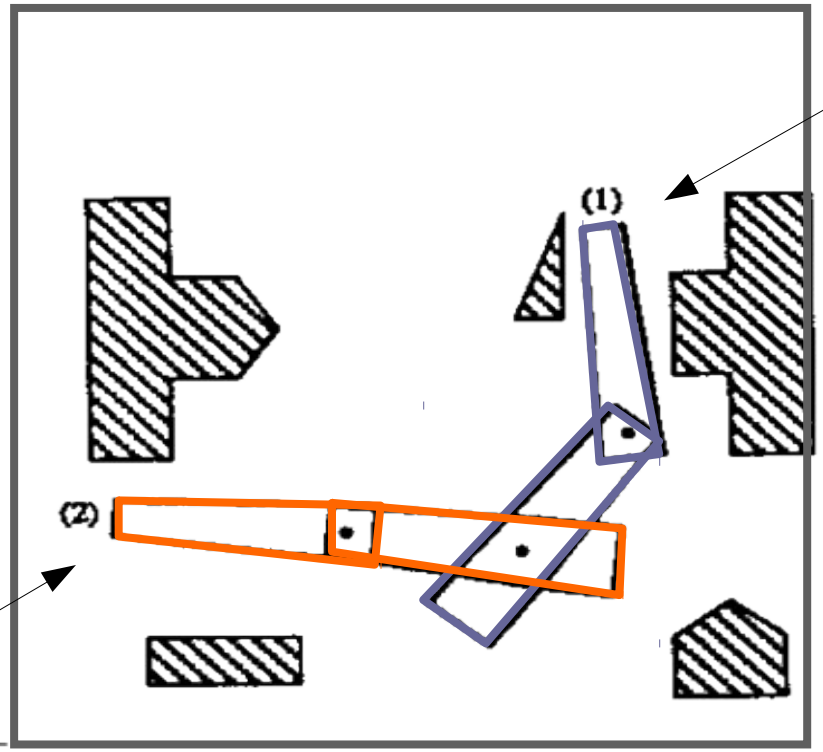
How do you plan a path for a robot from start to goal?

Goal configuration

# Problem we want to solve

Given:
– a point-robot (robot is a point in space)
– a start and goal configuration

Find:
– path from start to goal that does not result in a collision




Starting configuration

Goal configuration

# Problem we want to solve

Given:
– a point-robot (robot is a point in space)
– a start and goal configuration

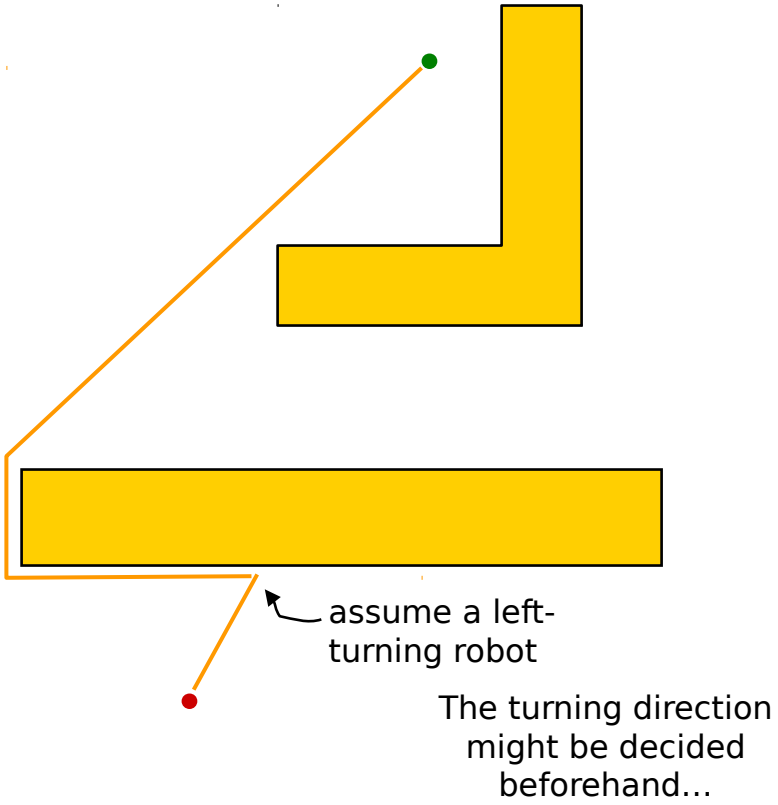
Find:
– path from start to goal that does not result in a collision

Assumptions:
– the position of the robot can always be measured perfectly
– the motion of the robot can always be controlled perfectly
– the robot can move in any directly instantaneously

# First attempt: BUGs!


Bug algorithms:
– assume only local knowledge of the environment is available
– simple behaviors: follow a wall; follow straight line toward goal

# First attempt: BUG 0

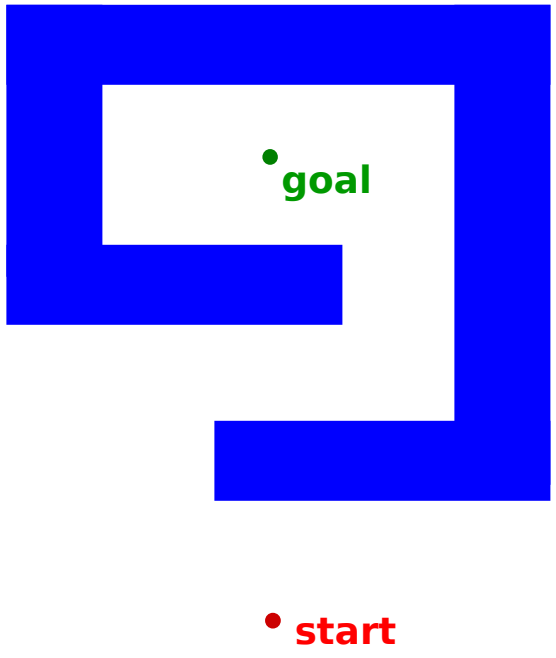



BUG 0:

1. head toward goal
2. if hit a wall, turn left
3. follow wall until a line toward goal will move you away from wall.

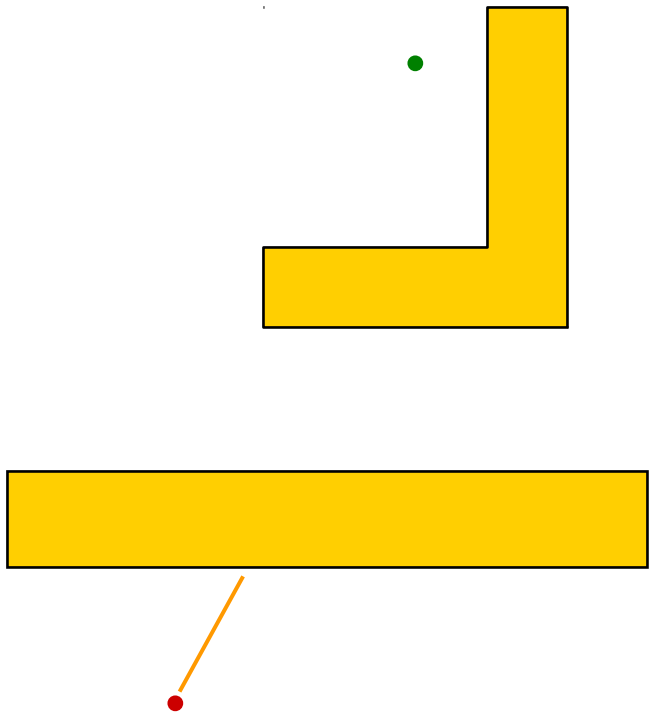
(assume we only have local sensing – we cannot sense position of walls we are not touching)

# First attempt: BUG 0




What happens here?

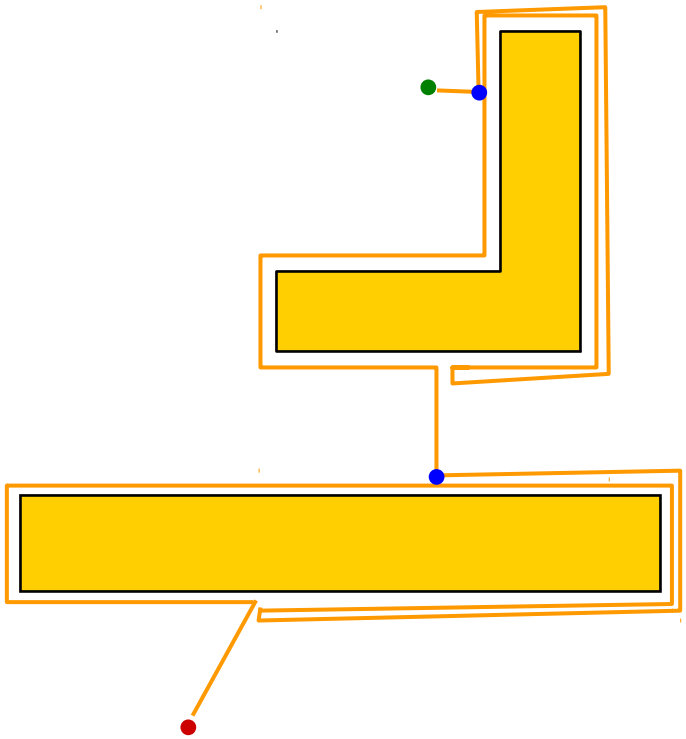
# Second attempt: BUG 1



BUG 1:

1. move on straight line toward goal
2. if obstacle encountered, circumnavigate entire obstacle and remember how close bug got to goal
3. return to closest point and continue on a straight line toward goal

# Second attempt: BUG 1



BUG 1:

1. move on straight line toward goal
2. if obstacle encountered, circumnavigate entire obstacle and remember how close bug got to goal
3. return to closest point and continue on a straight line toward goal

# BUG 1 Performance Analysis

How far does BUG 1 travel before reaching goal?

Best case scenario (lower bound): $D$

Worst case scenario (upper bound): $D + 1.5 \sum_i P_i$

Where

- D denotes distance from start to goal and
- P_i denotes perimeter of ith obstacle

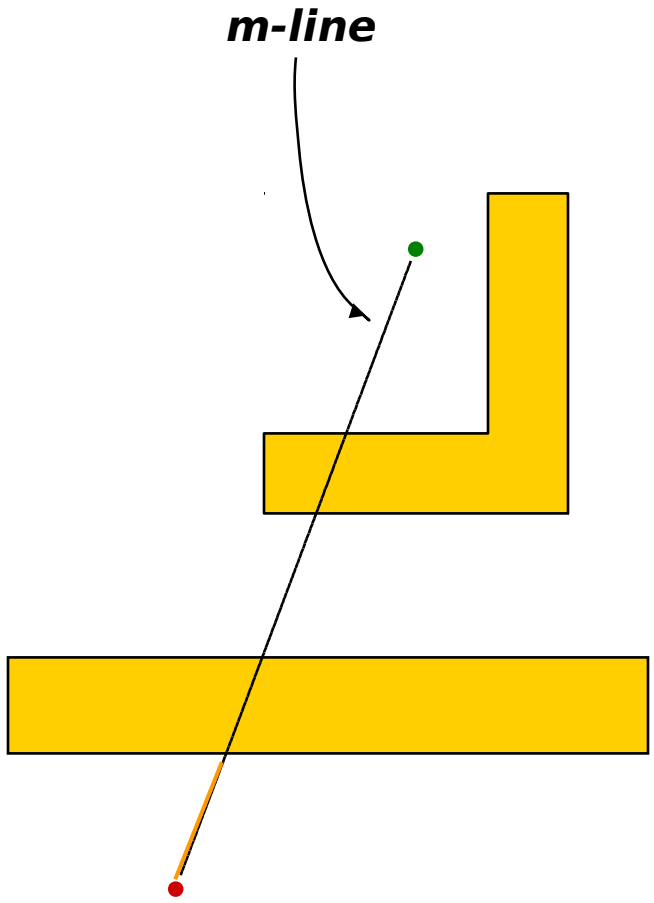
# BUG 1 completeness?

Is BUG 1 *complete?*
– is it guaranteed to find a path if one exists?

Yes? No?

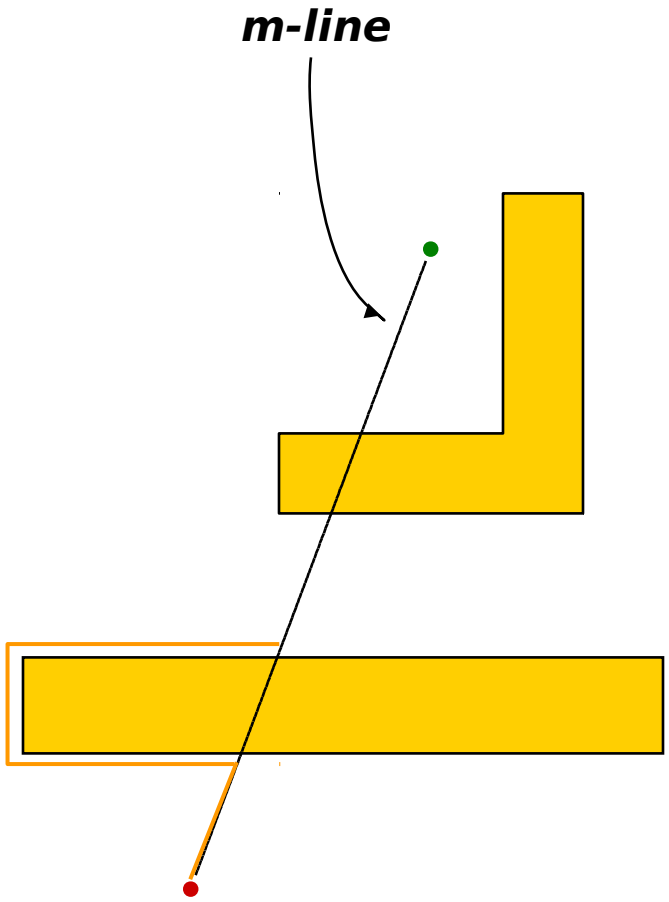
– how would you prove completeness (exercise for class)?

# Another bug: BUG 2




1. head toward goal on m-line

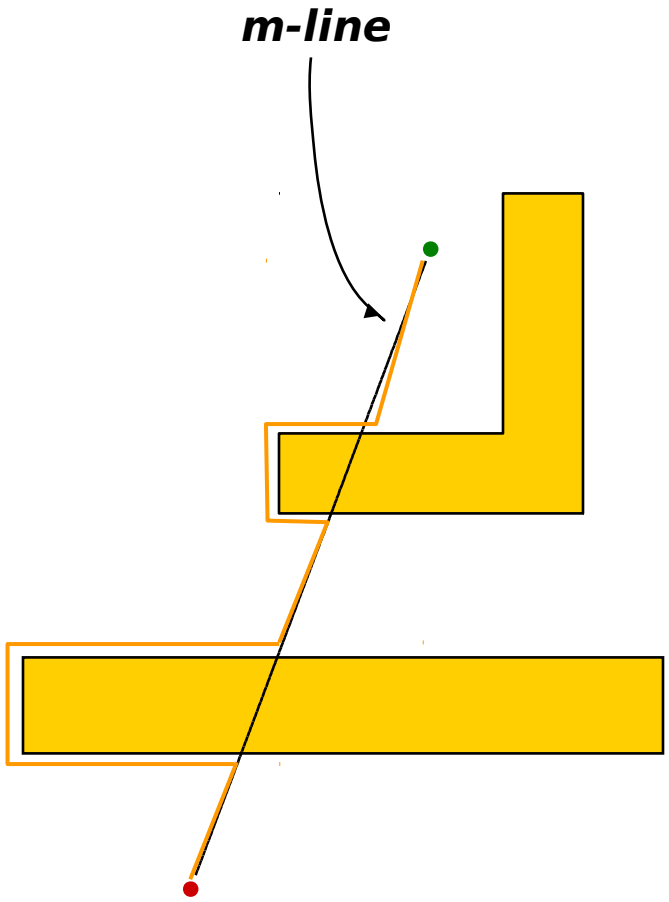
# Another bug: BUG 2




1. head toward goal on m-line

2. if you encounter obstacle, follow it until you encounter m-line again at a point closer to goal

# Another bug: BUG 2

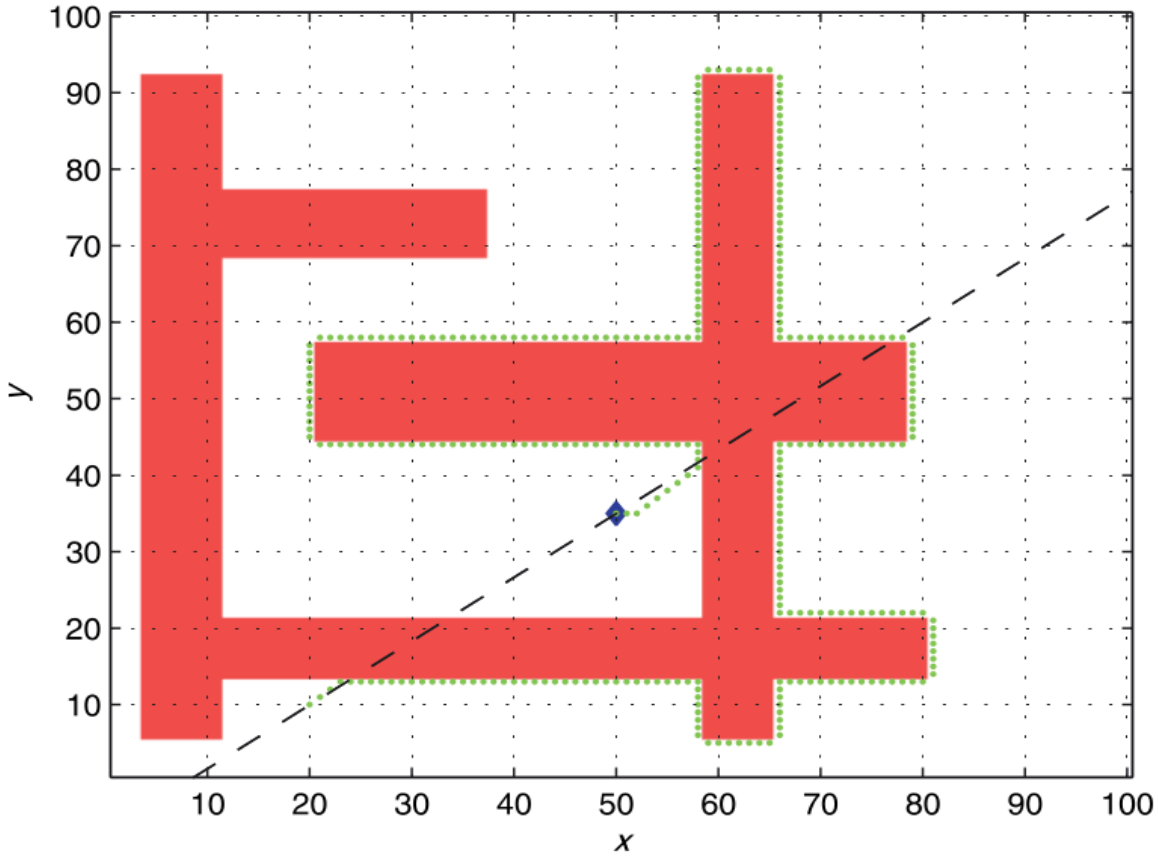



1. head toward goal on m-line

2. if you encounter obstacle, follow it until you encounter m-line again at a point closer to goal

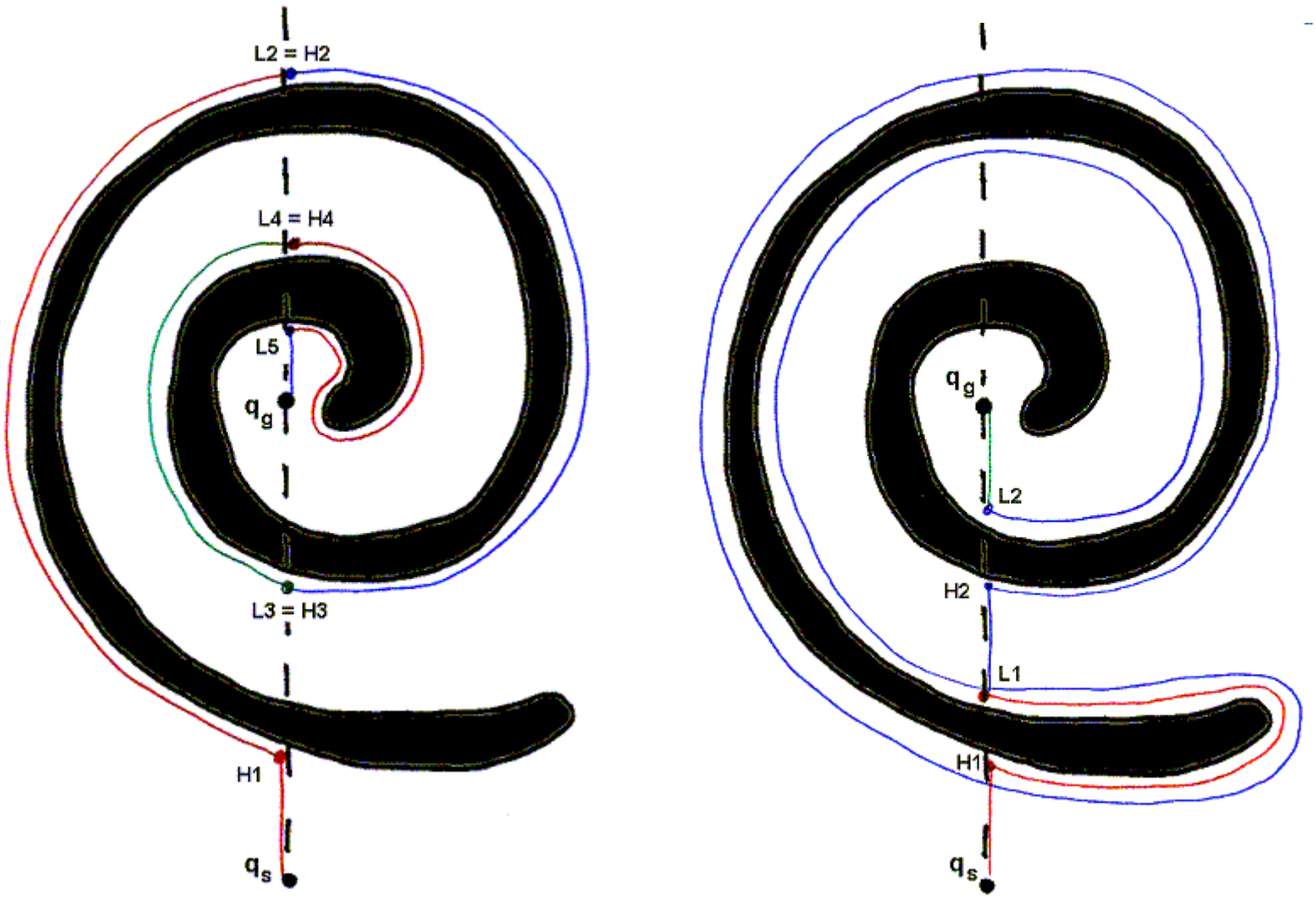
3. leave line and head toward goal again

# Another bug: BUG 2




Is BUG 2 complete?
– Why? Why not?

# Another bug: BUG 2



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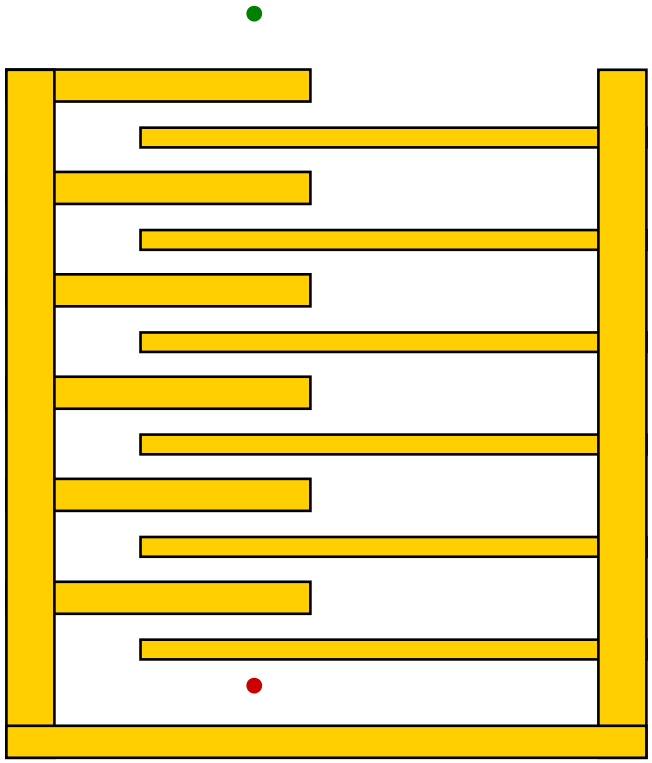


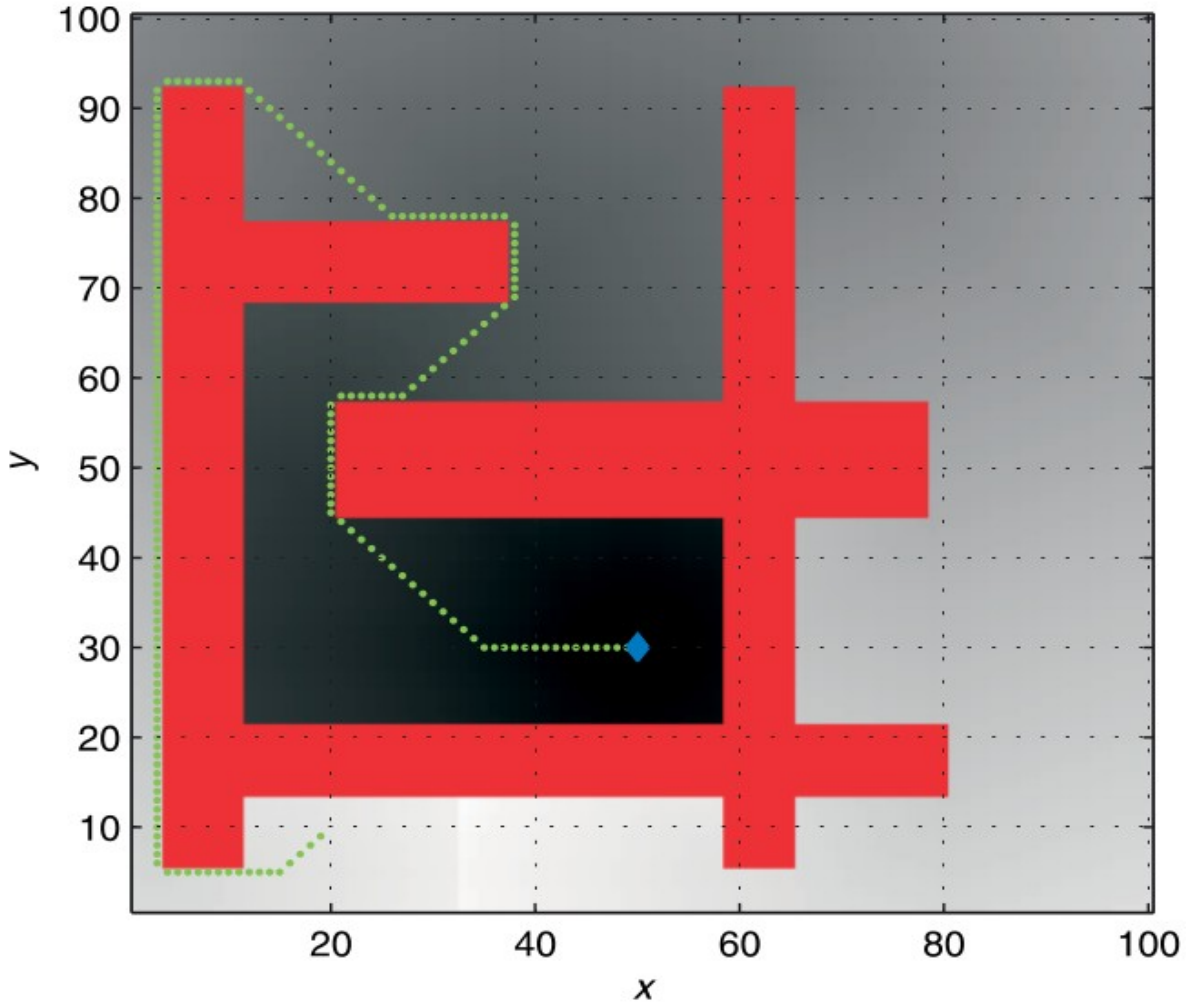
How bad can it get?

Lower bound: $D$

Upper bound: $D + \sum_i \frac{n_i}{2} P_i$

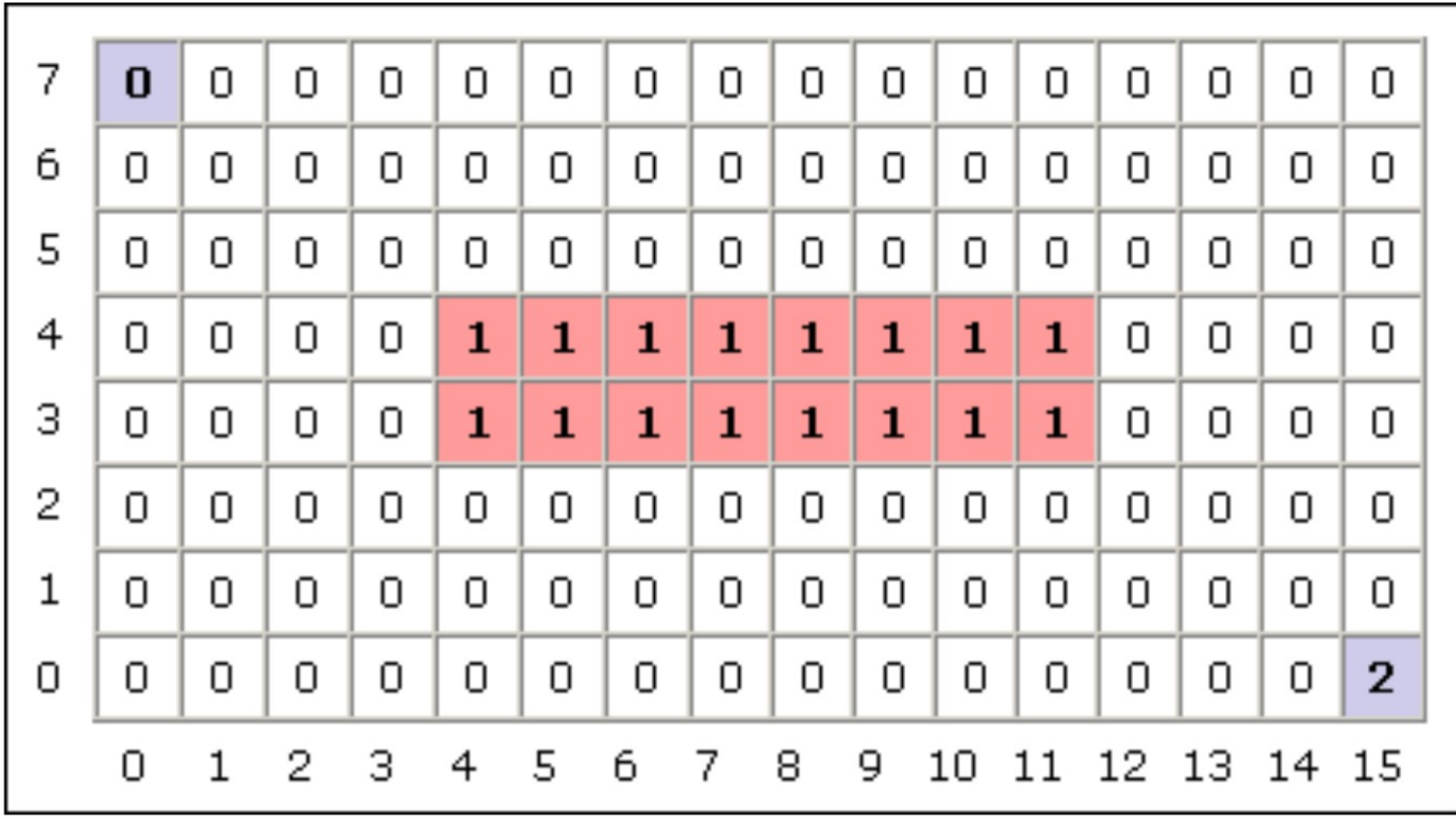
where n_i is the number of s-line intersections
In the ith obstacle.

# Wavefront planner (distance transform)




– intensity of a point denotes its (obstacle-respecting) distance from the goal

# Wavefront planner (distance transform)

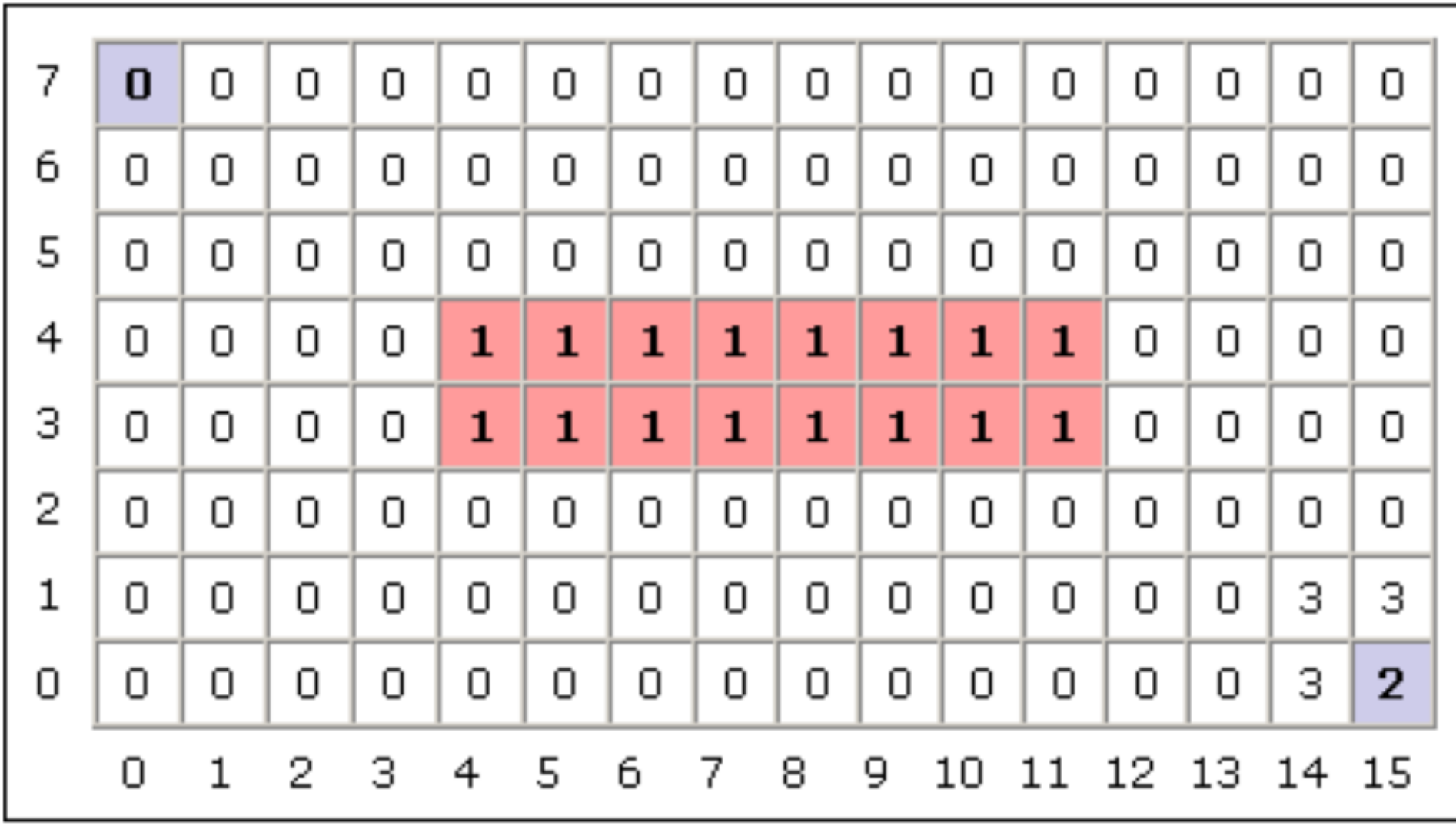



Idea:

– discretize the workspace into cells

– label each cell with distance from goal by expanding a “wavefront”

# Wavefront planner (distance transform)

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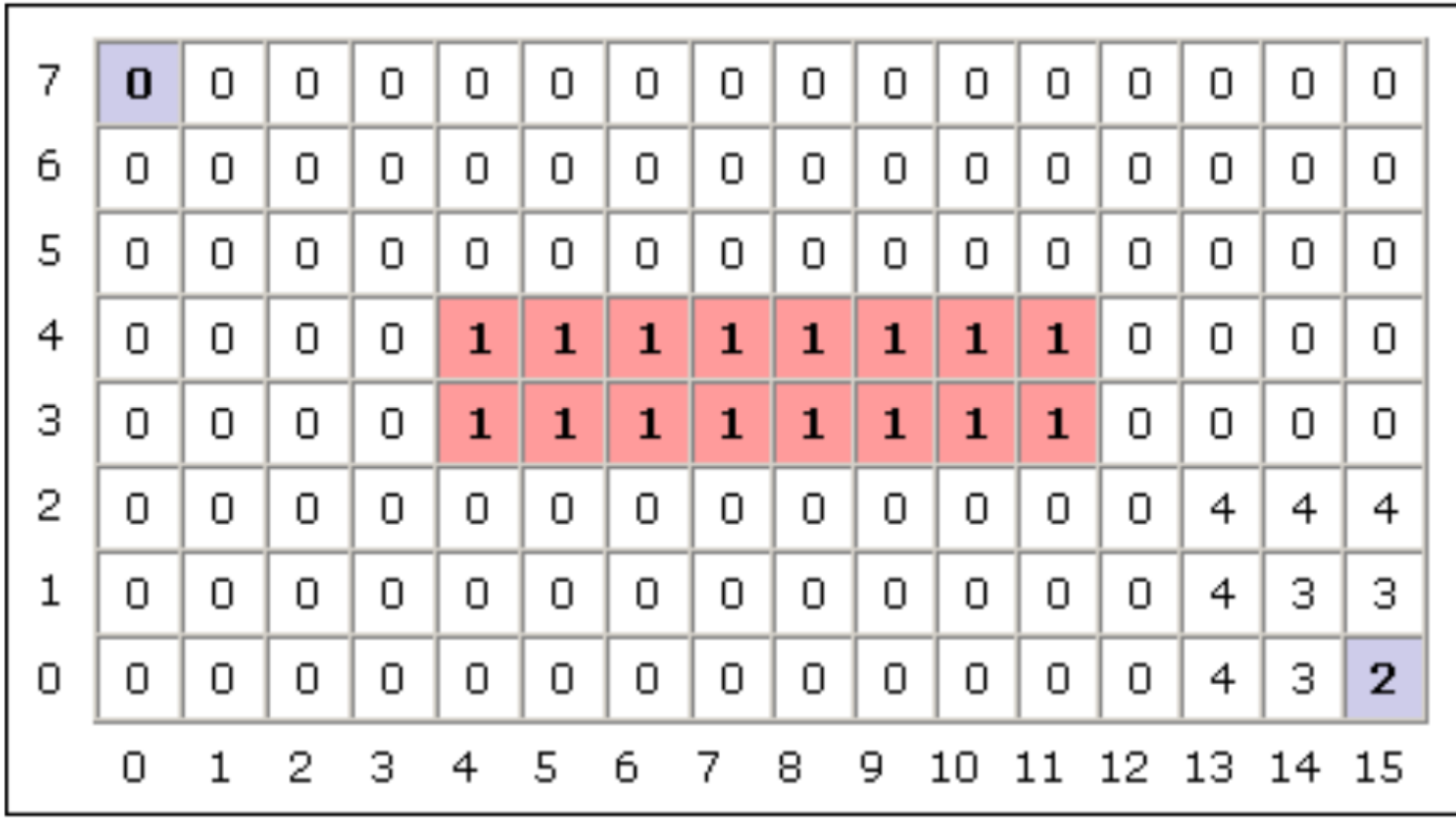


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# Wavefront planner (distance transform)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 3 | **2** |

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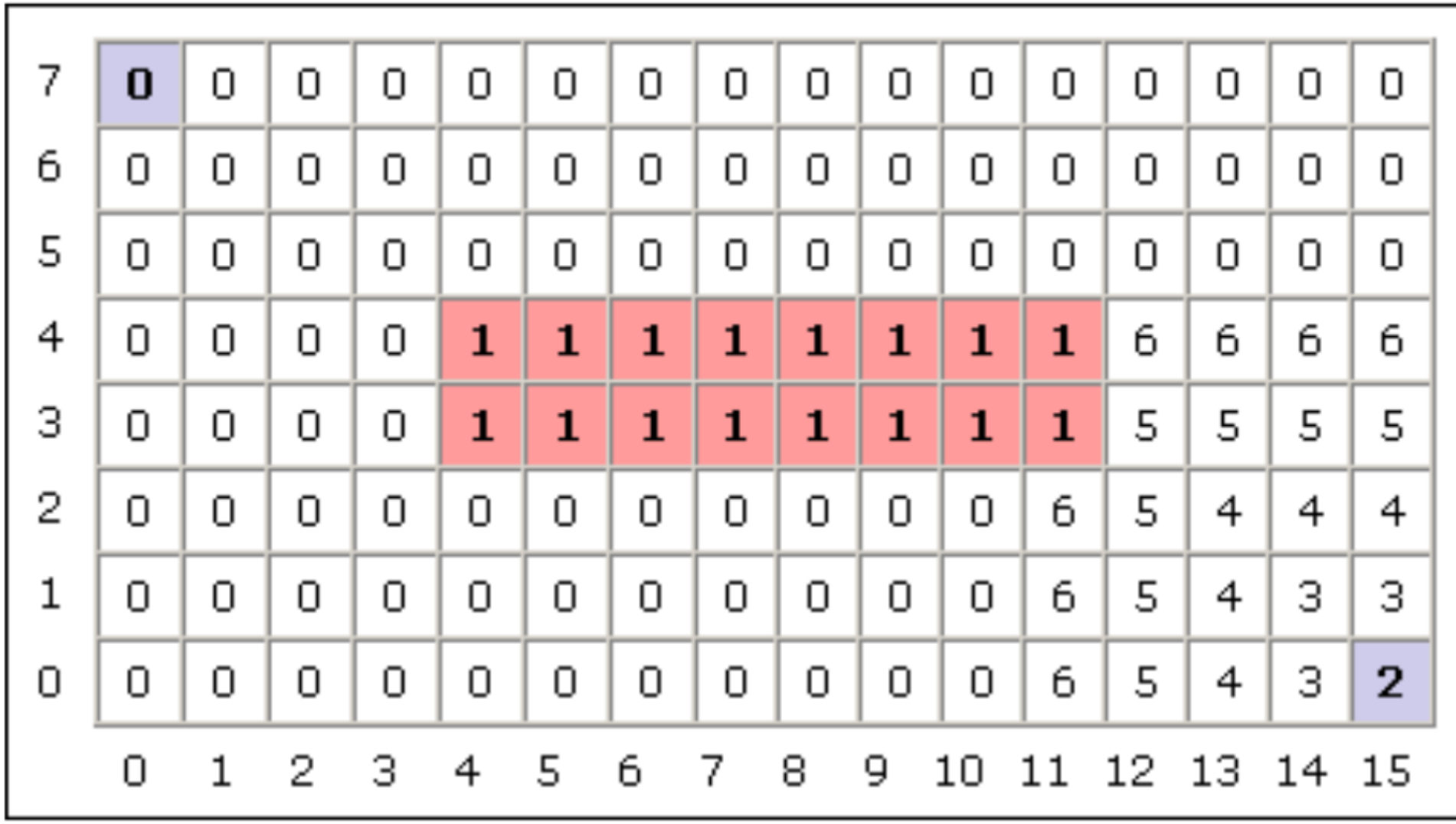
# Wavefront planner (distance transform)

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# Wavefront planner (distance transform)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 6 | 6 | 6 | 6 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 3 | **2** |

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# Wavefront planner (distance transform)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **18** | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 6 | 17 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 8 | 8 | 8 | 8 | 8 |
| 5 | 17 | 16 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 |
| 4 | 17 | 16 | 15 | 15 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 6 | 6 | 6 | 6 |
| 3 | 17 | 16 | 15 | 14 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 4 |
| 1 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 |
| 0 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | **2** |

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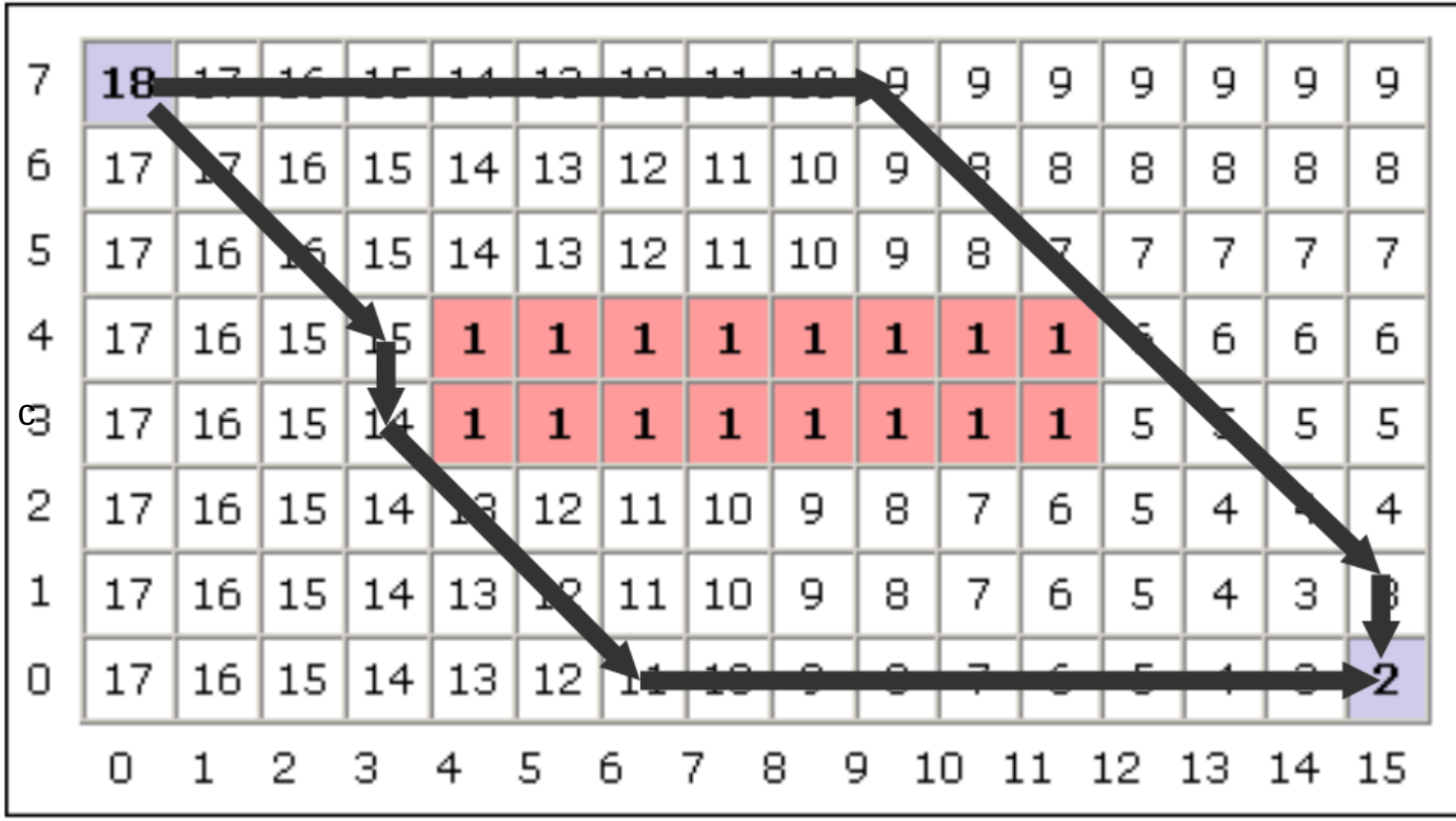


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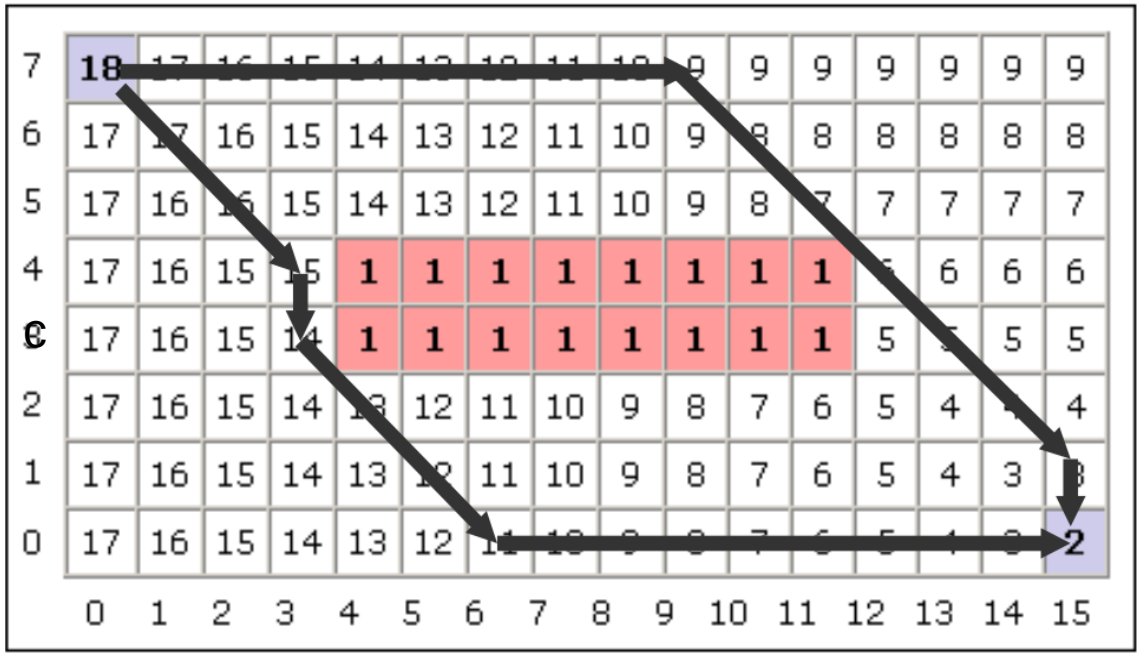
# Wavefront planner (distance transform)

Algorithm:

1. L={goal state}, d(goal state) = 2, d(obstacle states) = 1, d(rest of states) = 0
2. while L != 0
3. pop item i from L
4. for all neighbors j of i such that d(j)==0
5. d(j) = d(i)+1




L: list of nodes in wave front; initially just the goal state
d: distance function over nodes; initially zero everywhere except goal state

# Wavefront planner (distance transform)

Pros:
– complete
– optimal

Cons:
?