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Networks of Elastic Circuits

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with

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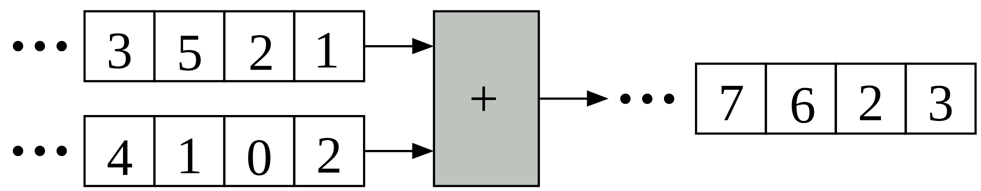
Latency Insensitive Design

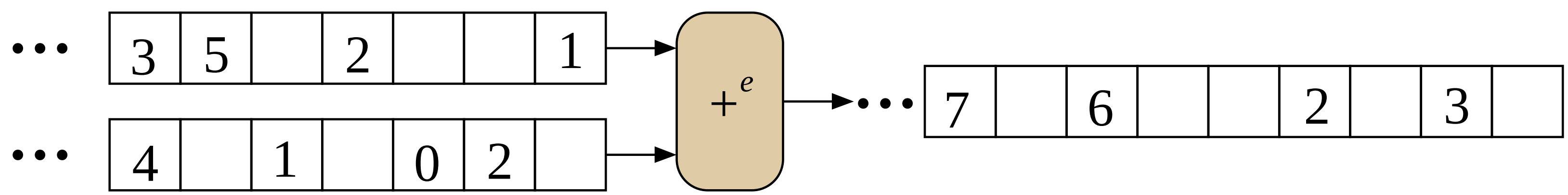
- Challenge in nanoscale technology: Implement a given functionality in a way that tolerates the latency changes of components and wires connecting them.
- Pioneering work: Carloni, McMillan, Sangiovanni-Vincentelli (CAV 1999)
- Intel project SELF (Synchronous Elastic Flow): Kishinevsky, Cortadella, Grundmann (TAU2005, DAC 2006)
- This presentation: Theoretical foundation for SELF

Elastic Circuits

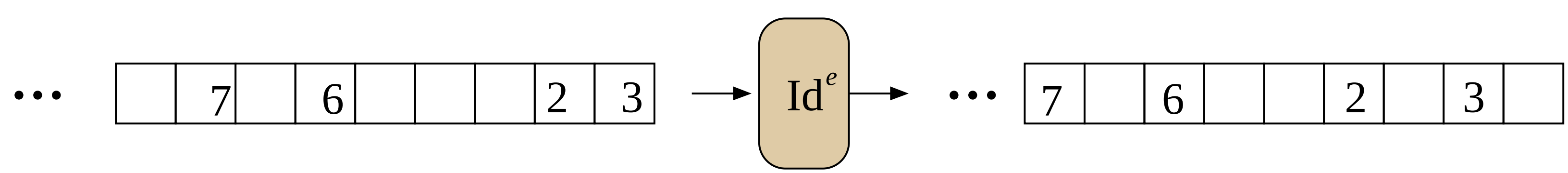
- Ordinary (non-elastic) adder



- Elastic adder

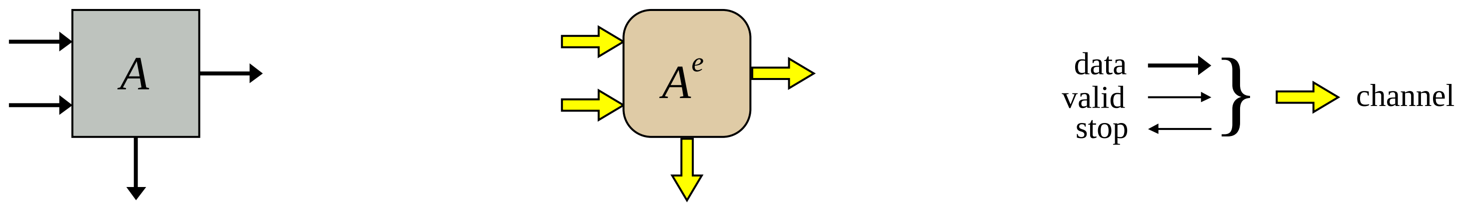


- Elasticization of a wire: Var. Latency Empty Elastic Buffer

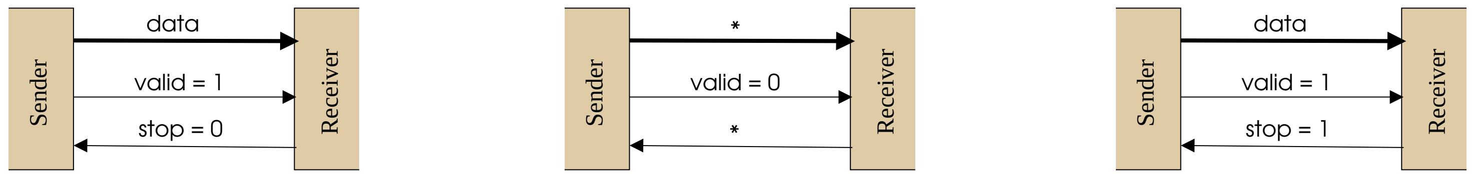


SELF Approach to Elasticization

- Wires of A become channels—triples of wires—in A^e.
  - X vs. $\langle X, \text{valid}_X, \text{stop}_X \rangle$



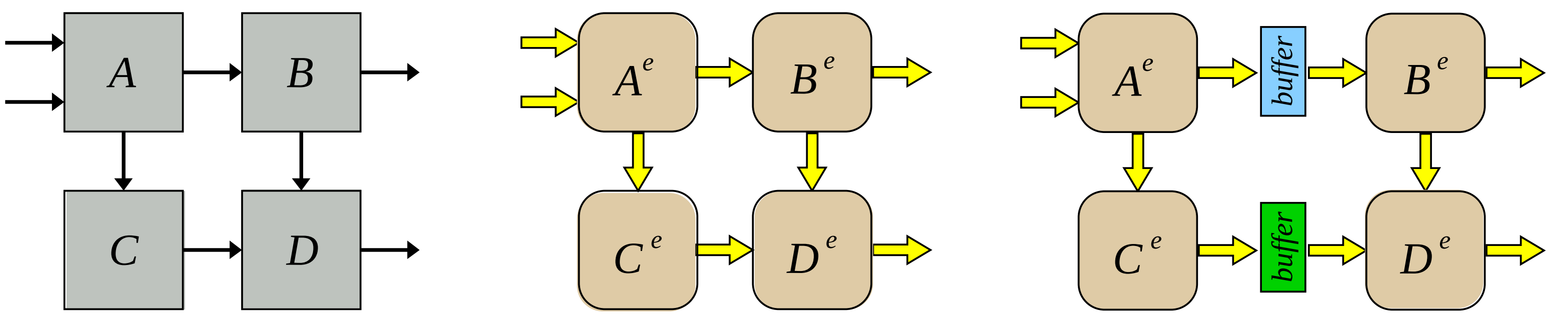
- States of SELF channels:



transfer *idle* *retry*

Questions

- Given a circuit A, how to construct its elasticization(s) A^e?
  - SELF does it
- If N is an ordinary network and we elasticize its components and connect channels accordingly, will we get an elasticization of N?
- If we insert an empty elastic buffer into a channel of an elastic network, will the resulting network be "equivalent" to the given one?



More Basic Questions

- What is precisely the "equivalence" of an ordinary and an elastic circuit?
- What is an elastic circuit?
- What is a circuit?

Ordinary Circuits and Networks

Systems

- Set of wires W
  - Example: for the system Adder, $W = \{\mathsf{in1}, \mathsf{in2}, \mathsf{out}\}$
- Set of W-behaviors $[\![W]\!]$: W-indexed records of streams
  - Example: $\sigma = \langle \sigma.\mathsf{in1}, \sigma.\mathsf{in2}, \sigma.\mathsf{out} \rangle$

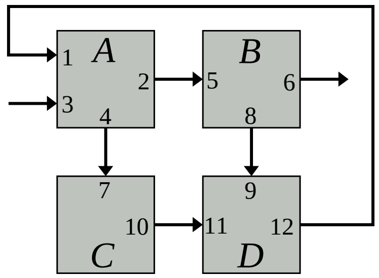
    $\sigma.\mathsf{in1} = \langle 2, 2, 2, \ldots \rangle \quad \sigma.\mathsf{in2} = \langle 1, 2, 3, \ldots \rangle \quad \sigma.\mathsf{out} = \langle 3, 4, 5, \ldots \rangle$
- A W-system is a set of W-behaviors
  - Example: $\mathsf{Adder} = \{\sigma \mid \sigma.\mathsf{out} = \sigma.\mathsf{in1} \oplus \sigma.\mathsf{in2}\}$

    $\langle 3, 4, 5, \ldots \rangle = \langle 2, 2, 2, \ldots \rangle \oplus \langle 1, 2, 3, \ldots \rangle$
  - Example: $\mathsf{Conn} = \{\sigma \mid \sigma.\mathsf{out} = \sigma.\mathsf{in}\}$

System Operations: Hiding, Composition, Networks

- $\mathsf{hide}_V(\mathcal{S}) = \{\sigma_{W-V} \mid \sigma \in \mathcal{S}\} \subseteq [\![W - V]\!]$
- $\mathcal{S}_1 \sqcup \mathcal{S}_2 = \{\sigma \mid \sigma_{W_1} \in \mathcal{S}_1 \wedge \sigma_{W_2} \in \mathcal{S}_2\} \subseteq [\![W_1 \cup W_2]\!]$
- Networks of systems:

$$\langle \mathcal{S}_1, \ldots, \mathcal{S}_m \,|\, u_1 = v_2, \ldots, u_n = v_n \rangle \;=$$
$$\mathsf{hide}_{\{u_1,\ldots,u_n,v_1,\ldots,v_n\}}(\mathcal{S}_1 \sqcup \cdots \sqcup \mathcal{S}_m \sqcup \mathsf{Conn}(u_1, v_1) \sqcup \cdots \sqcup \mathsf{Conn}(u_n, v_n))$$



$$= \langle A, B, C, D \mid 2 = 5, 4 = 7, 10 = 11, 8 = 9, 12 = 1 \rangle$$

Measuring Distance Between Streams (Behaviors)

- **Definition** $a \sim_n b$ iff $\text{prefix}(n, a) = \text{prefix}(n, b)$
- **Definition** $\sigma \sim_n \tau$ iff $(\forall w \in W)\ \sigma.w \sim_n \tau.w$
- Example:

$$\sigma.\text{in1} = \langle 2, 2, 2, \ldots \rangle \qquad \sigma.\text{in2} = \langle 1, 2, 3, \ldots \rangle \qquad \sigma.\text{out} = \langle 3, 4, 5, \ldots \rangle$$

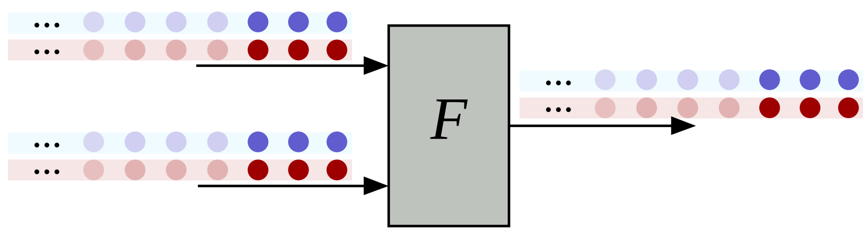
$$\tau.\text{in1} = \langle 2, 2, 2, \ldots \rangle \qquad \tau.\text{in2} = \langle 1, 2, 5, \ldots \rangle \qquad \tau.\text{out} = \langle 3, 4, 7, \ldots \rangle$$

$$\therefore\ \sigma \sim_2 \tau \qquad\qquad \therefore\ \sigma \not\sim_3 \tau$$

Machines (Circuits Abstractly)

Definition An (I, O)-machine is an $(I \cup O)$-system given by a function $F\colon [\![I]\!] \to [\![O]\!]$ satisfying the causality property

$$(\forall \sigma, \sigma' \in [\![I]\!])(\forall k \geq 0) \quad \sigma \sim_k \sigma' \implies F(\sigma) \sim_k F(\sigma')$$

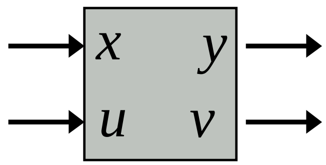


Outputs at the first k cycles are determined by inputs at the first k cycles.

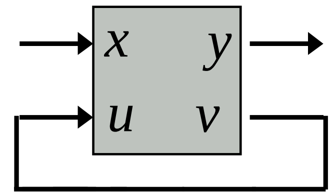
Modeling Combinational vs. Sequential Dependency

- Feedback: When is it a machine?

$\mathcal{S}$:

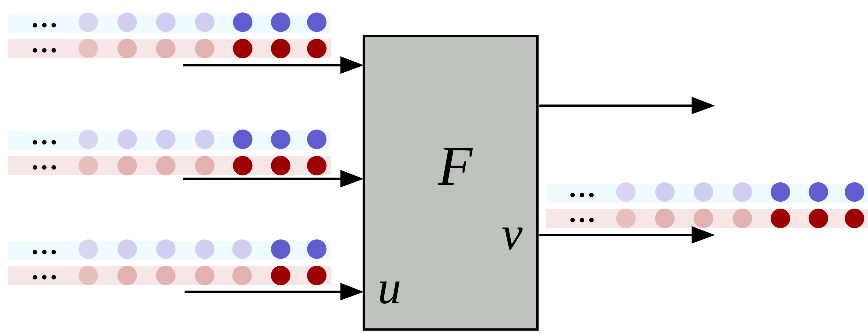


$\langle \mathcal{S} \mid v = w \rangle$:

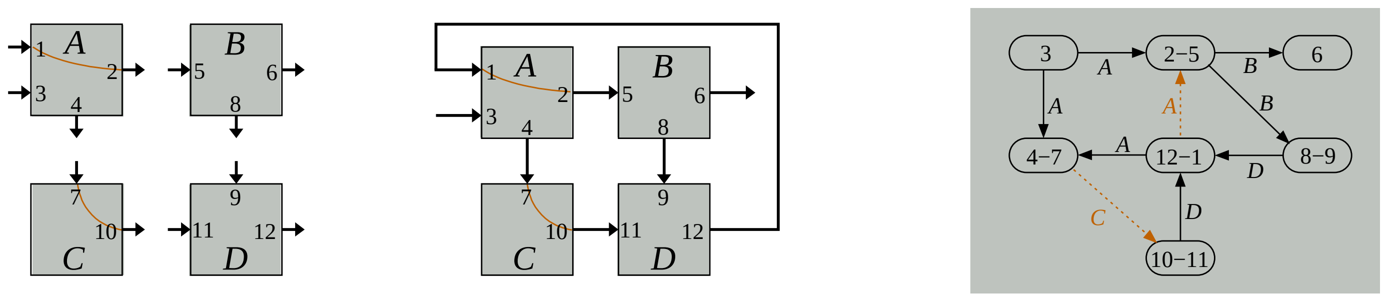


Definition An input-output pair (u, v) is sequential if

$$\left(\begin{array}{c} \forall \sigma, \sigma' \in [\![I]\!] \\ \forall k \geq 0 \end{array} \right) \quad \begin{array}{c} \sigma.u \sim_{k-1} \sigma'.u \\ \wedge \\ (\forall x \neq u)\ \sigma.x \sim_k \sigma'.x \end{array} \implies F(\sigma).v \sim_k F(\sigma').v$$



Combinational Loop Theorem



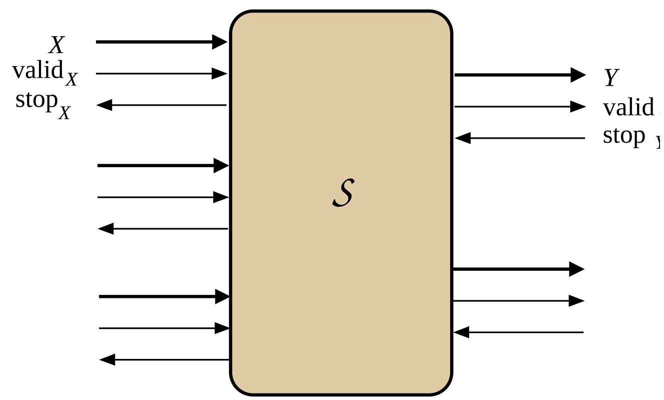
Definition $\Gamma(\mathcal{N})$: Vertices are wires of $\mathcal{N}$; directed edges drawn for non-sequential wire pairs.

Theorem If $\Gamma(\mathcal{N})$ is acyclic, then $\mathcal{N}$ is a machine.

Elastic Circuits and Networks

$[I, O]$-Elastic Machine

- Input-output structure
  - inputs: $I \cup \{\mathsf{valid}_X \mid X \in I\} \cup \{\mathsf{stop}_Y \mid Y \in O\}$
  - outputs: $O \cup \{\mathsf{valid}_Y \mid Y \in O\} \cup \{\mathsf{stop}_X \mid X \in I\}$



- Persistence
  - $\mathcal{S} \models \mathsf{G}\,(\mathsf{valid}_Y \wedge \mathsf{stop}_Y \Rightarrow (\mathsf{valid}_Y)^+)$ for every $Y \in O$

$[I, O]$-Elastic Machine (ctd)

- Transfer and token count

cycle	0	1	2	3	4	5	6	7	8	9	...
X	$*$	A	B	B	B	C	$*$	$*$	D	D	...
valid_X	0	1	1	1	1	1	0	0	1	1	...
stop_X	0	0	1	1	0	0	0	1	1	0	...
tct_X	0	1	1	1	2	3	3	3	3	4	...

- Transfer behavior ω^{T} (data from transfer cycles)
- $\omega^{\mathsf{T}}.X = (A, B, C, D, \ldots)$
- Components $\omega^{\mathsf{T}}.X$ of ω^{T} are perhaps finite sequences

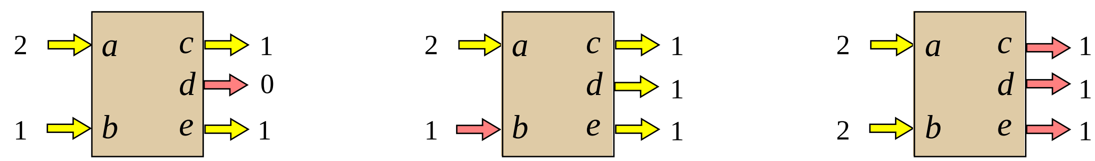
$[I, O]$-Elastic Machine (ctd)

- Liveness

$(\forall Y \in O)$ $\mathcal{S} \models \mathsf{G}\,(\mathsf{min\_tct}_O \geq \mathsf{tct}_Y \wedge \mathsf{min\_tct}_I > \mathsf{tct}_Y \Rightarrow \mathsf{F}\,\mathsf{valid}_Y)$

$(\forall X \in I)$ $\mathcal{S} \models \mathsf{G}\,(\mathsf{min\_tct}_{I \cup O} \geq \mathsf{tct}_X \Rightarrow \mathsf{F}\,\neg\mathsf{stop}_X)$

- Serve only the hungriest channels:



- Liveness guarantees that all transfer behaviors $\omega^{\mathsf{T}}.Z$ are infinite (in an "elastic environment")

$\therefore$ The transfer system $\mathcal{S}^{\mathsf{T}}$ $= \{\omega^{\mathsf{T}} \mid \omega \in \mathcal{S} \sqcup \mathsf{Env}_{I,O}\}$

$[I, O]$-Elastic Machine (ctd)

- Determinism

$$(\forall \omega_1, \omega_2 \in \mathcal{S}) \quad \omega_1^{\mathsf{T}}.I = \omega_2^{\mathsf{T}}.I \quad \Rightarrow \quad \omega_1^{\mathsf{T}}.O = \omega_2^{\mathsf{T}}.O$$

Definition $\mathcal{S}$ is an $[I, O]$-elastic machine if it has the input-output structure as described, and satisfies the persistence, liveness, and determinism conditions.

Theorem If $\mathcal{S}$ is an $[I, O]$-elastic machine, then $\mathcal{S}^{\mathsf{T}}$ is an (I, O)-machine.

- $\mathcal{S}$ is an elasticization of $\mathcal{M}$ when $\mathcal{M} = \mathcal{S}^{\mathsf{T}}$

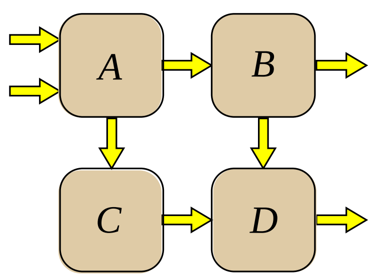
Elastic Networks

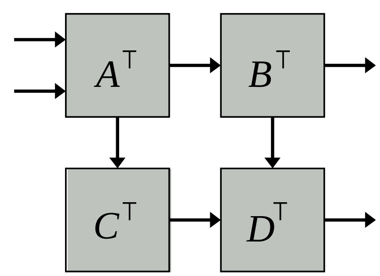
Suppose $\mathcal{S}_1, \ldots, \mathcal{S}_m$ are elastic machines.

$$\mathcal{N} = \langle\!\langle \mathcal{S}_1, \ldots, \mathcal{S}_m \,[\!]\, X_1 = Y_1, \ldots, X_n = Y_n \rangle\!\rangle$$

$$\triangleq$$

$$\langle \mathcal{S}_1, \ldots, \mathcal{S}_m \mid X_i = Y_i, \mathsf{valid}_{X_i} = \mathsf{valid}_{Y_i}, \mathsf{stop}_{X_i} = \mathsf{stop}_{Y_i} \ (1 \leq i \leq n) \rangle$$

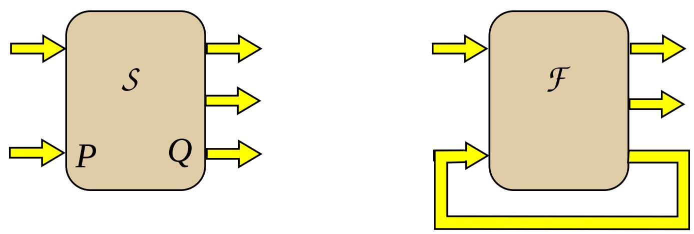




- Is $\mathcal{N}$ an elastic machine?
- Do we have $\mathcal{N}^\top = \langle \mathcal{S}_1^\top, \ldots, \mathcal{S}_m^\top \mid X_1 = Y_1, \ldots, X_n = Y_n \rangle$?

Elastic Feedback

$$\mathcal{F} = \langle\!\langle \mathcal{S} \,[\!]\, P = Q \rangle\!\rangle \;=\; \langle \mathcal{S} \mid P = Q, \mathsf{valid}_P = \mathsf{valid}_Q, \mathsf{stop}_P = \mathsf{stop}_Q \rangle$$



Definition An i/o channel pair (P, Q) sequential for $\mathcal{S}$ if

$$\mathcal{S} \models \mathsf{G}\,(\mathsf{min\_tct}_{I \cup O} \geq \mathsf{tct}_Q \wedge \mathsf{min\_tct}_{I - \{P\}} > \mathsf{tct}_Q \Rightarrow \mathsf{F}\,\mathsf{valid}_Q)$$

and the graph $\Gamma(\mathcal{F})$ is acyclic.

Elastic Network Theorem

- $\mathcal{N} = \langle\!\langle \mathcal{S}_1, \ldots, \mathcal{S}_m \,[\!]\, X_1 = Y_1, \ldots, X_n = Y_n \rangle\!\rangle$
- δ_i : a sequentiality interface for $\mathcal{S}_i$

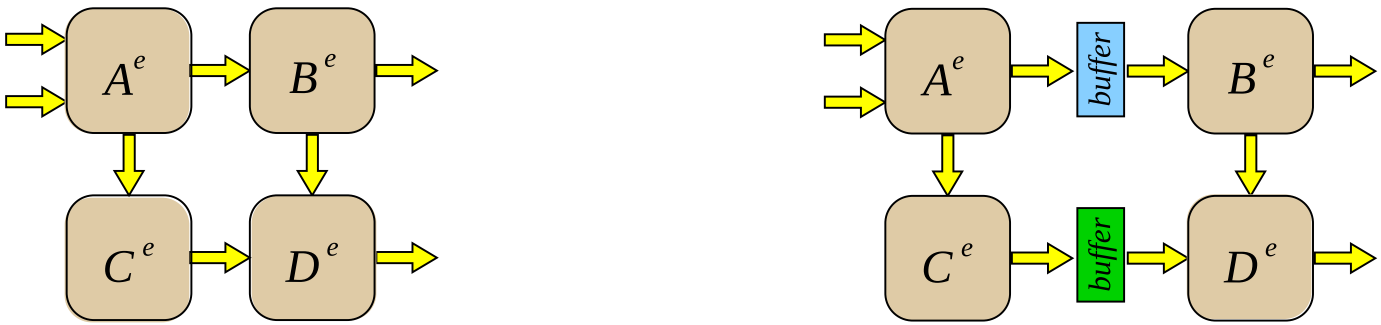
  $\delta_i(Z)$ = set of input wires "jointly sequential" wrt Z

Definition $\Delta(\mathcal{N})$: Vertices are channels of $\mathcal{N}$ ($\because$ X_j and Y_j are identified); a directed edge drawn for each pair $(P, Q) \in I_i \times O_i$ such that $P \notin \delta_i(Q)$.

- $\mathcal{N}' = \langle \mathcal{S}_1^{\mathsf{T}}, \ldots, \mathcal{S}_m^{\mathsf{T}} \,|\, X_1 = Y_1, \ldots, X_n = Y_n \rangle$

Theorem If $\Delta(\mathcal{N})$ is acyclic, then $\mathcal{N}$ is an elastic machine, $\mathcal{N}'$ is a machine, and $\mathcal{N}^{\mathsf{T}} = \mathcal{N}'$.

Inserting Empty Buffers



Theorem Suppose $\mathcal{N}_1$ and $\mathcal{N}_2$ are elastic networks obtainable from each other by insertion and deletion of empty elastic buffers. If $\Delta(\mathcal{N}_1)$ is acyclic, then

- $\Delta(\mathcal{N}_2)$ is acyclic
- $\mathcal{N}_1^{\mathsf{T}} = \mathcal{N}_2^{\mathsf{T}}$

What's Coming Next?

- Prove that SELF creates elastic circuits
- Weaken the definition of elasticity to include all existing "elastic" designs
- Extend theory to more complex SELF protocols

Background: Patient Systems
(Carloni, McMillan, Sangiovanni-Vincentelli)

- Behavior: for each wire, a stream in which each element is either a value or $\square$ ("bubble")
- Example:

elastic	X	$*$	A	B	B	B	C	$*$	$*$	D	D	$\ldots$
	valid_X	0	1	1	1	1	1	0	0	1	1	$\ldots$
	stop_X	0	0	1	1	0	0	0	1	1	0	$\ldots$
patient		$\square$	A	$\square$	$\square$	B	C	$\square$	$\square$	$\square$	D	$\ldots$

- Precise definition when a collection of such behaviors is a patient process
- Compositionality Theorem for patient processes; costruction of a patient process latency equivalent to a given circuit
- "Elastic" and "patient" are difficult to compare