CS 7805: Theory of Computation Homework 2

Problem 1: In class, I made the claim that not every non-regular language can be shown to be non-regular using the pumping lemma.

Consider the language $L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

(a) Show that the pumping lemma cannot be used to prove that L is not regular.

(b) Read about the Myhill-Nerode theorem, which is exercise 1.52 of your book. Note that the solution to the exercise appears at the end of the chapter. Use the Myhill-Nerode theorem to prove that L is not regular.

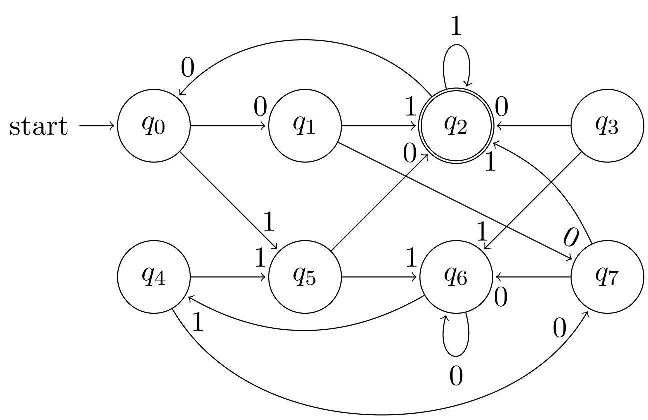
Problem 2: Prove that the languages recognized by NFAs are closed under complement.

Problem 3: The Myhill-Nerode theorem implies that for any regular language L, any DFA recognizing L has to have at least i states, where i is the index of L. In fact, there is a DFA of size i that accepts L. This is a minimal DFA recognizing L.

(a) Propose an algorithm for DFA minimization.

(b) Prove that your algorithm is correct.

(c) Use your algorithm to minimize the following DFA.



Problem 4: In class we studied finite-state automata that operate on strings of finite length. How about finite-state automata that operate on strings of infinite length? Define an *Infinite Input Finite Automaton* (IIFA) to be a tuple (Q, Σ, T, Q_0, F) where:

- Q is a finite set of states.
- Σ is the alphabet.
- $\delta : \Sigma \times Q \to \mathcal{P}(Q)$ is the transition function.

- $Q_0 \subseteq Q$ is a set of initial states.
- $F \subseteq Q$ is a set of accepting states.

Given an infinite string $s = s_0 s_1 \ldots$ over Σ, a *run* r of IIFA A on s is an infinite sequence of states $r = r_0, r_1, \ldots$ where $r_0 \in Q_0$ and $r_{i+1} \in \delta(r_i, s_i)$ for all $i \geq 0$. There is no final state, so we need a different notion of acceptance than we had with NFAs. Let $lim(r) = \{q : q = r_i$ for infinitely many i's $\}$. That is, $lim(r)$ is the set of states that appear infinitely often in run r. Run r is *accepting* if $lim(r) \cap F \neq \emptyset$, *i.e.*, some accepting state is visited infinitely often. Automaton A accepts string s if there is an accepting run r of A on s. The language of A, denoted $L(A)$, is the set of infinite strings accepted by A.

(a) Show that if A and B are IIFAs, then there is an IIFA C such that $L(C) = L(A) \cup L(B)$.

(b) Show that if A and B are IIFAs, then there is an IIFA C such that $L(C) = L(A) \cap L(B)$.

(c) Are nondeterministic IIFAs more expressive than deterministic IIFAs? Provide a proof.