CS 2800 Section 1 – Exam 1 – Spring 2012

Name: _____

Student Id (last 4 digits): _____

- Write down the answers in the space provided.
- Please write clearly. If we can't read what you write, we can't give you credit for it.
- You may use anything we covered in class or in the class notes. Everything else needs to be defined. If you have any questions, please ask!
- For programming questions, you are to define ACL2s functions. Use the design recipe described in class. Your grade will depend on it. In particular, write a purpose statement, use `check=` to define tests, use contracts, make sure the function terminates, etc.

Question	Points	out of
1		120
2		240
3		240
4		288
5		312
Total		1200

Good luck!

Question 1. (5 * 24 = 120 points)

Clearly mark the following statements as true or false. Please write "true" or "false."

1. Both `if` and `equal` are *lazy* functions.

2. There are as many satisfiable Boolean functions of n arguments as there are falsifiable ones.

3. Any recursively defined function `f(n)` whose body calls `f` only on argument `(- n 1)` is guaranteed to terminate.

4. For every element `e` of the ACL2 universe, there exist infinitely many expressions that evaluate to `e`.

5. A formula f is satisfiable exactly if $\neg f$ is not falsifiable.

Question 2. (240 points)

Define an ACL2 function `zip` that takes two lists `l1` and `l2` as inputs and *zips* them. That is, the result is a list of pairs (a pair is a list of length 2). The nth pair consists of the nth element of both lists. The length of `l1` is at least the length of `l2`. If `l1` is longer than `l2`, the result should contain the number 0 in place of the missing element.

Write an input contract that ensures that `zip` accepts exactly the input specified above (contract violation for anything else).

What can you say about the length of the result list in relation to the length of the input lists? Formalize this and the type of the output in the output contract.

Define at least 3 additional tests.

```
(check= (zip '(a b c) '(1 2 3)) '((a 1) (b 2) (c 3)))
(check= (zip '(a c d) '(1))     '((a 1) (c 0) (d 0)))
```

Question 3. (120 + 120 = 240 points)

(a) Someone suggests to introduce a new Boolean connective, $\dashv$, such that $a \dashv b$ abbreviates the expression $\neg(\neg a \wedge b)$. Formally define this connective, using a truth table. Do we really need this new connective, or can you express $a \dashv b$ using a *single* connective that we already know?

(b) Prove that the set $\{\neg, \dashv\}$ of connectives is a complete Boolean base.
Hint: Recall that $\{\neg, \vee\}$ is a complete Boolean base.

Question 4. (3 * 96 = 288 points)

For each of the following Boolean formulas, decide whether they are satisfiable, unsatisfiable, valid, or falsifiable. Among those four properties, list *all* that the formula satisfies.

1. $(\neg a \vee b) \Rightarrow ((c \Rightarrow \neg b) \vee a)$

2. $(b \oplus \neg c) \vee \neg(b \equiv c)$

3. $a \wedge (b \Rightarrow (\neg(a \vee c))) \wedge b$

Question 5. (72 + 120 + 120 = 312 points)

Formalize and analyze the following statements using propositional logic. Clearly explain whether the argument is valid, *i.e.*, whether the stated conclusions logically follow from the premises.

1. If I don't get a good grade, it means I didn't work hard. But I did work hard. Therefore, I do get a good grade.

2. The Tokyo stock exchange crashes if and only if the New York stock exchange crashes. When the Tokyo stock exchange crashes, the Shanghai stock exchange crashes. The New York stock exchange crashed. Therefore, the Shanghai stock exchange crashed.

3. Atoms are not the smallest particles in the universe if and only if Bill's theory is wrong. If atoms are the smallest particles in the universe, we can build a quantum computer. Bill's theory works! Therefore, we can build a quantum computer.